



2020

A Thesis Submitted To  
The College of Science and Engineering in Partial Fulfilment of The  
Requirements for The Degree of Master of Engineering Science  
(Mechanical) at Flinders University, Adelaide, Australia

**Submitted By**

Kushagra Mandwal

2197966 | mando102

**Supervisor**

Dr Amir Zanj

**Word Count**

*15700*

***Topic- ENGR\_9700 Masters Thesis***

Research Title

**Develop a model for the Hydraulic Active Suspension  
system to investigate the performance of the system**

**(Quarter Car Method)**

By

© Kushagra Mandwal

## **Declaration of Academic Integrity**

I certify that this thesis does not incorporate without acknowledgment any material previously submitted for a degree or diploma in any university and that to the best of my knowledge and belief it does not contain any material previously published or written by another person except where due reference is made in the text.

**Name: Kushagra Mandwal**

**Student Id: 2197966**

## *Abstracts*

The continuous technical progressions are necessary for every field to provide better quality and comfort to consumers. In this way, automotive industries are playing an important role. Where continuous inventions could be led by quality ride, vehicle safety improvements and provide luxury to customers. Also, the vibration is generated from the uneven condition of the road can cause injuries like fatigue, mental stress and back pain. To overcome these vertical forces and unevenness, the excellent suspension system is a necessary part of all vehicles.

The investigation of vehicle performance regarding vehicle suspension and increase safety of the passengers, many lumped parameter models of the suspension system such as the quarter car, half car and full car model are in use. But this research is based on the quarter car model (Single tyre suspension). Also, there are many types of the suspension system are using in the vehicles: passive, semi-active and fully active suspension. But, get topmost results, active suspension system would be the best.

The development of the full hydraulic active suspension system and its different part is the primary objective of this research. The developed model is based on the bond graph model technique. State-space equations are derived from the modelled bond graph. These equations are simulated with the help of MATLAB/Simulink software environment. For hydraulic unit following components: the reservoir, speed-controlled motor, positive displacement pump, accumulator, bypass or relief valve, four-way spool valve and hydraulic piston-cylinder arrangement has been modelled from the bond graph technique. For the suspension unit, the quarter car model of the suspension system has been used and modelled with the help of bond graph technique.

In respect to the chapter of this thesis, introduction and background of the suspension is outlined in chapter one. The reviewed of the previous relevant researches are written in the second chapter. While the various suspension system and theory behind it are in chapter three. Bond graph modelling technique and its essential elements are described in chapter four. The fifth chapter is about the mathematical representation of the passive and active suspension system (on quarter car model). Also, the initial behaviour of these systems is in this chapter. The whole system model, mathematical equations and brief theory of different components are mentioned in chapter six. The derived state-space equations from the system bond graph have used to design the MATLAB/Simulink model has been described in chapter seven. The results and discussion of the simulated results are explained in chapter eight. The concluding remark has been formed, and possible future scopes are suggested in chapter nine.

The results show that the hydraulic active suspension system could perform better and minimise the vertical acceleration and velocity better than the passive and semi-active suspension. But there are some minor errors in results that could be fixed with the use of the proper controller technique. Also, this work only for the left moving position of the direction control valve which could also be analysed for right moving position and controller could be used to get proper direction control in future works.

Fundamentally, this research work provides the physics model platform that could help to control engineers to design proper control algorithm to get the desired output from this system.

## **Acknowledgements**

Inspiration for presentation and motivation have always played a key role in the success of any endeavour.

I would like to express my sincere appreciations to my supervisor Dr Amit Zanj, for his practical advice and unceasing ideas that always helped me extremely in my research and for his constant support to prepare the project.

I want to show my deep sense of gratitude to Dr Stuart Wildy to encourage me for my hard work, for my project and support me with his deep knowledge. Also, I would like to show my gratitude to Julie Zanker (Student Administration Services Assistant) for her constant support.

I am immensely obliged to my friend Yesh Patel and Chintal Patel in developing the project and help me with their abilities.

I also wish to express my deepest thanks to my parents and my parents in law who support me morally and inspired me to face all the challenges and win all the hurdles in life. They always prayed for me throughout the time of research.

I am also thankful to my beloved fiancée who supports me every day and motivates me to do every task on time. Always there for me caring, cheering me up and stood by me through the peaks and valleys of my life.

Last but not the least, I am extremely grateful to God for the abundant blessings and unfailing love for me.

**This Thesis**

**Dedicate to My Beloved MOTHER**

## Table of Contents

<i>Abstracts</i> .....	iii
<i>Acknowledgements</i> .....	v
<i>List of Figure</i> .....	x
<i>List of Table</i> .....	xiii
<i>Nomenclatures</i> .....	xiv
<b>Chapter 1- Introduction</b> .....	1
1.1 Background .....	2
1.2 Problem Statement .....	2
1.3 Initial Scope of Research.....	3
1.4 Final Scope of Research.....	3
1.5 Reasons Behind to Change the Scope .....	3
1.6 Thesis Outline .....	4
<b>Chapter 2- Literature Review</b> .....	6
2.1 Introduction .....	7
2.2 Suspension Modelling System .....	7
2.1.1 Two Degree of Freedom Model (Quarter Car) .....	7
2.1.2 Multibody System Modelling .....	11
2.1.3 Kinematics Modelling.....	13
2.1.4 Polynomial Modelling .....	14
2.1.5 Physics-based Modelling (Bond Graph) .....	15
2.3 Gap Statement .....	17
2.4 Objectives.....	17
<b>Chapter 3- Suspension System Overview</b> .....	18
3.1 Introduction .....	19
3.2 Passive Suspension System.....	19
3.3 Semi-Active Suspension System.....	20

3.4	Active Suspension System .....	21
<b>Chapter 4- Bond Graph Background .....</b>		<b>24</b>
4.1	Introduction .....	25
4.2	Power Variables .....	26
4.3	Bond Graph Elements .....	27
4.3.1	Passive Elements (One port).....	27
4.3.2	Active Elements (Source of Effort and Source of Flow).....	32
4.3.3	Junctions .....	33
4.3.4	Two Port Elements.....	34
4.4	Causality.....	36
4.4.1	Sequence to Assigned the Causality .....	37
4.5	State Space Equations .....	38
4.5.1	State Variable Properties.....	38
4.5.2	Steps to Derive Equations.....	39
<b>Chapter 5- Mathematical Model of Suspension System .....</b>		<b>40</b>
5.1	Introduction .....	41
5.2	Passive Suspension Model .....	41
5.3	Active Suspension Model .....	46
<b>Chapter 6- System Mathematical Model and Bond Graph .....</b>		<b>51</b>
6.1	Introduction .....	52
6.2	Bond graph of the Different Parts .....	53
6.2.1	Positive Displacement Pump (PDP) .....	53
6.2.2	Accumulator and Bypass Valve.....	54
6.2.3	Spool Valve (Direction Control Valve) .....	54
6.2.4	Hydraulic Cylinder.....	55
6.2.5	Suspension System (1/4 Model) .....	57
6.3	Bond Graph of the Whole System.....	58

6.4	State Space Equations of the System .....	58
6.4.1	The Element Gives to the System.....	59
6.4.2	Taken from the System .....	61
6.5	Hydraulic Equations .....	62
6.6	Assumptions .....	62
6.7	Used Parameters .....	63
<b>Chapter 7- MATLAB and Simulink Model.....</b>		<b>65</b>
7.1	Introduction .....	66
7.1.1	Step Source .....	66
7.1.2	Integrator Block .....	67
7.1.3	Gain Block .....	67
7.1.4	Scope Block .....	68
<b>Chapter 8- Result and Discussion .....</b>		<b>69</b>
8.1	Results and Discussions .....	70
8.1.1	For Hydraulic Part.....	70
8.1.2	For Suspension Part .....	80
<b>Chapter 9- Conclusion and Future Scope .....</b>		<b>85</b>
9.1	Conclusion.....	86
9.2	Future Scope.....	86
<b>Reference .....</b>		<b>88</b>
<b>Appendices.....</b>		<b>95</b>
<i>MATLAB Programming .....</i>		<i>95</i>
<i>Volume and Momentum due to Pressure.....</i>		<i>95</i>
<i>Displacement and Momentum.....</i>		<i>98</i>
<i>Risk Assessment.....</i>		<i>100</i>

## *List of Figure*

Figure 1- Quarter Car (Two Degrees of Freedom Model) [5] .....	8
Figure 2- Sprung Mass Displacement Comparison of Two, Four and Seven DOF Models in Pitch Mode [5] .....	8
Figure 3- Sprung Mass Acceleration Comparison of Two, Four and Seven DOF Models in Pitch Mode [5] .....	9
Figure 4- Sprung Mass Displacement Comparison of Two, Four and Seven DOF Models in Roll Mode [5].....	9
Figure 5- Sprung Mass Acceleration Comparison of Two, Four and Seven DOF Models in Roll Mode [5].....	9
Figure 6- McPherson Suspension Nonlinear Model and Its Kinematics Model [23].....	13
Figure 7- Active Suspension Physical and Bond Graph Model [33].....	15
Figure 8- Typical Suspension System [41].....	20
Figure 9- Typical Semi-Active Suspension System [41].....	21
Figure 10- A comparison of the response characteristics of passive and active suspension systems [43] .....	21
Figure 11- Typical Active Suspension System [44] .....	22
Figure 12- (a) Symbol, (b) Mechanical Damper (c) Hydraulic System .....	31
Figure 13- Representation of Zero and One Junction.....	33
Figure 14- Representation of Zero Junction for Mechanical and Hydraulic System [47].....	33
Figure 15- One Junction Representation of Mechanical and Hydraulic System [47] .....	34
Figure 16- TF element for Hydraulic and Mechanical System [47].....	35
Figure 17- Electric Motor and its Gyator Element [47] .....	36
Figure 18- Causality Assignment [47].....	36
Figure 19- Mathematical Representation of Passive Suspension System [50].....	41
Figure 20- Bond Graph of Passive Suspension System.....	42
Figure 21- Sprung Mass Acceleration .....	43
Figure 22- Sprung Mass Velocity .....	43
Figure 23- Sprung Mass Displacement.....	44
Figure 24- Unsprung Mass Acceleration .....	44
Figure 25- Unsprung Mass Velocity.....	45
Figure 26- Unsprung Mass Displacement.....	45
Figure 27- Spring Velocity .....	46

Figure 28- Mathematical Representation of Active Suspension System [50] .....	46
Figure 29- Bond Graph of Active Suspension System (Actuator Force $F_c$ ) .....	47
Figure 30- Sprung Mass Acceleration .....	47
Figure 31- Sprung Mass Velocity .....	48
Figure 32- Sprung Mass Displacement.....	48
Figure 33- Unsprung Mass Acceleration .....	49
Figure 34- Unsprung Mass Velocity.....	49
Figure 35- Unsprung Mass Displacement.....	50
Figure 36- Spring Velocity .....	50
Figure 37- Layout of the System .....	52
Figure 38- Positive Displacement Pump and Bond Graph .....	53
Figure 39- Gas Accumulator, Bypass Valve and Bond Graph .....	54
Figure 40- Spool Valve (Direction Control Valve) and Bond Graph [47] .....	55
Figure 41- Hydraulic Actuator and Bond Graph .....	56
Figure 42- Quarter Car Suspension System and Bond Graph .....	57
Figure 43- Full System Bond Graph.....	58
Figure 44- MATLAB/Simulink Model of Hydraulic Active Suspension System.....	67
Figure 45- Flow Rate Generated from the Pump.....	70
Figure 46- Bypass Valve Flow Rate .....	71
Figure 47- Accumulator Pressure .....	72
Figure 48- Accumulator Flow Rate .....	72
Figure 49- Pressure at Spool One .....	73
Figure 50- Flow Rate at Spool One .....	73
Figure 51- Pressure at Spool Two.....	74
Figure 52- Flow Rate at Spool Two.....	74
Figure 53- Return Pressure to Sump from Chamber A .....	75
Figure 54- Return Flow Rate to Sump from Chamber A.....	75
Figure 55- Return Pressure to Sump from Chamber B.....	76
Figure 56- Return Flow Rate to Sump from Chamber B.....	76
Figure 57- Pressure in Chamber A.....	77
Figure 58- Flow Rate in Chamber A.....	77
Figure 59- Pressure in Chamber B.....	78
Figure 60- Flow Rate in Chamber B.....	78
Figure 61- Flow Rate Due to Inside Leakage in the Cylinder .....	79

Figure 62- Sprung Mass Acceleration .....	80
Figure 63- Sprung Mass Velocity .....	80
Figure 64- Sprung Mass Displacement.....	81
Figure 65- Unsprung Mass Acceleration .....	82
Figure 66- Unsprung Mass Velocity .....	82
Figure 67- Unsprung Mass Displacement.....	83
Figure 68- Spring Velocity .....	84
Figure 69- Tyre Spring Velocity.....	84
Figure 70- Volume Inside Accumulator .....	95
Figure 71- Momentum Due to Pressure at Spool One.....	96
Figure 72- Momentum Due to Pressure at Spool Two .....	96
Figure 73- Volume in Chamber A .....	96
Figure 74- Volume in Chamber B .....	97
Figure 75- Momentum Due to Pressure Return to Sump From A.....	97
Figure 76- Momentum Due to Pressure Return to Sump From B .....	97
Figure 77- Displacement of the Spring .....	98
Figure 78- Momentum of Sprung Mass.....	98
Figure 79- Momentum of Unsprung Mass.....	99
Figure 80- Displacement of the Tire Spring .....	99

## *List of Table*

Table 1- Comparison of Suspensions [44].....	23
Table 2- Elements of Bond Graph [46].....	26
Table 3- Causality Assignment for Bond Graph Elements [47] .....	26
Table 4- Bond Graph Power Variable [47].....	27
Table 5- Power and Energy Variable [48] .....	27
Table 6- The inertia and SI Unit [48].....	29
Table 7- The Capacitance and SI Unit [48] .....	31
Table 8- The Resistance and SI Unit [48].....	32
Table 9- Active Elements [48].....	32
Table 10- Parameters .....	64

## *Nomenclatures*

$\omega_p$	Driving Speed of Pump Shaft
$V_p$	Displacement of the Pump
$\rho$	Density of Fluid
$C_d$	Coefficient of Discharge
$E$	Bulk Modulus of Fluid
$K$	Stiffness of Suspension Spring
$K_t$	Stiffness of the Tyre
$B$	Damping Rate
$M_s$	Car Body Mass (Sprung Mass)
$M_{us}$	Wheel Mass (Unsprung Mass)
$M_p$	Piston Mass
$g$	Gravitational Constant
$E_{air}$	Bulk Modulus of Air Inside the Accumulator
$V_a$	Accumulator Volume
$V_A$	Volume of Cylinder Chamber A
$V_B$	Volume of Cylinder Chamber B
$D_p$	Diameter of Piston
$L_p$	Length of Piston
$\mu$	Coefficient of Viscosity
$C_c$	Clearance between Piston and Cylinder
$L_s$	Stroke Length
$Q_{in}$	Initial Flow Rate
$P$	Pressure at inlet of Direction Control Valve
$P_A$	Pressure in Cylinder Chamber A

$P_B$

Pressure in Cylinder Chamber B



# *Chapter 1- Introduction*



## 1.1 Background

A key issue in the design and manufacture of modern automobiles is passenger comfort and satisfaction. A vehicle suspension system is a mechanism that physically isolates the body of the vehicle from its wheels. Road or trajectory irregularities such as bumps, and potholes can cause a lot of vibrations making the ride very uncomfortable. Vibrations have harmful effects on the passenger, including induced back pains, hyperventilation, and osteoarthritis. They also have detrimental effects on the vehicle itself, such as disc slipping. The achievement of a high-performance suspension system requires the consideration of several performance characteristics related to force distribution, suspension and body movement [1]. The ideal suspension system should be capable of isolating the vehicle's body from the road and inertial disturbances related to cornering, accelerating and braking [2]. Besides, the suspension mechanism should be capable of minimizing the vertical force transmitted to the passenger's seat. To achieve this objective, it is necessary to minimize the vertical acceleration of the vehicle. An excessive wheel travel results in non-optimum altitude of the tires relative to the ground, leading to poor contact. The maintenance of good vehicle handling properties requires maintaining the optimum contact between the tires and the ground. Conventional vehicle suspension systems have conflicting characteristics and therefore fail to meet all the requirements.

## 1.2 Problem Statement

It is necessary to continuously analyse any system to improve its overall performance and get better results. There is a lot of possibilities to improve the performance of the suspension system of automobiles and provide more comfort and safety to the passengers during travelling. The passive suspension system is just giving some amount of output when sudden uneven road



condition arrived. So that to overcome this situation *Active Suspension system* had been invented. Although there are several types available of *Active Suspension system*, the hydraulic suspension system gives the top level of isolation to vehicle.

### 1.3 Initial Scope of Research

Initially, it is aimed to develop a mathematical model for the hydraulic suspension system to examine the behavior of the system. Also, tune the parameters for different road and loading conditions. These results would be compared with the getting result of the developed test rig of a hydraulic active suspension system.

### 1.4 Final Scope of Research

Due to COVID-19 and limited time the scope of research has changed drastically. It is changed by proposing the mathematical model for the hydraulic active suspension system for a single tire. Also, bond graph model is proposed of this system with hydraulic parts.

### 1.5 Reasons Behind to Change the Scope

1. Due to global pandemic situation (COVID-19), the university not allowed to set a personal meeting with the supervisor. It led to not get the proper consultation and results of this research.
2. Due to a limited budget, could not purchase the “*20-sim*” software used for the analysis of the bond graph in limited time.
3. Due to limited budget and time, it could not be developed the actual prototype model of the hydraulic suspension system to verify with the real-life behaviour.



## 1.6 Thesis Outline

**Chapter 1-** It is about to introduction and background of the suspension system and its importance in the Automobile sector. Also, Problem statement, scopes and limitations are described.

**Chapter 2-** The important studies are done in the past years are reviewed in this chapter. It is primarily about the different modelling techniques used to analyse the behaviour of the suspension system. After analysis, the best possible modelling technique would be adopted for this analysis.

**Chapter 3-** This chapter mainly about to theory behind the suspension system. Various type of suspension system is elaborated in this chapter. Also, the comparison between different suspensions is formed.

**Chapter 4-** It is about to history behind the bond graph. Here describe the different elements that are required to fully define the mechanical, hydraulic and electrical system in a real-life situation.

**Chapter 5-** In this chapter, the mathematical models of different suspension system of a quarter car model are used in the analysis are described. Also, construct a bond graph to generate the state space equation which is simulated in MATLAB/Simulink.

**Chapter 6-** This chapter is about to design the active suspension system Bond graph. Also, based on this model, state-space equations are derived. The different parts need to design an active hydraulic suspension system are elaborated in this chapter.

**Chapter 7-** Derived state-space equation on chapter-6 would be used to design a Simulink model of the suspension system. Here, used a MATLAB user-defined function used to design the whole system.



**Chapter 8-** This chapter is dedicated to simulation results and discussion of the *Hydraulic Active Suspension System*.

**Chapter 9-** Conclusion of this research is present in this chapter. Also give suggestions about possible future scopes.



## *Chapter 2- Literature Review*



## 2.1 Introduction

This chapter carefully reviews all the researches done in the past to develop a model for the active hydraulic suspension system to able to examine the performance of such system and tune parameter to get better performance. For the development of such dynamic model following techniques are used: Mathematical, Analytical, Polynomial and Physical (Bond Graph). For two degree of freedom model lumped parameter approach is commonly used [3]. Following paragraphs describe different suspension model system to investigate the real-life behavior of suspensions such as ride dynamics and vehicle handling.

## 2.2 Suspension Modelling System

To consider the actual suspension system for mathematical analysis, it is considered according to its DOF (Degree of Freedom) model. It is the following types: Two degrees of freedom system, four degrees of freedom system and seven degrees of freedom system [4]. Also, for linear system analysis quarter car system has been using whereas for nonlinear analysis, multibody system modelling is used.

### 2.1.1 Two Degree of Freedom Model (Quarter Car)

In the field of the active suspension system analysis, the majority of researches based on the quarter car (2 DOF) model which represent a suspension of a single wheel. For this system, axle displacement and vehicle body displacement for an individual wheel are considered as a 2 degree of freedom [5]. This model consists of spring, damper and actuator. For passive suspension system force actuator set to be zero. According to Sharp and Crolla [6] the quarter car suspension model provides the most basic feature that important to represent the problem to control the load variation of wheel and suspension force that applied between the wheel and



body mass. Sharp and Crolla [6] mentioned the following advantages of the quarter car model over the more complex models (Half car model and full car model).

1. Few design parameters used to describe the whole suspension system.
2. It is defined by a few performance parameters.
3. It has a single input which provides easy computation of performance
4. Relationship between design and performance is easy to understand.

However, due to simplicity and 2 DOF, it is not accurate as like a higher degree of freedom models [7].

*Removed due to copyright restriction*

*Figure 1- Quarter Car (Two Degrees of Freedom Model) [5]*

Based on the above free body diagram, Mahala, et al. [5] derived following equation of motion for quarter car active suspension model.

*Removed due to copyright restriction*

Mahala, et al. [5] analysis showed that seven degrees of freedom model simulated the roll and pitch perfectly. While the pitch is simulated accurately from four degrees of freedom model. According to Mahala, et al. [5] the 2 DOF and 4 DOF model shows the estimated suspension and car body system. Due to these limitations, it cannot analyze the real behavior of the suspension and vehicle in every situation. Figure 2 to 5 shows the pitch and roll behavior of the sprung mass for two, four and seven DOF analysis of Mahala, et al. [5].

*Removed due to copyright restriction*

*Figure 2- Sprung Mass Displacement Comparison of Two, Four and Seven DOF Models in Pitch Mode [5]*



*Removed due to copyright restriction*

*Figure 3- Sprung Mass Acceleration Comparison of Two, Four and Seven DOF Models in Pitch Mode [5]*

*Removed due to copyright restriction*

*Figure 4- Sprung Mass Displacement Comparison of Two, Four and Seven DOF Models in Roll Mode [5]*

*Removed due to copyright restriction*

*Figure 5- Sprung Mass Acceleration Comparison of Two, Four and Seven DOF Models in Roll Mode [5]*

Mohammadpour and Scherer [8] investigated quarter car models using an analysis method based on parameter variation. Their simulation results of the parameter variations established that the system's resonance frequency was highly dependent on the damping parameters. This means that the parameters of the damping system have a great impact on ride comfort. They presented that comfort can be enhanced through the use of a controller for the damper.

Alleyne and Hedrick [9] examined the effect of Coulomb friction of a quarter car hydraulic suspension system. Also, introduced an adaptive algorithm which delivers performance without any convergence of the parameters and Luenberger-type observer was explained. Their simulation result for both systems gives the same and satisfactory results.

Using a non-linear two degree of car model Dong, et al. [10] showed that it was very difficult to maintain a top-level of ride comfort, handling, and control of the vehicle's body simultaneously using the conventional suspension system.



Agharkakli, et al. [11] obtained a quarter car mathematical model of the active and passive suspension system. The linear-quadratic control technique has been implemented in this study and compared both suspensions in different road conditions.

Mitra, et al. [2] used a quarter car model for an active suspension of a passenger car to boost the vehicle's holding ability and ride comfort. The results showed an improvement in the system's response in terms of reduced overshoot of the sprung mass acceleration and displacement in comparison with a passive system. They concluded that the behaviour of the active system was improved than that offered by a passive system.

Kuber [12] used computer simulation (MATLAB and Simulink) to model the performance of an active quarter car suspension system. To improve the performance of the system they used a controller with manual tuning. They compared the performance specifications of the active system to that of a passive system and concluded that the proper tuning can reduce the settling time for equivalent road inputs and the ability to lower the vertical displacement of the body.

Ahmed, et al. [13] designed an active quarter car model based on a hydraulic actuator for a passenger car. The aim was to enhance the vehicle's holding ability and the ride dynamics, their results indicated that the active system can improve ride comfort even at the resonant frequency. Using a step input of 0.8 m, the displacement of the sprung mass reduced by 25%. This indicated an improvement in the ride dynamics as well as the sprung mass acceleration reduced by 74.64%. A major problem of this system was its power consumption. Results showed that power consumption was at its minimum when driving on a smooth road. Driving through rough roads was observed to demand more power from the system as the hydraulic pump worked much harder.



### 2.1.2 Multibody System Modelling

When two or more than two rigidly imperfect bodies are joint together and make an assembly called a multibody system [14]. The multibody system modelling is a computer-based process to generate the equation of motions of the system. Duysinx, et al. [15] developed an approach to model, simulate and optimize the mechatronic system. This semi-active analysis simulated and optimized the Audi A6 suspension. Duysinx, et al. [15] used two different approaches for modelling and optimization: Simulink based symbolic model of chassis, which is equipped with hydraulic actuator and controller; Another one, finite element based multibody model.

Schumann and Anderson [16] presented the simulation study of hydro-gas active suspension system based on the full car multibody nonlinear system. They used velocity degree of freedom model technique to reduce the vehicle body roll. For regenerating computer-based multibody equation of motion for nonlinear model Schumann and Anderson [16] used the principle equation of Jourdain. Simulation result indicating that for 2 mm and 12 mm steering input the body roll decreased by 50%. Also, their study showed control of active suspension is depends on the lateral force generated by tire that affect the roll moment.

Alexandru and Alexandru [17] compared the behavior of the half car active and passive suspension system under multibody system modelling (ADAMS). Both suspensions were analyzed under the bumpy road condition. Also, they integrated the FEM multibody model and build a quick CAD model to perform the analysis and twin the parameters according to situations. After the virtual analysis, Alexandru and Alexandru [17] built a test rig based on the linear hydraulic actuator, FLEX test system and published the result in a future paper.

He and McPhee [18] applied a multidisciplinary (All-in-one) approach to design the active suspension based mechatronic vehicle. They designed it in multibody dynamic software and simulated in “GA-A’GEM-MATLAB” environment. Their result indicated that the



multidisciplinary design process of active suspension system gives a better overall performance compared to conventional LQG design process. This A-i-O method is possibly the best method to design and analyze the half car active suspension model for the flexible body [18].

Hamersma and Els [19] aimed to investigate the performance of the semi-active suspension system of ABS vehicle on the rough road. With antilock braking system, suspension performance improved significantly. They modelled full car model of SUV on Adams and simulated on MATLAB Simulink. The result showed that the braking performance significantly affected by the suspension system and vehicle stopping distance was reduced by up to 9 meters.

Feng, et al. [20] took a co-simulation approach to analyze the behaviors of the active suspension system and compare it with a passive suspension system. Their research based on the design process of hydro-pneumatic active suspension system which used on rough road condition. This co-simulation investigation divided into three parts: First, by use of ADAMS Feng, et al. [20] designed a multibody dynamic model of full car. Second, combined PID and Fuzzy logic controllers on the S domain of Simulink to achieve the top vehicle handling and ride quality during high speed on the off-road. Thirdly, proposed a new algorithm to control and integrated with multi-body system model to achieve more real-world result.

According to Yang and Xu [21], multibody system modelling has the following advantages and limitations:

1. For full car or half car analysis, it could be the best technique.
2. With the help of the multibody system model, a higher degree of freedom model could be constructed and analyze easily.
3. It could be the co-simulation-based process where combine more than one simulation techniques such as ADAMS+MATLAB/Simulink.



4. It required more time and data to analyze the behavior of the system.

### 2.1.3 Kinematics Modelling

According to Joo, et al. [22] for position analysis of the system; relation of the movement can be solved on each point. With the help of the differentiation of kinematics equations, the equations of acceleration and velocity could be easily found. These obtained equations would be used for kinematics modelling of the active suspension system. Hurel, et al. [23] designed a two-dimensional model of McPherson suspension to propose the kinematic model. They considered a vertical motion of chassis (Unsprung mass) and revolving and translating moment of wheel assembly (Sprung mass). Their result showed the improvements in the parameters of kinematics such as chamber angle and width of the track could not be analyzed in conventional modelling.

*Removed due to copyright restriction*

#### *Figure 6- McPherson Suspension Nonlinear Model and Its Kinematics Model [23]*

Reddy, et al. [24] comprehensively analyze the position kinematics of McPherson and double-wishbone suspension systems. They used two main elements: parameters of Rodrigues to derive the algebraic equations that represent the kinematics' mechanisms and to solve these equations Reddy, et al. [24] used a Gröbner computation method. The univariate equation showed the 64 degrees of freedom kinematics solution for both suspensions. According to the author of this paper is the first published journal which represents a three-dimensional kinetic model.

Fallah, et al. [25] proposed a new MacPherson nonlinear model for ride control. Their analysis based on sprung mass acceleration and integrate it with the suspension linkage kinematics. The result of the variations of the kinematic parameters is based on the linear model



road disturbances and it compared with MacPherson model in ADAMS. Their results are showing great agreement in both cases.

Habibi, et al. [26] has studied and design a McPherson suspension system for the front wheel and optimized it with the help of a Genetic Algorithm. They studied a three-dimensional kinematic approach and derived a set of the equation to define behavior under rolling for McPherson suspension system. Their study could be used for designing an optimum level of suspension and enhance the behavior of the kinematics of current systems. The obtained result could be applied in the enhancement of the stability during the cornering of the vehicle.

Dutta and Choi [27] presented a Macpherson MR damper suspension system and evaluated its response time under the kinematics and dynamic modelling. With the help of kinematics properties, the governing equations and Lagrange's equations are formulated. They analysed the suspension parameter such as kingpin angle, camber angle and track width alteration with the help of the governing equations that could not be analyzed from the linear conventional system. Their result shows that the camber angle and track width decreased considerably if applied an input current to the MR damper. Also, it is shown that the controlling of input current can control the kinematics and dynamic behavior of the suspension system. They develop robust control approaches for the proposed Macpherson MR damper suspension model (nonlinear) to improve the comfort during ride and stability of steering in future research.

#### 2.1.4 Polynomial Modelling

For estimation and prediction of the response values of the system over the input parameter values uses a polynomial modelling method. It is a great method to determine the input factor that drives responses and in which direction. Jian, et al. [28] presented a polynomial inverse model to control the MR damper that situated with the semi-active suspension system. The



result showed that modelling with the help this method can reject the disturbance on both medium and low frequency. Also, minimize the vertical acceleration of the vehicle body under different road conditions.

Sabino, et al. [29] focused on the performance improvement of the quarter car suspension system situated with a magnetorheological hydraulic damper. Its behavior investigated by the characteristics of force-velocity and force-displacement. The 6<sup>th</sup> order polynomial model used to check the behavior of the nonlinear damper. Their result showed that the magnetorheological damper car suspension gives the performance better than its counterparts.

### 2.1.5 Physics-based Modelling (Bond Graph)

According to Dridi, et al. [30], the quarter car suspension can be a heterogeneous system with mechanical, electrical and mechatronic components. This makes the modelling of the dynamic behavior of such a system quite complex. For the representation of the models and control laws for such a system, using the bond graph is the best approach. This method is based on the notion of energy transfer between subsystems [31]. It combines two variables namely effort and flow. Gonzalez-A and Madrigal [32] represent the passive and active suspension steady-state response under the bond graph modelling technique. They mainly proposed derivative causality and basic junction structure. Creed, et al. [33] created a linear model for the full car active suspension model on the bond graph to improve the ride comfort for the passenger. Their study mainly focused on the vertical movement and avoid complex lateral and longitudinal dynamics. Also, the model successfully validated the situation of corner, bump and brake.

*Removed due to copyright restriction*

*Figure 7- Active Suspension Physical and Bond Graph Model [33]*



According to Deur, et al. [34] bond graph has the following advantages:

- i) It's not only useful for the simulation of mechanical systems but it's also a useful tool for synthesis, diagnostics and support analysis.
- ii) It has causal properties and graphical aspects.
- iii) It can be modified by the addition of many elements without starting from the beginning giving fast evolution.
- iv) Its modelling language is unified and multidisciplinary.

Mitra and Benerjee [35] develop and applied a methodology which is a combination of damping and stiffness parameters. Here, they constructed a four degree of freedom model for a single wheel to study effects on the human body. Also, they compared the Bond graph result with SIMULINK which found a similar result and very near to real-life behaviour.

Emami, et al. [36] used the bond graph technique combined with simulation to model the performance of an active hydropneumatic suspension mechanism. The hydro-pneumatic system usually operates using two different fluids upon each other such as gas over fluid. Since such kind of systems combines mechanical and hydraulic subsystems, the bond graph approach was considered as the best choice.

Adibi-asl and Rideout [37] used a bond graph technique to measure the behaviour of seven degrees of freedom model. After analysis, they found that bounce and pitch have reduced significantly, and little improvement measured in rolling. Also, compared the behaviour of the active and passive linear suspension system.

Tiwari and Mishra [38] used a "Symbol's Shakti" software to simulate the dynamic response of the quarter car suspension model. They simulated the active and passive suspension



system at different operation speed under the bond graph model. From their analysis, minimum deflection of wheel and upright body velocity had been accomplished.

### 2.3 Gap Statement

After reviewing all studies done in the past showing that the modelling of the suspension system could be done by many techniques. But it can be identified that there is not enough study done on the physical-based modelling of the hydraulic active suspension system. There are many opportunities toward the investigation of the performance of the active hydraulic suspension system for commercial vehicles and easily tune parameter to get better performance.

### 2.4 Objectives

The following objectives would be considered for this research:

1. The development of the bond graph model of two-degree of freedom of the hydraulic active suspension system.
2. Formulate the equation of motions (State Space equations) of the two-degree freedom model of the active suspension system from the bond graph.
3. Formulate the hydraulic equations of the hydraulic system.
4. Design the Simulink model based on the state-space equations.
5. Examine the behavior of the hydraulic active suspension system.



## *Chapter 3- Suspension System Overview*



### 3.1 Introduction

According to Deng and Lai [39], suspension systems can be classified into three broad categories. These include passive, semi-active and full-active suspension systems. Passive suspension systems are constructed from conventional components with spring and damping characteristics that are time-invariant. Springs are passive elements and they can only store energy corresponding to a part of the suspension cycle while dampers dissipate energy. Passive suspension systems have no direct supply of external energy. Semi-active suspension mechanisms utilize springs and damping elements whose characteristics can be altered using an external controller [40]. Fully active or simply active systems use actuators to produce the desired forces in the suspension mechanism. The actuators used are usually hydraulic cylinders that derive power from an external supply.

### 3.2 Passive Suspension System

The control of the dynamics of many vehicles' vertical motion, roll (tilting) and pitch (spinning) is achieved with passive suspension mechanisms. Passive is a term that suggests that the elements used in the suspension mechanism do not provide energy to the system. The system is made up of a spring and a damper which are then mounted at each of the vehicle's wheels (Figure 8). The purpose of the spring is to support the body of the vehicle and to absorb and store energy. The damper on the other hand (also called shock absorber) has the task of dissipating the vibrational energy stored in the spring and to control impulses from the road that are transmitted to the vehicle. Besides, the suspension mechanism isolates the sprung mass from the Unsprung mass vibration which provides directional stability during cornering [40].



*Removed due to copyright restriction*

*Figure 8- Typical Suspension System [41]*

The passive suspension system has the following drawbacks due to its structure:

1. The damping and stiffness are limited which restrict its parameters' range.
2. Due to the fixing of the parameters suspension system cannot meet the actual condition of the applied load from the passengers, speed and road conditions.
3. The natural frequency of the system is inversely proportional to the travel stroke. The travel stroke would be large if the frequency is reducing.

### **3.3 Semi-Active Suspension System**

The semi-active suspension system represents an improvement in the passive suspension system. The two mechanisms have some similarities but the semi-active can produce better performance characteristics. This type of suspension mechanism has a spring and a damper that can be controlled to dissipate energy. According to Deng and Lai [39], some semi-active systems utilize a controllable spring and a passive damper due to the limited ability of the controllable damper to yield a controlled force while dissipating energy. An advantage of the semi-active system is its lower operating cost compared to the active system since it consumes only a small amount of energy. Figure 9 shows a semi-active suspension system with a controllable damper. The semi-active suspension system with a controllable damper can only alter the damping coefficient of the shock absorber and cannot add energy to the system [42]. The most commonly used semi-active control devices include magneto-rheological (MR) dampers. This device is capable of yielding high damping forces while consuming very little energy.

*Removed due to copyright restriction*

*Figure 9- Typical Semi-Active Suspension System [41]*

### 3.4 Active Suspension System

As can be observed from figure 11; in the active suspension system, both the passive components (spring and damper) are replaced with a force actuator. The force actuator is capable of both energy dissipation and energy addition into the system, unlike a passive damper mechanism. With this mechanism, a force can be applied independently of the relative displacement or the velocity. According to Yerrawar and Arakerimath [1], this model yields better results even with a compromise between vehicle stability and ride comfort, as long as the correct control strategy is adopted. Figure 10 shows a comparison of the performance of both passive and active suspension systems.

*Removed due to copyright restriction*

*Figure 10- A comparison of the response characteristics of passive and active suspension systems [43]*

The control for active suspension systems can be achieved from the use of four different types of control mechanisms. These include the use of magnetorheological damper, electromagnetic control, solenoid actuation, and hydraulic actuation. The fully active suspension system thus requires different components or elements such as actuators, sensors, accelerometers, control units, servo valves, and high-pressure tanks for the control fluid. This makes fully active systems more expensive than either the semi-active or passive systems. However, their performance is superior as demonstrated in figure 10 above. The active suspension system can be configured in two different ways depending on the linking between the active part (controller) and the passive part (spring and damper). If the active and passive components are



linked in parallel, this results in the high-bandwidth configuration. On the other hand, series linking results in a low-bandwidth configuration. The major advantage of the high-bandwidth over low-bandwidth configuration is the ability to achieve control over the suspension system in case the actuator fails to work satisfactorily.

*Removed due to copyright restriction*

*Figure 11- Typical Active Suspension System [44]*

According to Zhou [44], the active suspension system is controlled by computer has three conditions:

1. The required force is generating from the power source.
2. The components are capable to implement the required force and continuously work.
3. To find the required control mode for microcomputer processes, a large number and variety of sensors are used.

Due to all these conditions, the active suspension system takes the technical knowledge of electrical and mechanical fields. Table 1 is showing the comparison factors between three suspension systems [44].

*Table 1- Comparison of Suspensions [44]*

<b>Suspension Type</b>	<b>Passive Suspension</b>	<b>Semi-Active Suspension</b>	<b>Active Suspension</b>
<b>Controlling Element</b>	Shock absorber	Variable damper	Hydraulic and Electrical system
<b>Acting Principle</b>	Damping principal (constant)	Continuously adjustable Damper	Adjustable force vehicle body and wheel
<b>Controlling Technique</b>	-	Electro-hydraulic (Automatic)	Fluid/ magnetic/ electronic controlled
<b>Bandwidth</b>	-	<20 Hz	>15 Hz
<b>Consumed Energy</b>	-	Small	High
<b>Dynamic Characteristics</b>	-	Medium	Good
<b>Production Cost</b>	Low	Medium	High



## *Chapter 4- Bond Graph Background*



## 4.1 Introduction

This chapter about to describe the bond graph method and its related theoretical knowledge. There are different methods of defining the physical system, but the bond graph is one of the best graphical methods to use for describing the system. It is based on the structure of the considered system and subsystems (Mechanical, Electrical and Hydraulic, etc) in different energy domain. With the use of bond graph method more complicated systems could be solved more easily. Paynter [45] was the first scientist who proposed this new modelling technique in 1961. After a few years, the Karnopp, Rosenberg and Margolis were re-presented it with the more advanced version.

The bond graph method consists of bond and element, where two elements are connected all together with the bond. It shows the relation between effort and flows from the attached elements which are the power variables. Half arrow and stroke at the end present in each bond which describes the effort and flows directly between the elements. Also, the effort between two elements directed by the causality which demonstrate the stroke position. Table 2 shows the different elements and the maximum acceptable ports. Also, Causality assignments of bond and element showing in table 3.

*Table 2- Elements of Bond Graph [46]*

No of Ports	Elements	Symbol	Use
<b>1</b>	Inertia	$I$	p-type variable storage
	Capacitance	$C$	q-type variable storage
	Resistance	$R$	Dissipate the Energy
	Source of Flow	$S_f$	Sources
	Source of Effort	$S_e$	
<b>2</b>	Gyrator	$GY$	Present mathematical relation of flow and effort
	Transformer	$TF$	
<b>3</b>	Zero-junction	$0$	Generalized loop Laws and mode of Kirchhoff
	One-junction	$1$	

*Table 3- Causality Assignment for Bond Graph Elements [47]*

*Removed due to copyright restriction*

## 4.2 Power Variables

When two multiport are connected passively than the interaction of power between them are presented. The universal scheme used to describe the different power variables in the bond graph method. Here various multiport in the different physical domain is defined by a common language known as effort and flow variables shown in Table 4. Their power and energy variables situated in table 5. The product of effort and flow always gives the Power for the certain port:

$$P(t) = e(t) * f(t)$$

**Table 4- Bond Graph Power Variable [47]**

Domain	Effort $e(t)$	Flow $f(t)$
Mechanical Translation	Force $F$ (N)	Velocity $v$ (m/s)
Electrical	Voltage $u$ (V)	Current $i$ (A)
Hydraulic	Pressure $P$ (Pa)	Flow rate $Q$ (m <sup>3</sup> /s)

**Table 5- Power and Energy Variable [48]**

Domain	$p(t)$	$q(t)$
Mechanical Translation	Momentum ( $p$ ) (Ns)	Displacement ( $x$ ) (m)
Hydraulics	Momentum due to pressure ( $p_p$ ) (Pa-s)	Volume ( $V$ ) (m <sup>3</sup> )
Electrical	Flux linkage $\lambda$ (Vs)	Charge $q$ (C)

### 4.3 Bond Graph Elements

#### 4.3.1 Passive Elements (One port)

Merzouki, et al. [47] mentioned that the passive elements convert the gain energy into the dissipated energy (R-element), C-elements store energy in the form of potential while the I-element store it in the form of kinetic energy. These are known as one port elements because it can connect with another system by one port. Below shows the arrangement of the 1-port passive elements.

*Removed due to copyright restriction*

##### 4.3.1.1 I-Storage Element (Inertance)

I-element of the bond graph is called if it followed the dynamic equation which relating the integral of effort (Moment) and flows. This can be used to model the system who transfer the



received energy into kinetic power without any dissipation into the environment [47]. For the mechanical system, I-element could transform the gain power into the kinetic energy. In the mechanical and hydraulic system, the I-element used to show the effects of inertia or mass. The given relation for I-element between Effort and Flow [47].

*Removed due to copyright restriction*

$$F(f, p) = 0;$$

### ***Inertia Element for Mechanical System:***

*Removed due to copyright restriction*

From the above diagram:

$$P_{\omega} = J\omega$$

For linear motion

$$P = m \frac{dx}{dt}$$

**Note:** This I-element presents; the inductance mass of system or Moment of Inertia. In the suspension system, I-element represents the mass of the car body and mass of axle.

### ***Inertia Element for Hydraulic System:***

*Removed due to copyright restriction*

From the above diagram, the inertia element of the hydraulic system is

*Removed due to copyright restriction*



From the above expression, the Inertia of the fluid is

$$I = \frac{\rho l}{A}$$

Where

$\rho$  = Density of the Fluid

$l$  = Length of the Pipe

$A$  = Cross-sectional Area of the Pipe

$P_h$  = Pressure Momentum ( $= p/A$ )

According to Karnopp, et al. [48] following are the inertias' constitutive relations and units in the linear case (Table 6).

**Table 6- The inertia and SI Unit [48]**

<i>Energy Domain</i>	<i>Linear Relation</i>	<i>Linear Units in SI System</i>
General Variable	$f = p/I$	$I = p/f$
Mechanical Translation	$v = p/m$	$m = \text{N}\cdot\text{s}^2/\text{m} = \text{kg}$
Hydraulic System	$Q = p_p/I$	$I = \text{N}\cdot\text{s}^2/\text{m}^5$
Electrical	$i = \dot{\lambda}/L$	$L = \text{henrys (H)}$

#### 4.3.1.2 C-Storage Element (Capacitance)

According to Merzouki, et al. [47] if dynamic equation defines the effort and flow time integral (displacement) is called as a Capacitance element of the bond graph. It converts Potential energy from the received power without any energy loss. This is system has examples such as mechanical spring, torsion bars, electrical capacitor and reservoir.



*Removed due to copyright restriction*

$$F(e, q) = 0;$$

In the mechanical system,

*Removed due to copyright restriction*

$$F = Kx$$

So that the Potential Energy is given by,

*Removed due to copyright restriction*

Where,

$K$  = Spring Stiffness

***In Hydraulic System,***

*Removed due to copyright restriction*

Here,

$$C = \frac{A}{\rho g}$$

Where,

$A$  = Cross-Sectional Area of the Cylinder (or Tank)

$\rho$  = Fluid Density ( $\text{kg/m}^3$ )

$g$  = Gravitational Force.

Table 7 showing the linear relations in different energy domains with its SI units [48].

*Table 7- The Capacitance and SI Unit [48]*

<i>Energy Domain</i>	<i>Linear Relation</i>	<i>Linear Units in SI System</i>
General Variable	$e = q/C$	$C = q/e$
Mechanical Translation	$F = k X$	$k = N/m$
Hydraulic System	$P = V/C$	$C = m^5 / N$
Electrical	$e = q/C$	$C = \text{Farad (F)}$

#### 4.3.1.3 R-Storage Element (Resistive)

It is the element of the bond graph which defined the static function between effort and flow for the single port. To model the energy dissipation in any system this element used. In some cases, it could be negative which means added power to the system like dry friction. The bond graph symbol, Mechanical damper and Hydraulic fluid flowing in the pipe show in figure 12 [48] and [47].

*Removed due to copyright restriction*

*Figure 12- (a) Symbol, (b) Mechanical Damper (c) Hydraulic*

$$F(e, q) = 0;$$

For a linear and nonlinear system this relation could be the following:

1. For friction in mechanical system:  $F = R_m \dot{x}$
2. Bernoulli law for the hydraulic system:  $\Delta P = R_h Q$

Karnopp, et al. [48] mentioned the linear relation and units of the resistive element in different energy domains are shown in table 8.


**Table 8- The Resistance and SI Unit [48]**

<i>Energy Domain</i>	<i>Linear Relation</i>	<i>Linear Units in SI System</i>
General Variable	$e = Rf$	$R = e / f$
Mechanical Translation	$F = bv$	$b = \text{N-s/m}$
Hydraulic System	$P = RQ$	$R = \text{N-s/m}^5$
Electrical	$e = Ri$	$R = \text{ohm } (\Omega)$

#### 4.3.2 Active Elements (Source of Effort and Source of Flow)

The elements which supply power to the system defined as the sources or active element of the system. This could be two types: one who gives the effort ( $Se$ ) to the system in the form of Power or Force such as the hydraulic pressure pump and gravity force exerted on the mass. On the other hand, the element which gives the flow ( $Sf$ ) to the system in the form of Flow rate or Velocities like positive displacement pump and road velocity. The power direction of active elements always near to the junctions (0 or 1). Table 9 displays the symbols and relationship of sources.

**Table 9- Active Elements [48]**

<i>Energy Domain</i>	<i>Symbol and Direction</i>	<i>Relation</i>
General Variable	$Sf \xrightarrow{e}$	<i>Flow is given</i>
	$Se \xrightarrow{e}$	<i>Effort is given</i>
Mechanical Translation	$Sv \xrightarrow{e}$	<i>Velocity is given</i>
	$S_F \xrightarrow{e}$	<i>Force is given</i>
Hydraulic System	$S_Q \xrightarrow{e}$	<i>Flow rate is given</i>
	$S_P \xrightarrow{e}$	<i>Pressure is Given</i>



### 4.3.3 Junctions

To make the connection with a single port element such as  $R$ ,  $C$  and  $I$ , the junctions are used in the bond graph method. If all element connected with 0-junction then they have the same effort and if attached with 1-junction which means the common flow in all elements. Figure 13 shows the bond graph representation of zero and one junction [47].

*Removed due to copyright restriction*

(a)

(b)

*Figure 13- Representation of Zero and One Junction*

#### 4.3.3.1 Common Effort Junction (0-Junction)

When the same effort (such as Pressure and Force etc) in all elements which connected with a junction is known as a zero junction or common effort junction. From figure 14(a), the sum of the flow of all bonds are equal to zero and the effort of all bonds would be equal to zero [47].

$$f_1 - f_2 - f_3 + f_4 = 0$$

$$e_1 = e_2 = e_3 = e_4 = 0$$

*Removed due to copyright restriction*

(a)

(b)

*Figure 14- Representation of Zero Junction for Mechanical and Hydraulic System [47]*

In figure 14(a), the zero junction represents the situation where a single force is acting while the sum of many velocities would be zero. On the other hand, for the hydraulic system (figure 14(b)) zero junction characterizes the preservation of volume flow rate where different pipe or cylinder join with each other.



### 4.3.3.2 Common Flow Junction (1-Junction)

As name indicated that if flows are common in all bonds and sum of all effort would be zero than this is known as one junction. From figure 15, the following equations are derived.

$$f_1 = f_2 = f_3 = f_4 = 0$$

$$e_1 - e_2 - e_3 + e_4 = 0$$

*Removed due to copyright restriction*

*Figure 15- One Junction Representation of Mechanical and Hydraulic System [47]*

In the mechanical system (Figure 15 (a)) the equilibrium condition of the forces related to the similar velocity of the whole system. While in the hydraulic system, the sum of the pressure of all ports is equal to zero and the flow rate is the same for all ports. For figure 15 (b), the equation of the one junction:

$$Pe - \Delta P_1 - P_2 - P_{atm} = 0$$

Where,

$$\Delta P_1 = Pe - P_1$$

$$\Delta P_2 = P_1 - P_{atm} \quad (\text{Atmospheric pressure considered as a gauge pressure} = 0)$$

### 4.3.4 Two Port Elements

There is an infinite number of two elements but for bond graph, there are two types: Transformer and Gyrator.



#### 4.3.4.1 TF Element (Transformer)

The transformation of the energy from one domain to other the transformer element is used. Its relations indicated that effort inlet is allegorically linked with outlet efforts while the flow inlet is with outlet flow. The effort of the first port is proportional to the effort of the second port and the flow of the second port is proportional to the first port [47]. Following figure 16 indicated the *TF*-element relationship between two elements.

*Removed due to copyright restriction*

(a)

(b)

*Figure 16- TF element for Hydraulic and Mechanical System [47]*

$$e_1 = m e_2$$

$$f_2 = m f_1$$

Where  $m$  = Transformer modulus

For figure 16 (a) the transformer element equation of piston-cylinder where  $A$  is the cross-section area of the piston.

$$P = F/A$$

$$v = Q/A$$

#### 4.3.4.2 GY Element (Gyrator)

According to Merzouki, et al. [47] conversion of the effort of any side of the bond into the flow of any side of the bond, the gyrator element is used. The electric motor (figure 17) is the best example which converts electric power into the mechanical rotation. It is also called transducers which has the following constitutive equations to define the relations.



$$e_1 = r f_2$$

$$e_2 = r f_1$$

Where  $r =$  ratio of the gyrator

*Removed due to copyright restriction*

*Figure 17- Electric Motor and its Gyrator Element [47]*

#### 4.4 Causality

Merzouki, et al. [47] mentioned that to simulate the physical behaviour of any system need to decide which effort or flow variable are to be calculated. In this way first need to decide the cause and effects which defined by the *causality*. In the bond graph, the two or more systems are interconnected then they exchange the power represents the bond. To simulate the model, it is necessary to decide the sequence of the variables (*effort and flow*) which need to be computed. For this process, the block diagram causal is introduced (Figure 18). The direction of flow or effort in the bond the single vertical stroke is used. Table 3 mentioned the causality assignment on different ports. The effort is known if causality is near to the element and if away from the element is gives the flow. For the derivation of state-space equations, the causality assignments play an important role.

*Removed due to copyright restriction*

*Figure 18- Causality Assignment [47]*



#### 4.4.1 Sequence to Assigned the Causality

In power bond only one end can be defined as the effort therefore only one end could be the causal stroke. Due to this limitation, the following are the procedure to assign the causality in the whole model elements and ports [47].

1. For zero junction all efforts are equal while all flows are equal for the one junction. Therefore, for zero junction only one bond can give the effort to it and for one junction only bond can give flow. So that, if one bond's causality is known then the causality of others automatically defined. This bond gives the effort or flow to the junction known as a strong bond.
2. Like junctions, if the causality of one side of the two-port element is defined then another side automatically defined.
3. The *Sources* always have one causality, like for the source which gives the effort to the system then causality of it is always near to power arrow. While for the source which gives the flow to the system then its causality always away from the power arrow. It is necessary to assign the causality first to all sources which are in the system.
4. After that select the *Compliance* and *Inertance (Inertia)* element to assign the integral causality. These elements have already predefined causality. The compliance element always give effort to the system so its causality always away from it. The inertance (inertia) element always gives the flow to the system so that its causality always near to it. Repeat this process till all *C* and *I* elements are assigned.
5. At the end the causality assignment for *Resistance* elements because it has both types. If causality is assigned near to it then it will give the flow to the system while it is away from it gives the effort to the system. Also, it indicates the resistance or inductance parameters for the system.



## 4.5 State Space Equations

To simulate any system than it is the first to define the differential equations. According to Merzouki, et al. [47] there are many principles to derive the equations: equations of Lagrange, the principle of virtual work, Newtonian approach and principle of Hamilton, etc. But the bond graph method is an easy way to derived state-space equations for all energy domains. These equations can be divided into linear and nonlinear form.

Linear relation:

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

Nonlinear relation:

$$\dot{x} = F(x, u)$$

$$y = G(x, u)$$

Where  $x$  is the state vector,  $u$  is input vector such as effort or flow source of the bond graph. While  $y$  is the output vector and  $A$ ,  $B$ ,  $C$  and  $D$  defined the appropriate dimensions matrices. The  $x$  is defined by  $p$  and  $q$  are the energy variables of Inertance and Compliance elements, respectively.

$$x = \begin{bmatrix} p \\ q \end{bmatrix}$$

### 4.5.1 State Variable Properties

1. In bond graph diagram is not show the state vector but the power variables always display.

$$\dot{x} = \begin{bmatrix} \dot{e} \\ \dot{f} \end{bmatrix} = \begin{bmatrix} \dot{p} \\ \dot{q} \end{bmatrix}$$



2. The number of integrally casual  $C$  and  $I$  elements is always equal with the dimensions of the state vector.
3. The model order is  $N_0 = N - N_I$ . Where  $N =$  number of  $C$  and  $I$  elements and  $N_I =$  derivative causality.

#### 4.5.2 Steps to Derive Equations

1. First write the structural rules or equation that related to the  $O$ ,  $I$ ,  $TF$  and  $GY$  junctions.
2. For each  $I$ ,  $C$  and  $R$  elements, write the related equations.
3. To find the final state-space equations of any system, combine these different rules and equations of different elements.



# *Chapter 5- Mathematical Model of Suspension System*



## 5.1 Introduction

This chapter is about to describe the mathematical model and equations of the different suspension system. According to Zhou [44], the design of vibration control could start from the mathematical representation of different suspension systems and find the requirement for design. After that select one or more methods to design and attached to it with the simulation software to identify the designed model meet the requirements. Hence, for investigation of the whole system required to derive the mathematical equations first. Following the models which are considered only for quarter car mathematical representation.

## 5.2 Passive Suspension Model

passive suspension mathematical model shows in figure 19.  $M_s$  represents the vehicle mass (including passengers' weight),  $M_{us}$  is for the wheel and axle mass,  $C_s$ ,  $K_s$  and  $K_t$  denotes the damper, spring and tire constant, respectively. Whereas, the  $X_s$  and  $X_{us}$  represent the state variable of sprung and un-sprung mass, respectively and  $X_r$  for the vertical road input. Figure 16 represents the two degrees of freedom vehicle as a body-wheel mass model which attached with spring and damper. With the help of this arrangement, could derive the following equations of motions from Newton's law of motion [49].

$$M_s \ddot{X}_s = -C_s (\dot{X}_s - \dot{X}_{us}) - K_s (X_s - X_{us})$$

$$M_{us} \ddot{X}_{us} = C_s (\dot{X}_s - \dot{X}_{us}) + K_s (X_s - X_{us}) - K_t (X_{us} - X_r)$$

*Removed due to copyright restriction*

*Figure 19- Mathematical Representation of Passive Suspension System [50]*



The above mathematical representation is converted into the Bond Graph (Figure 20) and formed the state space equation for MATLAB/Simulink to simulate the behaviour of the system. There are four equation has been derived for Sprung Mass, Unsprung Mass, Tyre Stiffness and Spring Stiffness.

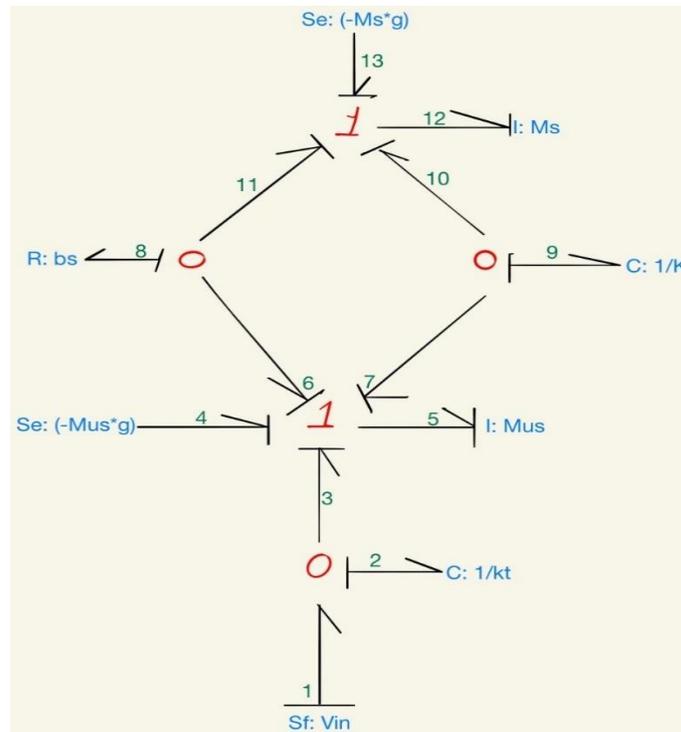
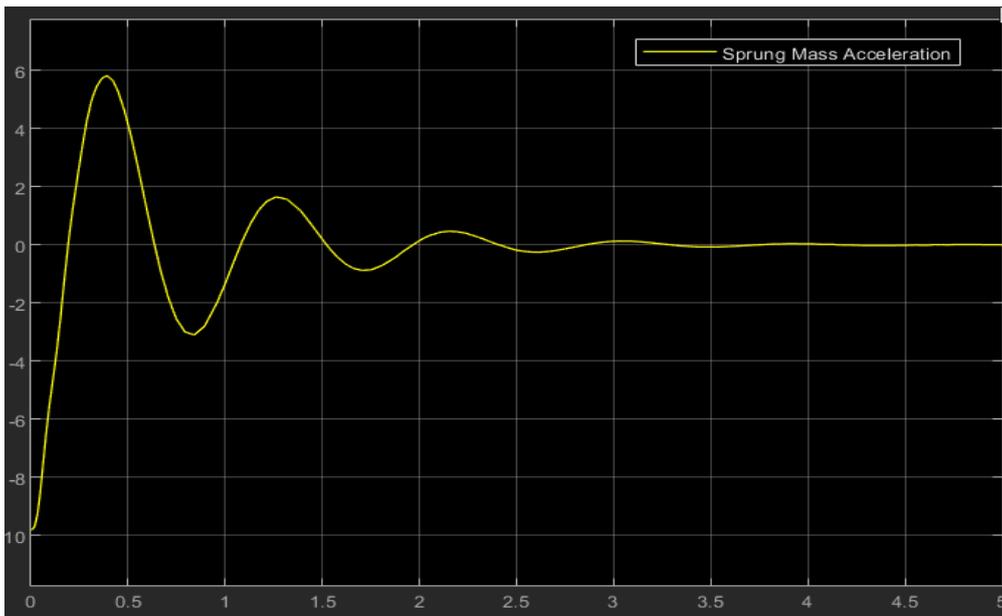
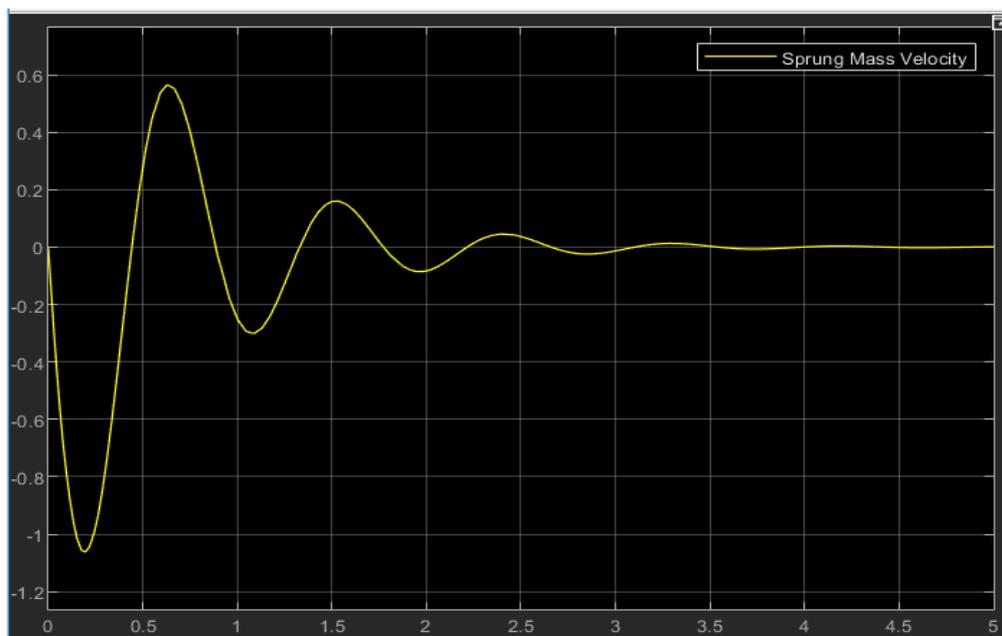


Figure 20- Bond Graph of Passive Suspension System

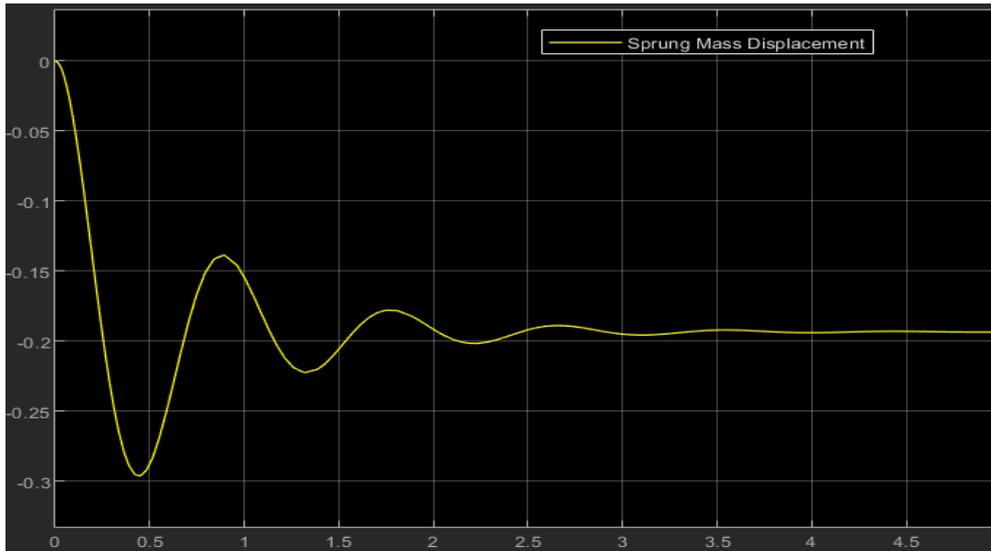


*Figure 21- Sprung Mass Acceleration*

It can be seen in figure 21 that the car body accelerated to the  $6 \text{ m/sec}^2$  upward and it is first compressed to downward  $10 \text{ m/sec}^2$ . It is coming to the normal position after 3.5 seconds. While in figure 22 the sprung mass velocity first compressed the body to  $1 \text{ m/sec}$  and then it is expanded to  $0.6 \text{ m/sec}$  and back to the neutral position after 4 seconds.

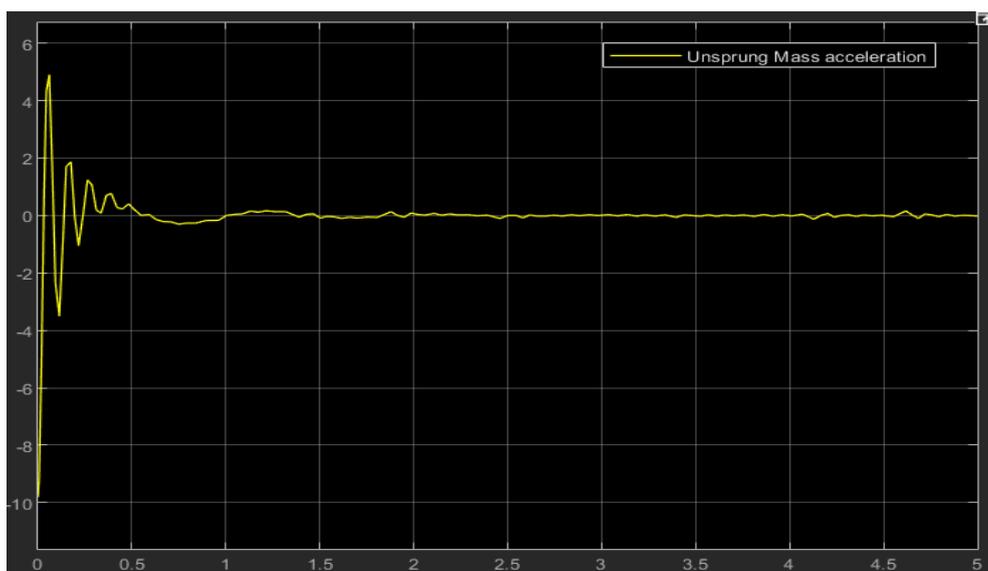


*Figure 22- Sprung Mass Velocity*

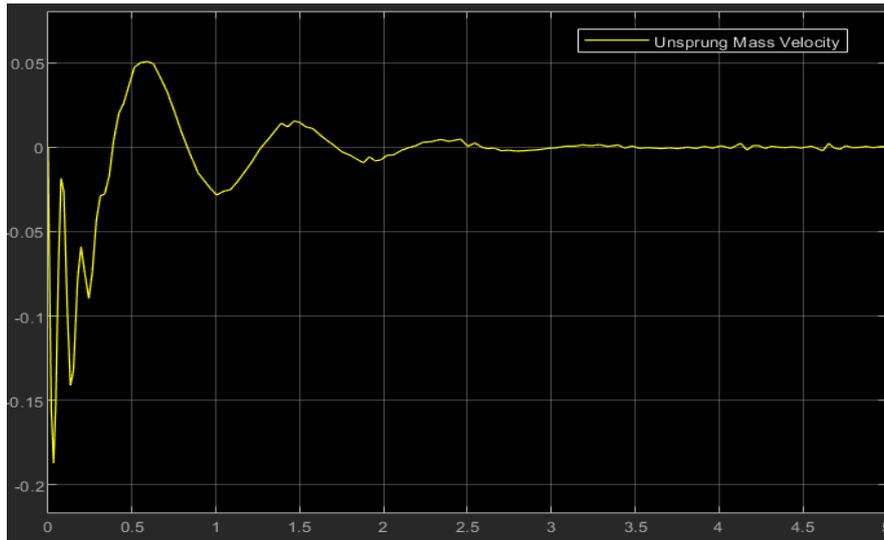


*Figure 23- Sprung Mass Displacement*

Figure 23 shows the sprung mass displacement where it can be seen that displacement started from the neutral position of the body and compressed downward till 0.3 meters and it is resting on nearly 0.2 meters downward after 3.8 seconds. On the other hand, figure 24 is about to acceleration of the un-sprung mass which starts accelerated from zero and goes downward till  $10 \text{ m/sec}^2$  and comes to upward  $5 \text{ m/sec}^2$ . There can be seen the less amount of accelerations happened in the body after 2 seconds and continuously occurred.

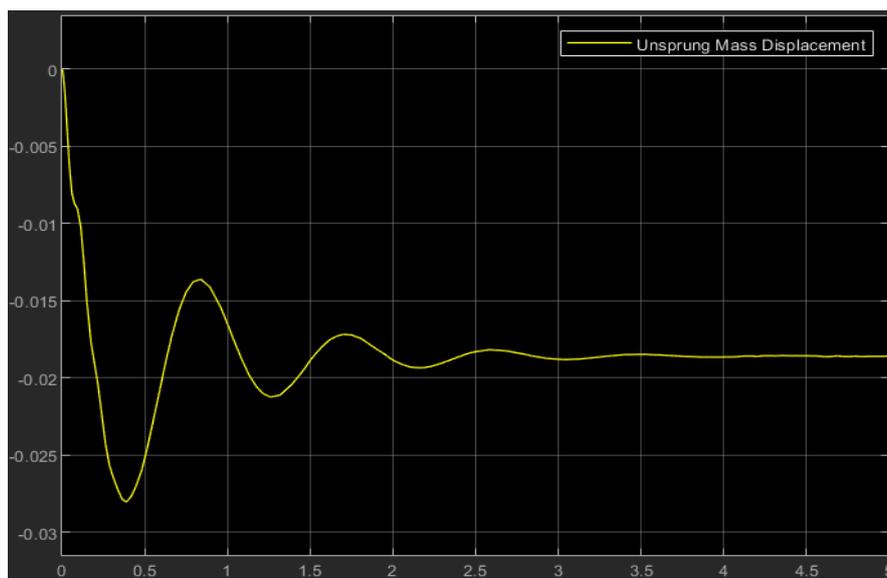


*Figure 24- Unsprung Mass Acceleration*

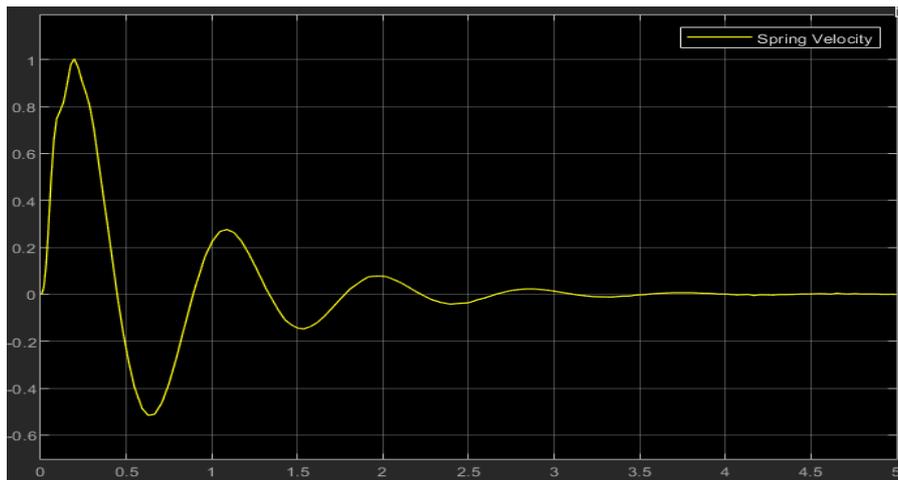


*Figure 25- Unsprung Mass Velocity*

Figure 25 is about the velocity of the un-sprung mass where it can be seen that the velocity started from the zero and goes to the downward till 0.2 m/sec which compresses the body and expanded till 0.05 m/sec. After 4 seconds there are negligible fluctuations continually occurred. In figure 26 can be seen that the un-sprung mass displacement has the same behaviour as the sprung mass in figure 23. Displacement of the un-sprung mass compressed the axle downward till 0.03 meters and resting on 0.02 downward after 4 seconds.



*Figure 26- Unsprung Mass Displacement*



*Figure 27- Spring Velocity*

It can be seen from figure 27 where the velocity of spring when a sudden uneven force applied on the body and it gives the input to the body to maintain constant location or overcome this applied force. Here the spring velocity going upward 1 m/sec and compressed spring till 0.6 m/sec and after 4 seconds it is back to the normal position.

### 5.3 Active Suspension Model

Figure 28 showing the arrangement of the active suspension system for a single tyre. This system has the same parameter except for force actuator which denoted by  $F_d$ . From the below diagram following equation of motions can be derived [49].

*Removed due to copyright restriction*

*Figure 28- Mathematical Representation of Active Suspension System [50]*

$$M_s \ddot{X}_s = -C_s (\dot{X}_s - \dot{X}_{us}) - K_s (X_s - X_{us}) + F_d$$

$$M_{us} \ddot{X}_{us} = C_s (\dot{X}_s - \dot{X}_{us}) + K_s (X_s - X_{us}) - K_t (X_{us} - X_r) - F_d$$

The equivalent bond graph present in figure 29 of the above mathematical representation. derived the state space equation for MATLAB/Simulink to simulate the behaviour of the

system and considered force actuator value as a constant. There are four equation has been derived for Sprung Mass, Unsprung Mass, Tyre Stiffness and Spring Stiffness.

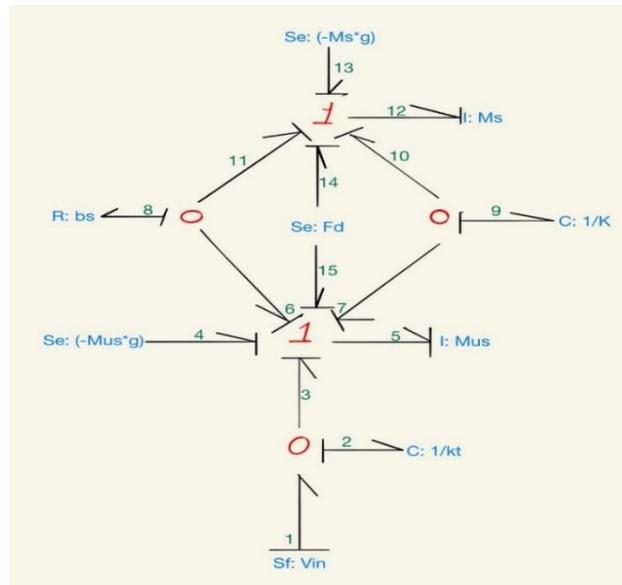


Figure 29- Bond Graph of Active Suspension System (Actuator Force  $F_c$ )

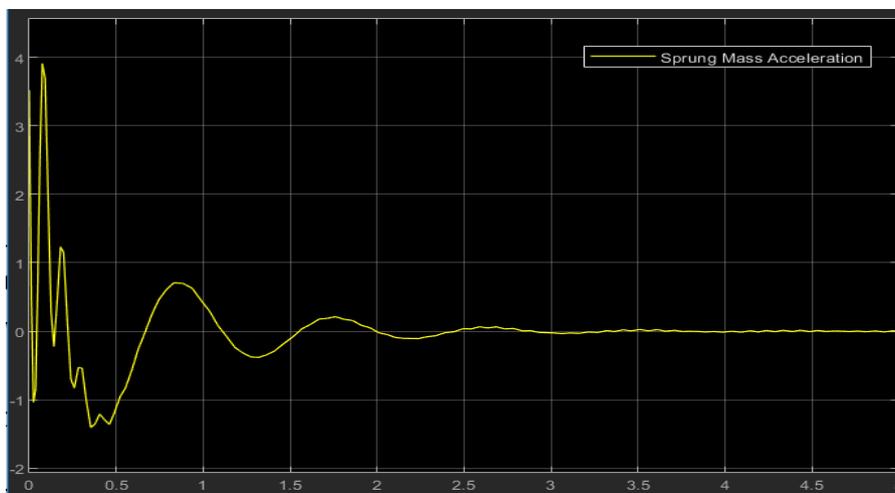
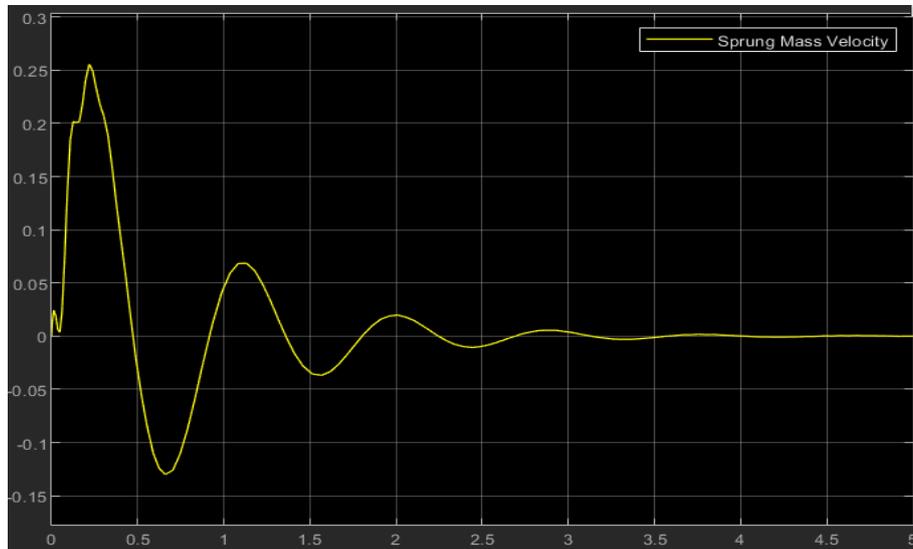


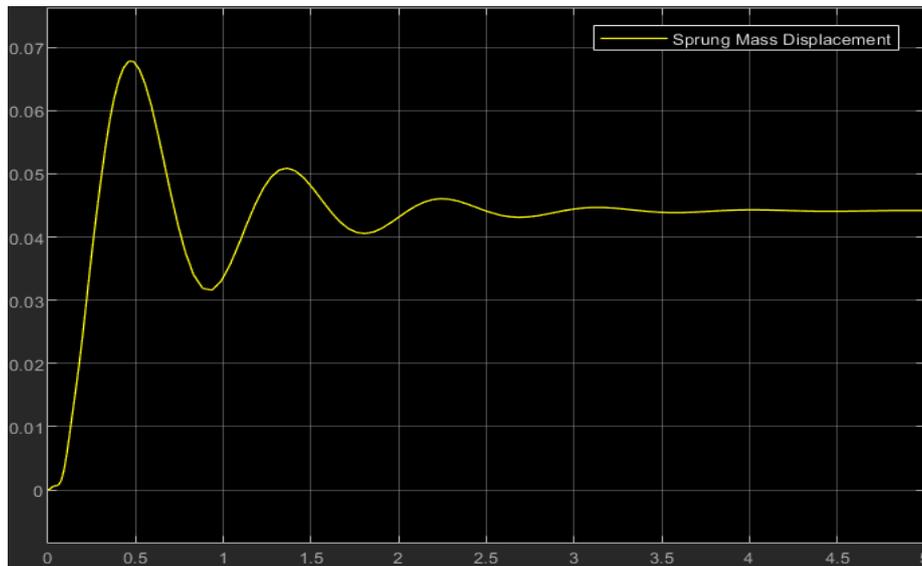
Figure 30- Sprung Mass Acceleration

Figure 30 is showing the acceleration of the sprung mass of the vehicle. Where it can be seen that the acceleration of the body is coming to the normal position after 3.5 seconds which is better than the passive suspension counterparts (Figure 21). Here in figure 30, the acceleration upper limit is  $4 \text{ m/sec}^2$  and downward is  $1 \text{ m/sec}^2$ . While figure 31 is about to the behaviour of

velocity of the body which starts from 0 and going upward till 0.25 m/sec than it is compressed the body to 0.15 m/sec and back to the normal position after 3.8 m/sec.



*Figure 31- Sprung Mass Velocity*

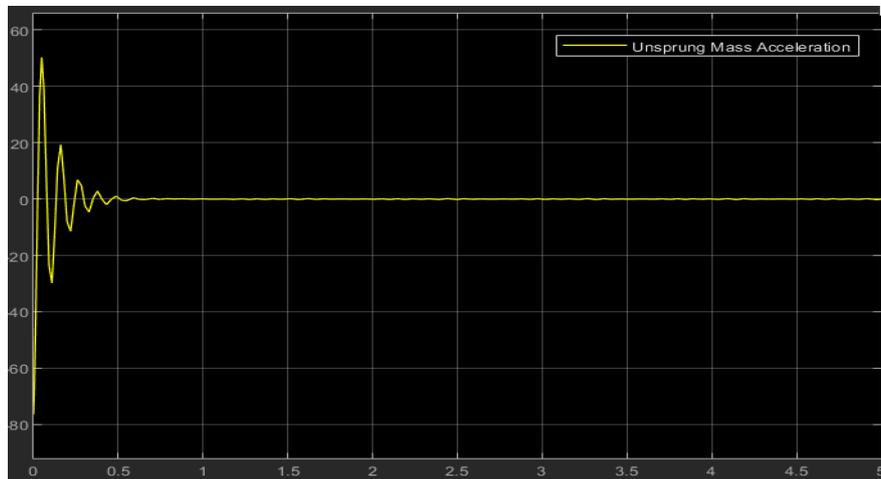


*Figure 32- Sprung Mass Displacement*

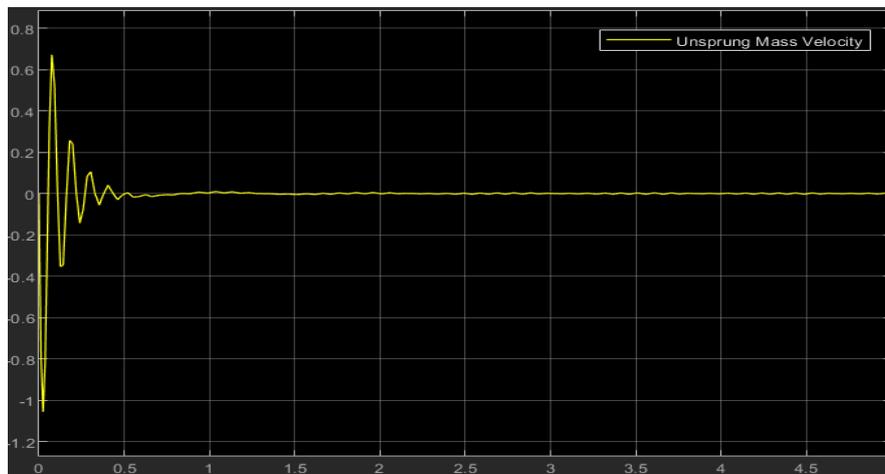
Figure 32 shows the displacement behaviour of the sprung mass. It can be seen that the displacement of the body starts from the neutral position and going upward till 0.07 meters and vibration are resting on 0.045 meters. The un-sprung mass acceleration is seen in figure 33,



where the acceleration of the un-sprung mass comes to the rest after 1 second which showing better working behaviour of the suspension. Acceleration upper and lower limit are  $50 \text{ m/sec}^2$  and  $70 \text{ m/sec}^2$ , respectively.

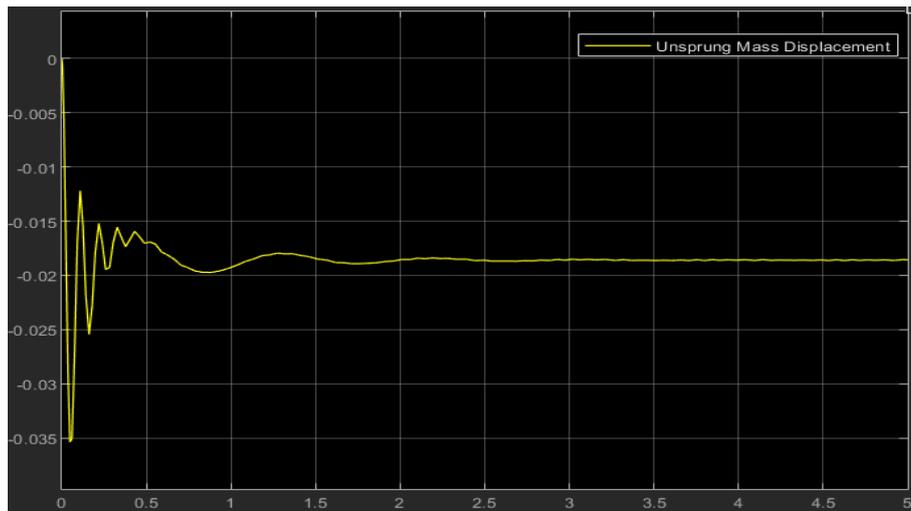


*Figure 33- Unsprung Mass Acceleration*

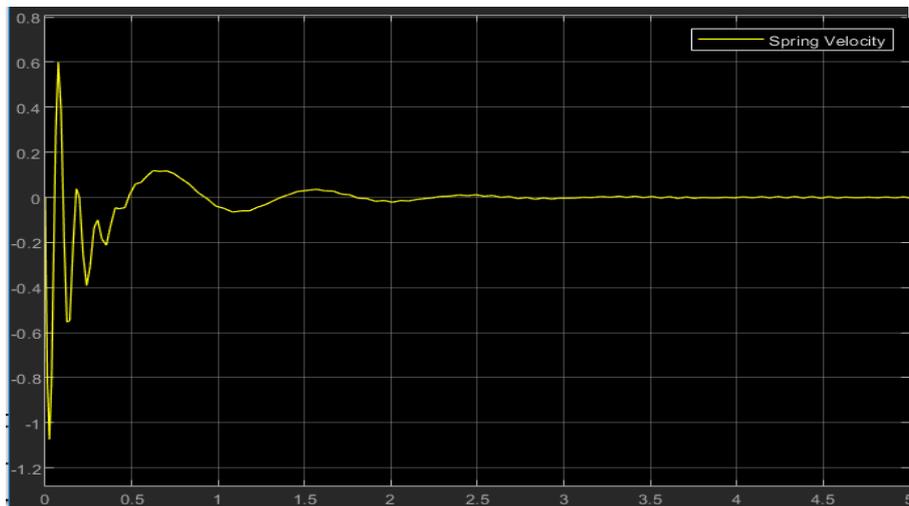


*Figure 34- Unsprung Mass Velocity*

Figure 34 describes the behaviour of the velocity of the un-sprung mass where the velocity is in between  $0.8$  to  $-1.1 \text{ m/sec}$  and resting after  $0.7$  seconds which is worked better than passive counterpart (Figure 25). Displacement of the un-sprung mass can be seen in figure 35, where the displacement started from the zero and compressed the mass to  $0.035$  meters and resting on  $0.02$  meters downwards after  $3$  seconds.



*Figure 35- Unsprung Mass Displacement*



*Figure 36- Spring Velocity*

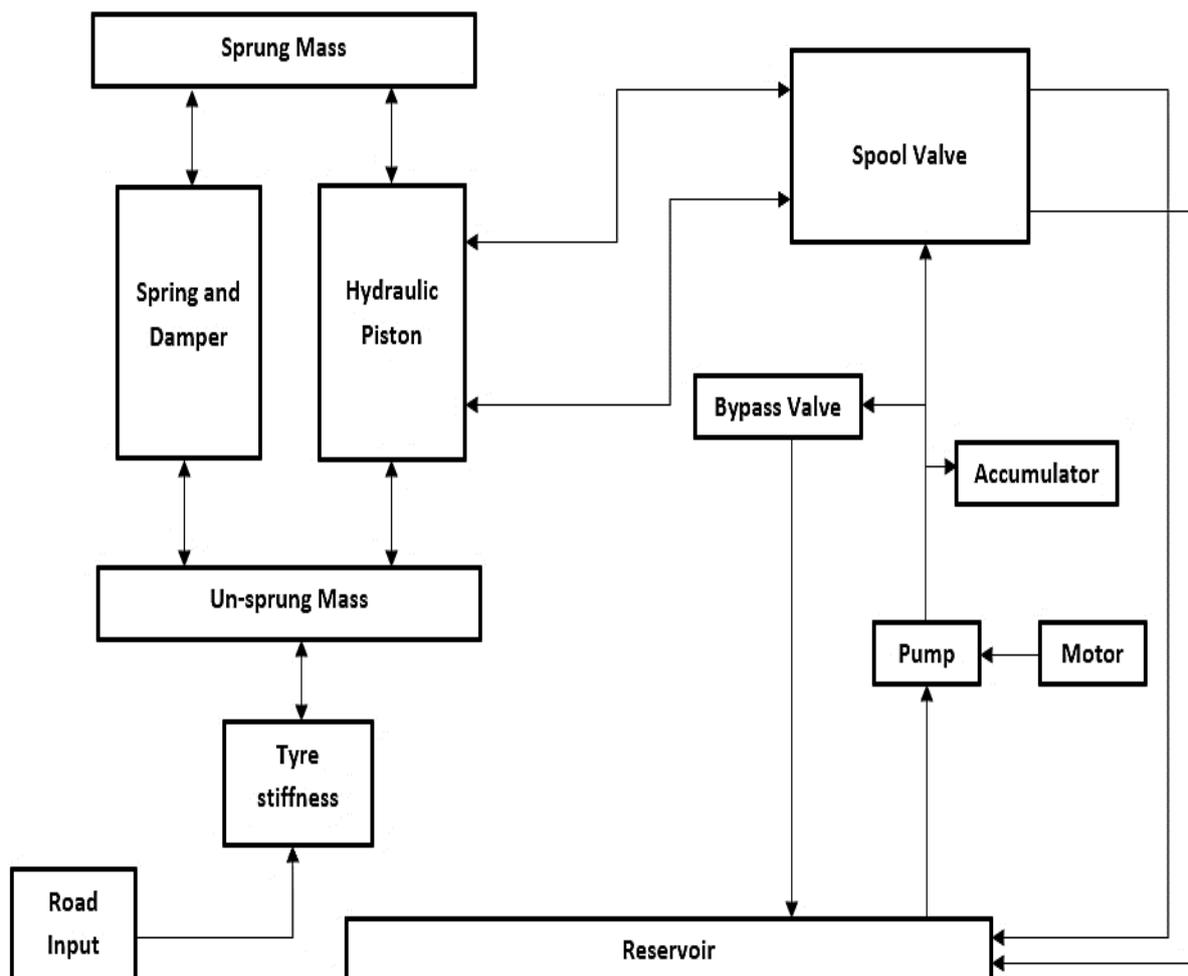
Figure 36 showing the spring velocity as an input to the sprung and un-sprung mass when sudden uneven conditions arrived. Here it can be seen the spring velocity is upward till 0.6 m/sec and downward till 1.05 m/sec. the suspension spring vibrations are resting after 3 seconds.



# *Chapter 6- System Mathematical Model and Bond Graph*

## 6.1 Introduction

This chapter is about to model the bond graph of an active hydraulic suspension system and its different parts. The initial model idea was adopted from *chapter-3* of [47]. Where the author constructed a bond graph for “*Hydraulic Servo-Actuator system*” which modified for active suspension system in this project. This system has the following parts: reservoir, pump, accumulator, Bypass valve, Spool valve, hydraulic cylinder-piston and suspension system. This thesis mainly focusing on the quarter car model of suspension where sprung mass and un-sprung mass are considered. Figure 37 shows the layout diagram of the hydraulic active suspension system.



*Figure 37- Layout of the System*



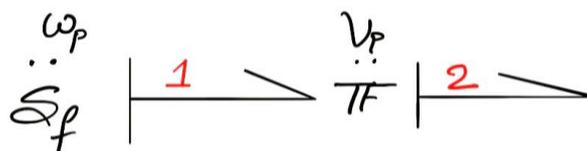
## 6.2 Bond graph of the Different Parts

As hydraulic suspension system consists of different parts and processes of energy conversion. So, it would be great to model every single part separately and combine them according to the system requirements. The following conditions describe the different parts, their brief theory, bond graph and elements equations.

### 6.2.1 Positive Displacement Pump (PDP)

Li, et al. [51] mentioned, to convert mechanical rotational energy into hydraulic energy the positive displacement pump used. In the bond graph, it is modelled as a hydraulic power source and used as a *Source of Flow (Sf)*. It is assuming that the rotations of pump constant and steady. The *Transformer TF* is converting the rotations speed of the pump shaft into the required flow rate of the fluid. The  $V_p$  is the pump displacement and used as *transformer modulus*. From figure 38, the following relations could be derived between *effort* and *flow* of the input and output for the *TF* elements.

*Removed due to copyright restriction*



**Figure 38- Positive Displacement Pump and Bond Graph**

$$f_2 = f_1 * V_p$$

$$e_1 = e_2 * V_p$$

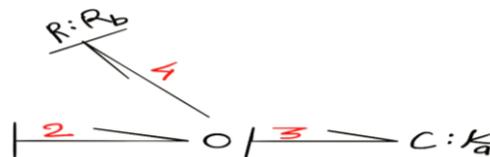
Here,  $e_1$  and  $f_1$  are showing the pressure and flow rate of the PDP and  $e_2$  and  $f_2$  are for the torque and angular velocity of the motor.



### 6.2.2 Accumulator and Bypass Valve

The accumulator is used to add compliance to the hydraulic system. It is also used to capture the potential and kinetic energy which is dispersed in the form of heat. This modelled system was used as a pre-charged gas accumulator. Due to the high pressure and flow rate, it should be necessary to add bypass valve to the system. It could be activated when the pressure inside the system is reaching the set limit of pressure. Figure 39 shows the *compliance* ( $C:K_a$ ) element for accumulator gives the pressure to the system that is taken from the pump and maintain it to the required limit. While the bypass valve is represented by the *nonlinear resistance* ( $R:R_b$ ). Due to the quick response time compared to the other hydraulic component, the dynamics of the bypass valve can be neglected [51]. To get the proper result from the bypass valve is required to maintain a large number of resistances which can block the flow from it. For this required to used orifice equation-2 to get the resistance equation.

*Removed due to copyright restriction*



*Figure 39- Gas Accumulator, Bypass Valve and Bond Graph*

### 6.2.3 Spool Valve (Direction Control Valve)

To control the direction of the flow rate of the hydraulic system the direction control valve is the best option. As from figure 40, the supply of pressurised fluid from the accumulator is connected to the *Pump* ( $P$ ) port while *Tank* ( $T$ ) port which return the fluid into the tank or reservoir. The port  $A$  (or  $1$ ) and  $B$  (or  $2$ ) are connected with the respective side of the hydraulic cylinder which placed between the body and axle mass [47]. According to Li, et al. [51], this

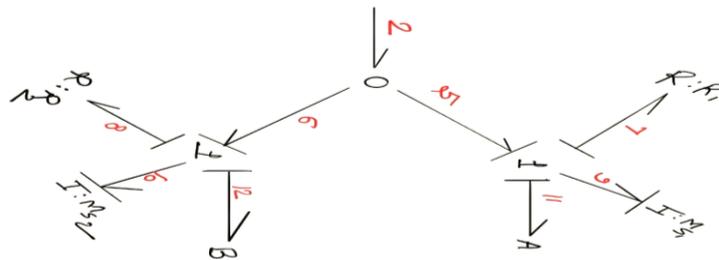


spool valve has only two working positions: left and right. These locations are maintained by the controller which get the feedback of locations of the sprung and unsprung mass. When its location in the left side, the flow of the fluid is directed from port  $P$  to  $B$  (or 2) and return from port  $A$  (or 1) to  $T$  and this changed wise versa when it is for the right working position. This positions can be implemented from model the *four resistance* ( $R_1, R_2, R_3$  and  $R_4$ ) elements in the bond graph. To model this resistance on the spool it is considered as an orifice and used orifice equation-2. If consider the flow from  $P$  to  $A$  is closed then it should be essential to set the sufficiently large number of respective *resistance* ( $R_1$ ) that can block the flow from this direction.

**Orifice Equation:**

$$Q = A * C_d \sqrt{\frac{2\Delta P}{\rho}} \quad (1)$$

*Removed due to copyright restriction*



**Figure 40- Spool Valve (Direction Control Valve) and Bond Graph [47]**

**6.2.4 Hydraulic Cylinder**

This analysis used a linear hydraulic cylinder which is shown in figure 38. In the hydraulic system, it is the primary aim to use a cylinder to convert hydraulic energy into mechanical



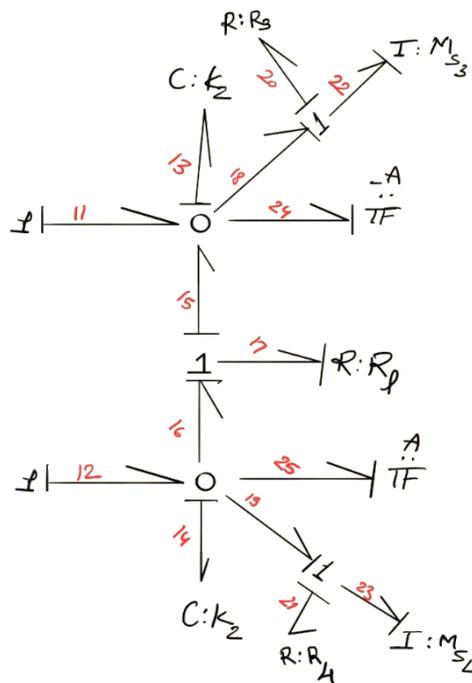
energy. Conversion of this phenomenon *Transformer TF* used in the bond graph and piston area  $A$  is the *transformer module*. As indicated the figure 38, the *port 11 and 12* are the inlet and outlet of the cylinder when spool moves in the right direction and it is vice versa inlet direction. The internal loss into the cylinder is modelled as a *resistance  $R_l$*  element while the two *Compliance  $C_1$*  and  $C_2$  are used to present the fluid compressibility in two chambers. This compliance would be change because the volume of chambers will be changed due to piston movement. The following are the *Transformer* element equations.

$$e_{26} = A * e_{24}$$

$$f_{24} = A * f_{26}$$

Here, *Force* =  $e_{26}$ , *Pressure* =  $e_{24}$ , *Flow rate* =  $f_{24}$  and *Velocity* =  $f_{26}$ .

*Removed due to copyright restriction*



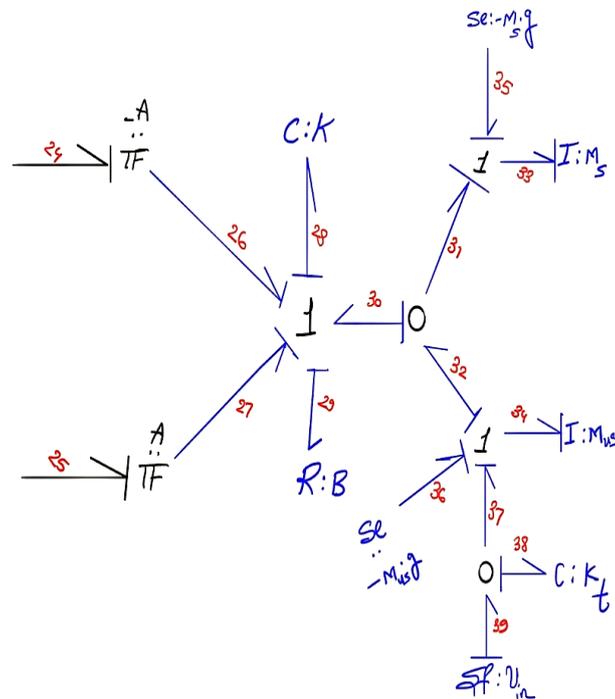
**Figure 41- Hydraulic Actuator and Bond Graph**



### 6.2.5 Suspension System (1/4 Model)

To analyse the suspension system single tyre would be the best. Figure 42 is the passive suspension system which includes the body mass  $M_s$ , axle mass  $M_{us}$ , spring  $K$ , damper  $B$ , Tyre stiffness  $K_t$ . The hydraulic part which is force generation device is connected with this passive suspension system to get better isolation for body and axle mass. Here, piston mass is considered with body mass because the cylinder is connected with it. The road velocity is considered as a source of flow for the system. Two sources of efforts are  $-M_s g$  and  $-M_{us} g$  acting on the sprung mass and unsprung mass, respectively. Both mass are considered as *inertia* element, suspension spring and Tyre stiffness are consider as *compliance* element and *resistance* for a damper for the bond graph.

*Removed due to copyright restriction*



**Figure 42- Quarter Car Suspension System and Bond Graph**



### 6.3 Bond Graph of the Whole System

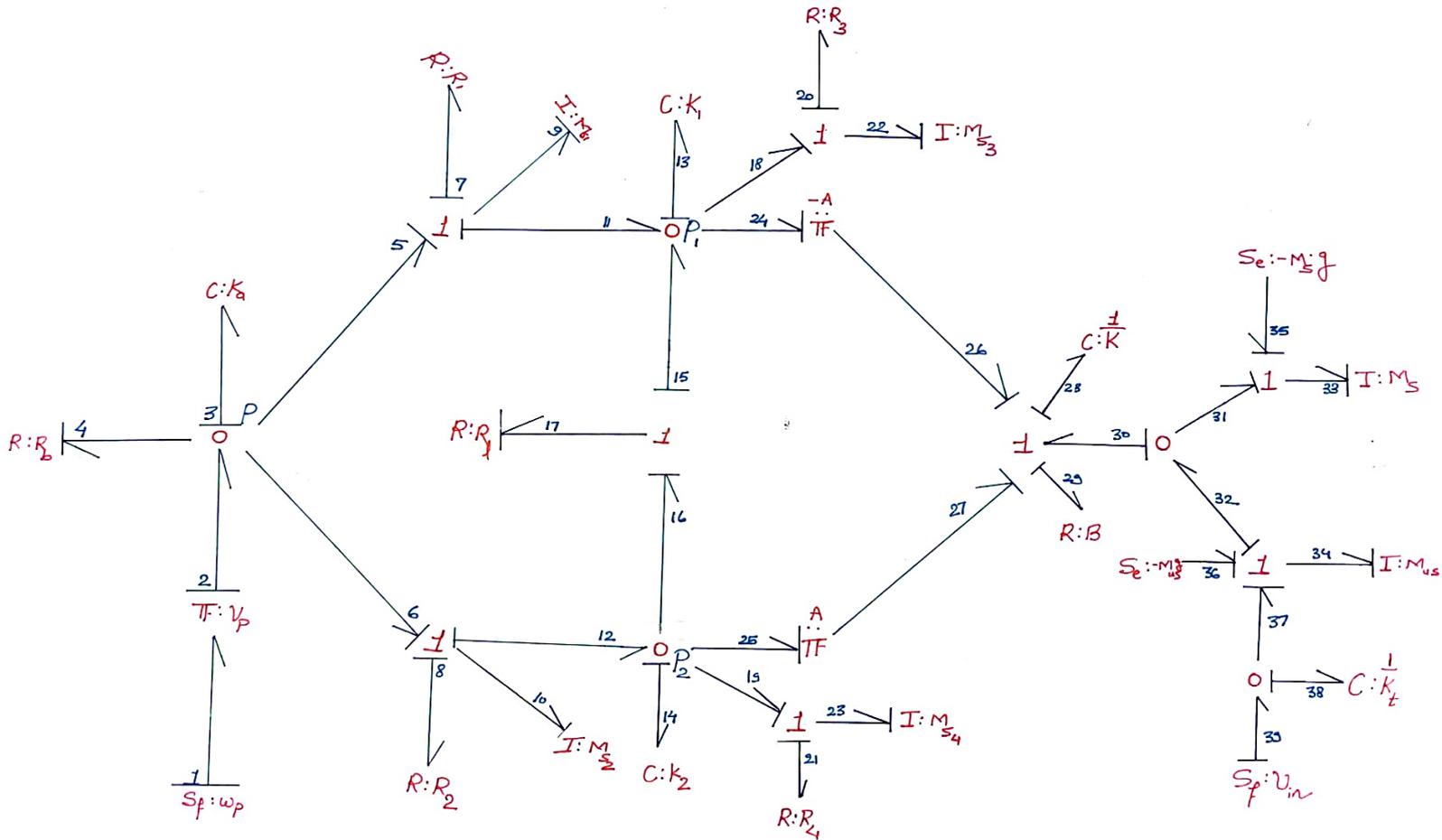


Figure 43- Full System Bond Graph

### 6.4 State Space Equations of the System

The state-space equations are derived from figure 43 for the whole system. Two basic steps would be followed to derive the equations. From the bond graph eleven equation could be derived for:  $C_3, I_9, I_{10}, C_{13}, C_{14}, I_{22}, I_{23}, C_{28}, I_{33}, I_{34}$  and  $C_{38}$ . The state-space equation of all compliance of hydraulic system gives the *Flow rate* ( $\dot{q} = f = Q$ ) and *compliance* of suspension system gives *Velocity* ( $\dot{q} = f = v$ ). While the *Inertance* element gives the *Pressure* ( $\dot{p} = e = P$ ) for the hydraulic system and *Force* ( $\dot{p} = e = F$ ) for the suspension system.



## 6.4.1 The Element Gives to the System

### 6.4.1.1 Source of flow and efforts

1. The rotational speed of the pump shaft- *Sf*:  $\omega_p$
2. Road velocity- *Sf*:  $v_{in}$
3. The gravitational force acting on sprung mass- *Se*:  $-M_s g$
4. The gravitational force acting on un-sprung mass- *Se*:  $-M_{us} g$

### 6.4.1.2 Transformers

1. For pump-  $Q_2 = \omega_p * V_p$
2. For cylinder (for port A and B)-

$$F_{26} = -A * P_{24} \text{ and } Q_{24} = -A * v_{26}$$

$$F_{27} = A * P_{25} \text{ and } Q_{25} = A * v_{27}$$

### 6.4.1.3 Compliance of Accumulator

$$P_3 = q_3 / C_3$$

### 6.4.1.4 The Resistance of Bypass Valve

$$Q_4 = R_4 * P_3$$

### 6.4.1.5 Spool Valve

1. The inertia of fluid flow from port one to the inlet of chamber A

$$Q_9 = p_9 / I_9$$

2. The inertia of fluid flow from port two to the inlet of chamber B

$$Q_{10} = p_{10} / I_{10}$$

3. Bernoulli resistance of port one when fluid flow to chamber A



$$P_7 = R_7 * Q_9$$

4. Bernoulli resistance of port two when fluid flow to chamber B

$$P_8 = R_8 * Q_{10}$$

#### 6.4.1.6 Cylinder

1. The inertia of fluid flow to the sump from chamber A

$$Q_{22} = p_{22} / I_{22}$$

2. The inertia of fluid flow to the sump from chamber B

$$Q_{23} = p_{23} / I_{23}$$

3. Bernoulli resistance of fluid flow to the sump from chamber A

$$P_{20} = R_{20} * Q_{22}$$

4. Bernoulli resistance of fluid flow to the sump from chamber B

$$P_{21} = R_{21} * Q_{23}$$

5. Compressibility of the fluid in chamber A

$$P_{13} = q_{13} / C_{13}$$

6. Compressibility of the fluid in chamber B

$$P_{14} = q_{14} / C_{14}$$

7. The resistance of leakage in the cylinder

$$Q_{17} = (P_{14} - P_{13}) / R_{17}$$



### 6.4.1.7 Suspension System

1. Compliance of spring

$$F_{28} = q_{28} / C_{28}$$

2. Compliance of Tyre stiffness

$$F_{38} = q_{38} / C_{38}$$

3. The inertia of the sprung mass

$$v_{33} = p_{33} / I_{33}$$

4. Inertia of un-sprung mass

$$v_{34} = p_{34} / I_{34}$$

5. Resistance of damper

$$F_{29} = R_{29} * (v_{34} - v_{33})$$

### 6.4.2 Taken from the System

1. Accumulator:

$$Q_3 = \dot{q}_3 = Q_2 - Q_4 - Q_9 - Q_{10}$$

2. For Spool Valve:

$$P_9 = \dot{p}_9 = P_3 - P_7 - P_{13}$$

$$P_{10} = \dot{p}_{10} = P_3 - P_8 - P_{14}$$

3. For Cylinder:

$$Q_{13} = \dot{q}_{13} = Q_9 + Q_{17} - Q_{22} + A * (-v_{34} - v_{33})$$

$$Q_{14} = \dot{q}_{14} = Q_{10} - Q_{17} - Q_{23} - A * (-v_{34} - v_{33})$$

$$P_{22} = \dot{p}_{22} = P_{13} - P_{20}$$

$$P_{23} = \dot{p}_{23} = P_{14} - P_{21}$$



## 4. For Suspension:

$$v_{28} = \dot{q}_{28} = v_{34} - v_{33}$$

$$v_{38} = \dot{q}_{38} = v_{in} - v_{34}$$

$$F_{33} = \dot{p}_{33} = F_{28} + F_{29} + F_{26} + F_{27} - F_{35}$$

$$F_{34} = \dot{p}_{34} = F_{38} - F_{36} - F_{28} - F_{29} - F_{26} - F_{27}$$

## 6.5 Hydraulic Equations

## 1. Compliance Elements:

$$C = V/E$$

## 2. Inertance (Inertia) Elements:

$$I = \rho l/A$$

## 3. Resistance Elements:

From orifice equation 1

$$R = 2 * (C_d * A)^2 / \rho$$

## 4. The resistance of Leakage in Cylinder:

$$R = (12 * u * L_p) / (\pi * D_p * C_c^3)$$

## 6.6 Assumptions

During developing a bond graph of the system, the following assumptions are taken.

- The *Positive Displacement Pump* supplied a constant pressure and flow rate of the fluid throughout the process.



- It is assumed a gauge pressure ( $P_{res} = 0$ ) for the reservoir.
- Assumed no leakage in the pump.
- Assumed laminar flow for leakage through the clearance between the piston and cylinder.
- The bandwidth of the direction control valve is higher than the bandwidth of the cylinder and symmetric [52].
- Pressure from spool outlets to inlets of the cylinder was assumed to be negligible.
- Due to improvement in sealing technology, it is assumed no external leakage from the piston rod.
- Assumed negligible viscous friction between the piston rings because of the proper lubrication techniques.
- The cylinder is assumed to be rigid therefore no compliance elements acting on cylinder walls. This is because the cylinder stiffness is more than of the flowing fluid.
- The used fluid is ideal, incompressible and non-viscous.
- Due to considering ideal geometry of valve, sharp edges of orifice and zero-clearance assumed no internal leakage [52].

## 6.7 Used Parameters

Table 10 is about to mention the used parameters' name, values and unit. These parameters are taken from [51] [52] [53] and [54].

*Table 10- Parameters*

<b>Parameter</b>	<b>Value</b>	<b>Unit</b>
Driving Speed of the Pump Shaft	1470	rpm
Displacement of the Pump	3.8038e-6	m <sup>3</sup> /rad
Density of Fluid	850	kg/m <sup>3</sup>
Coefficient of Discharge	0.61	-
Bulk Modulus of Fluid	1.6e9	Pa (N/m <sup>2</sup> )
Stiffness of Suspension Spring	16812	N/m
Stiffness of the Tyre	190000	N/m
Damping Rate	1000	N sec/m
Car Body Mass (Sprung Mass)	290	kg
Wheel Mass (Unsprung Mass)	59	kg
Piston Mass	2.5	kg
Gravitational Constant	9.81	m/sec <sup>2</sup>
Bulk Modulus of Air Inside the Accumulator	1.2e9	N/m <sup>2</sup>
Accumulator Volume	1.13e-4	m <sup>3</sup>
Volume of Cylinder Chamber A	5.30e-4	m <sup>3</sup>
Volume of Cylinder Chamber B	5.85e-4	m <sup>3</sup>
Diameter of Piston	0.0517	m
Length of Piston	0.050	m
Coefficient of Viscosity	1e-3	N sec/m <sup>2</sup>
Clearance between Piston and Cylinder	8.3e-5	m
Stroke Length	0.35	m
Road Velocity Input	60	km/hr



# *Chapter 7- MATLAB and Simulink*

## *Model*

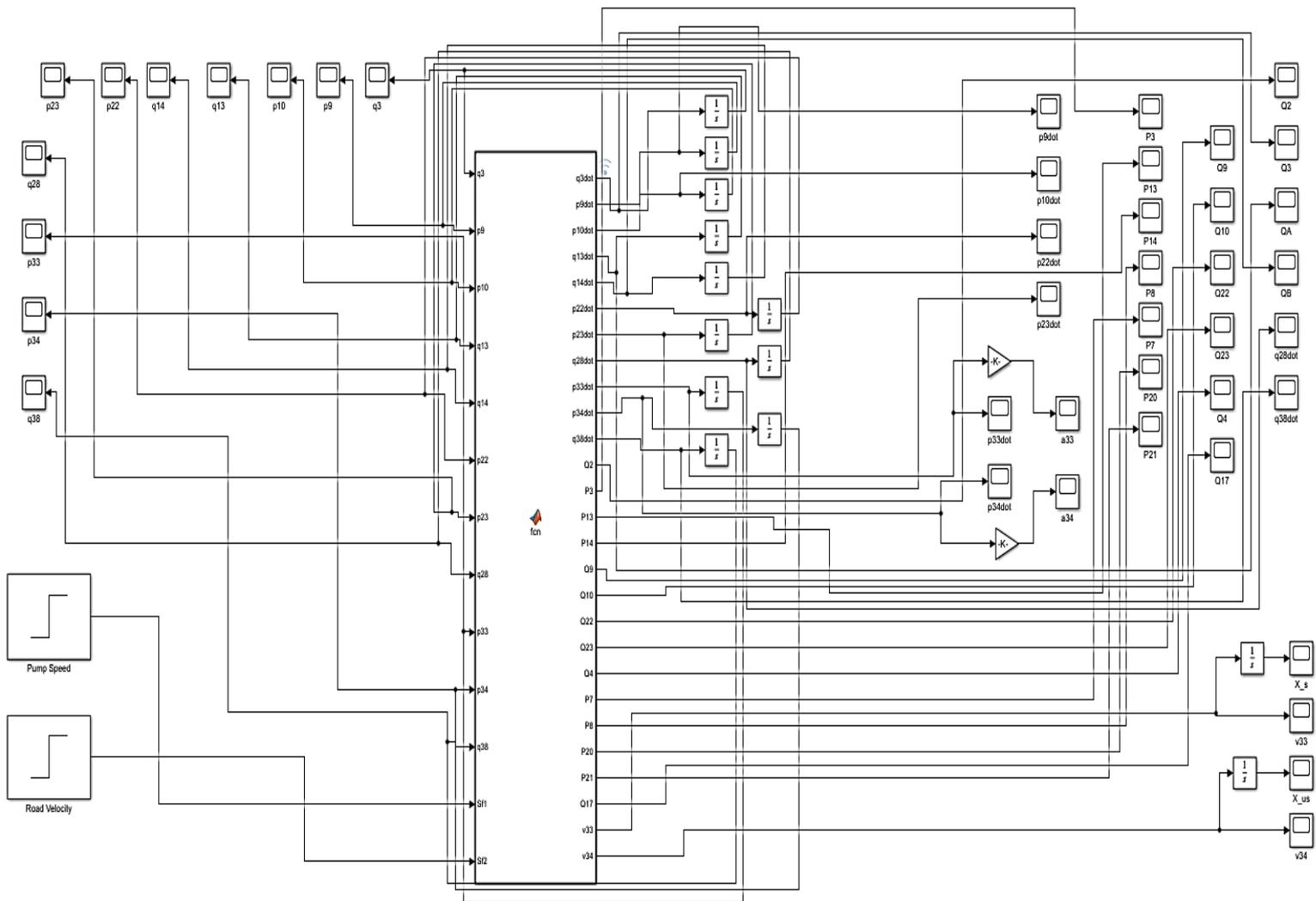


## 7.1 Introduction

According to Chaturvedi [55], the MATLAB is the computer-based high-performance language which is a combination of the visualisation and computations has developed by MathWorks in 1984. The programming of any system in MATLAB/Simulink is easy where both problems and results are expressed in the mathematical symbols. While the Simulink is the subsection of the MATLAB software which is used to model, analyse and simulate any dynamic system. It is providing the graphical interface to construct any model with the help of *Library* where ready-to-use blocks are available [55]. To analyse the modelled system in MATLAB/Simulink user-defined functions would be the best option. As the name indicated that it is created from the user and used it as a build-in function. It has mainly two parameters: input arguments and output variables. It could be any size of matrices, vectors and scalars. Input and output parameters could be any number including zero. This used the input parameters to perform the calculation and transferred out from the output parameters. Figure 44 shows the user-defined function of *Hydraulic Active Suspension System* and coding of the model is written in the appendices. To develop a full model following library blocks are used.

### 7.1.1 Step Source

To provide the input to any system the step block is the easy and best source block. This block provides the step between two definable levels at a definite period. Here model the driving speed of motor and road velocity in this system step source used. Where the driving speed of the pump starts from zero and goes to the 153.938 rad/sec (1470 rpm). While the road velocity from zero to 60 km/hours.



**Figure 44- MATLAB/Simulink Model of Hydraulic Active Suspension System**

### 7.1.2 Integrator Block

As the name indicated that to integrate any output parameter integrator block is used in Simulink. The Integrator block yields the estimation of the vitality of its information signal as for time. In figure 44 the integrator block used to integrate the output of the *user-defined function* and reconnect it with the input of the user-defined function.

### 7.1.3 Gain Block

To multiply or divide the input which comes from the user-defined function gain block is used. The gain and input value can be vector, matrices or scalar. In figure 44 the gain block is used



to divide the values of the sprung mass and un-sprung mass with the output of the user-defined function to plot the graphs of acceleration of sprung and un-sprung mass, respectively.

#### 7.1.4 Scope Block

To display any result of the system Scope block is used. It can be multiple axes which are one per port. It is allowed to change the time and input values which are required to show. In figure 44 scopes are used to show graphs of the different results of flow rate, pressure, acceleration, velocity and displacement parameters.



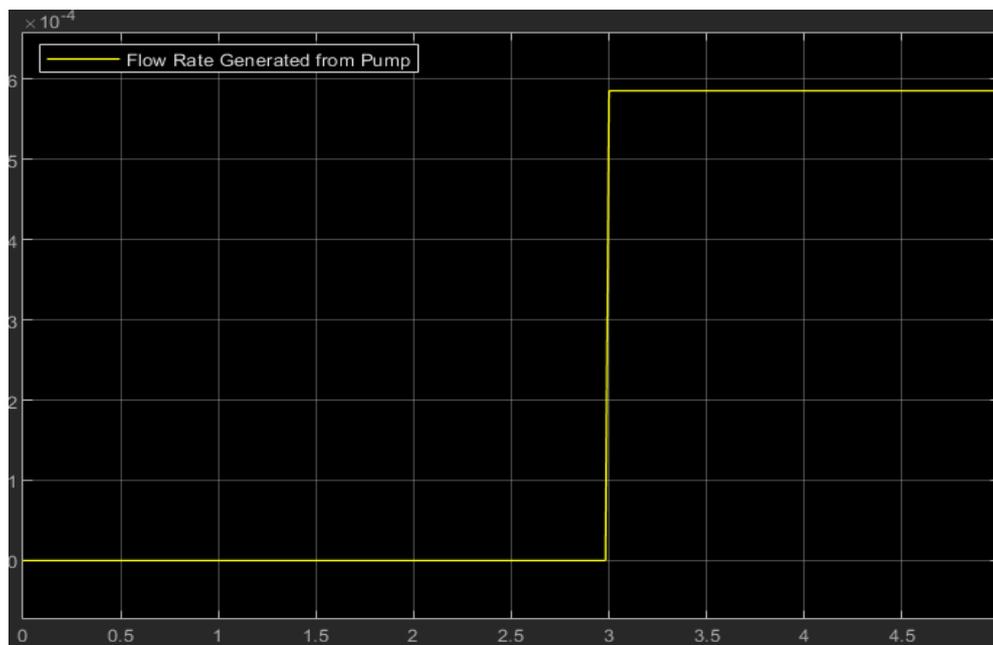
## *Chapter 8- Result and Discussion*

## 8.1 Results and Discussions

To primary aims of analysis to investigate and improve the behaviour of any system. The main motive of any physical system to transfer the *Power* between components. Here, used physical method of analysis can have more benefits over others. Bond graph method is the graphical representation to the model of any system where the different ports are connected with bonds that shows the energy transfer between same domain systems or it can be between multi-domain system [56]. Also, its graphical nature can make more visualising and easier to understand the complex system like *Hydraulic Active Suspension System*. From bond graph technique it could be easily derived the *State Space Equations* of any system. Also, modelling with the use of bond graph can provide ways to the designer to adjust different parameters and easily add more elements later to do advancements. The following paragraphs will present the results and discussions of the hydraulic and suspension parts graphs.

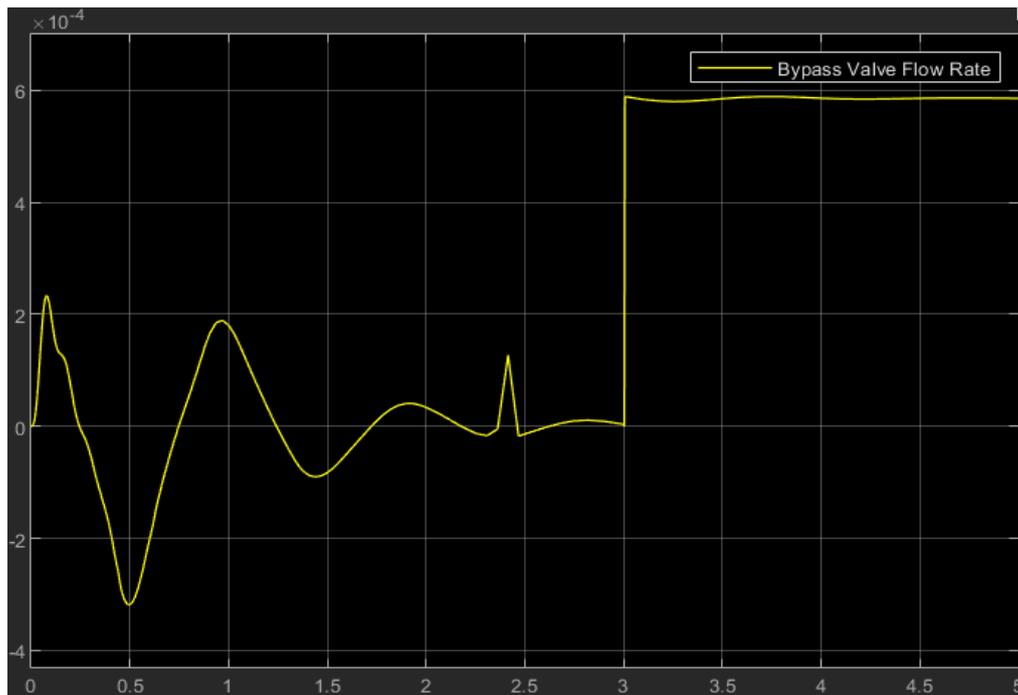
### 8.1.1 For Hydraulic Part

#### 8.1.1.1 Pump, Bypass Valve and Accumulator



*Figure 45- Flow Rate Generated from the Pump*

Figure 45 shows the flow rate generation from a positive displacement pump which starts to rotate from zero and continuously rotate at 1470 rpm. As it can be seen in figure 45, at 1470 rpm (from the  $S_f$  as a step function source) the pump generating enough flow rate that required to maintain the proper input for the actuator. Here, the pump generated the 0.6 litres per second flow rates.



*Figure 46- Bypass Valve Flow Rate*

Figure 46 shows that the flow rate of the bypass valve, where it fluctuated and increased by 0.6 litres per second when the system load increased. The bypass valve working is depending on the pressure fluctuation in the spool valve. Because if flowrate and pressure values are more than the spool valve working values than the access flowrate and pressure can be released from the bypass valve.

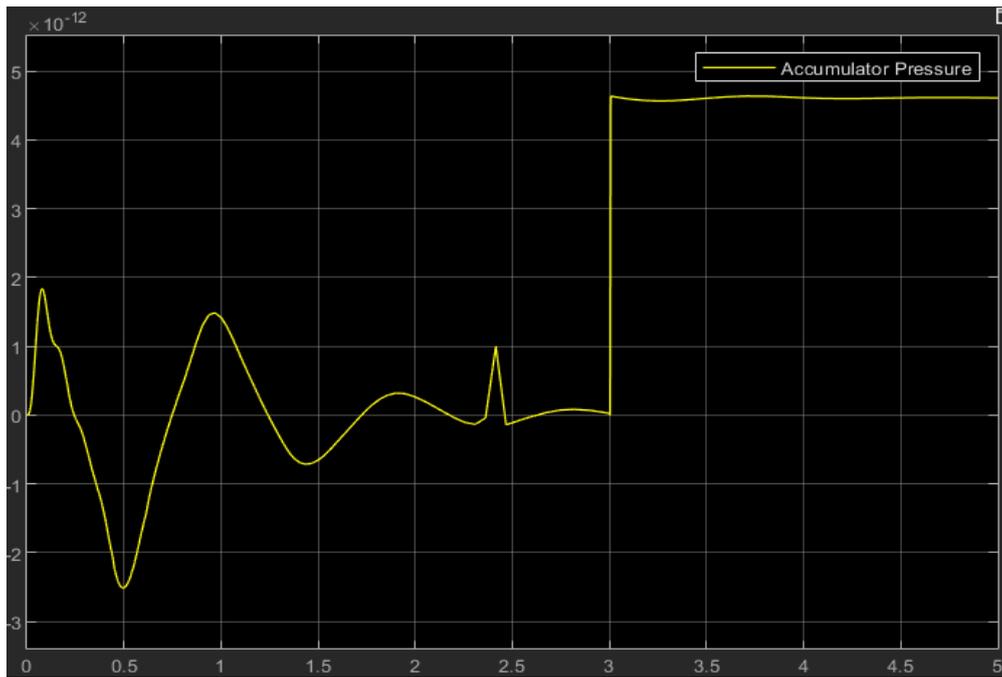


Figure 47- Accumulator Pressure

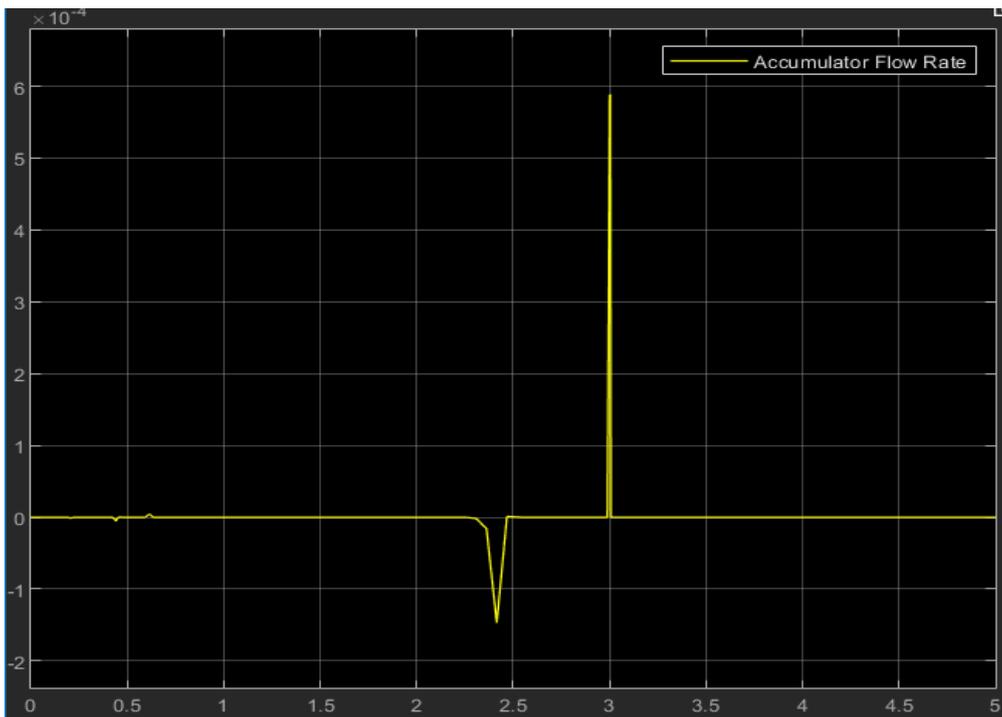


Figure 48- Accumulator Flow Rate

Figure 47 and 48 displays the behaviour of the accumulator. it shows that the flow rate and pressure fluctuations happened when the sudden load is coming to suspension and the



accumulator increasing the flow rate (maximum 6 litres/second) and back to normal when no load on the suspension. The inside pressure fluctuation is very nominal (approximately  $4.5 \times 10^{-12}$  Pa) which shows that the accumulator works correctly and remove access pressure to maintain constant pressure into the whole system.

### 8.1.1.2 Spool Valve (Direction Control Valve)

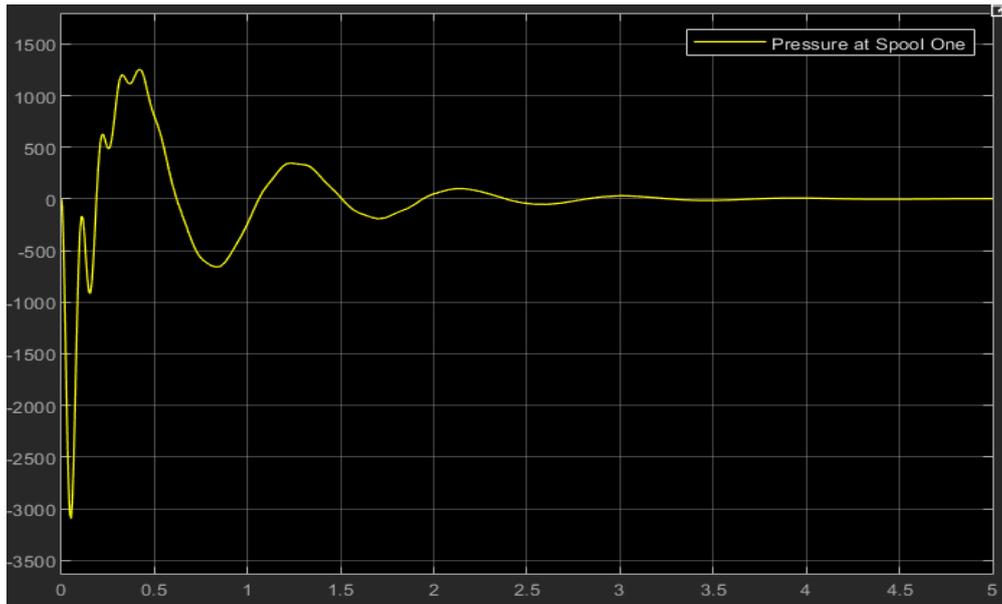


Figure 49- Pressure at Spool One

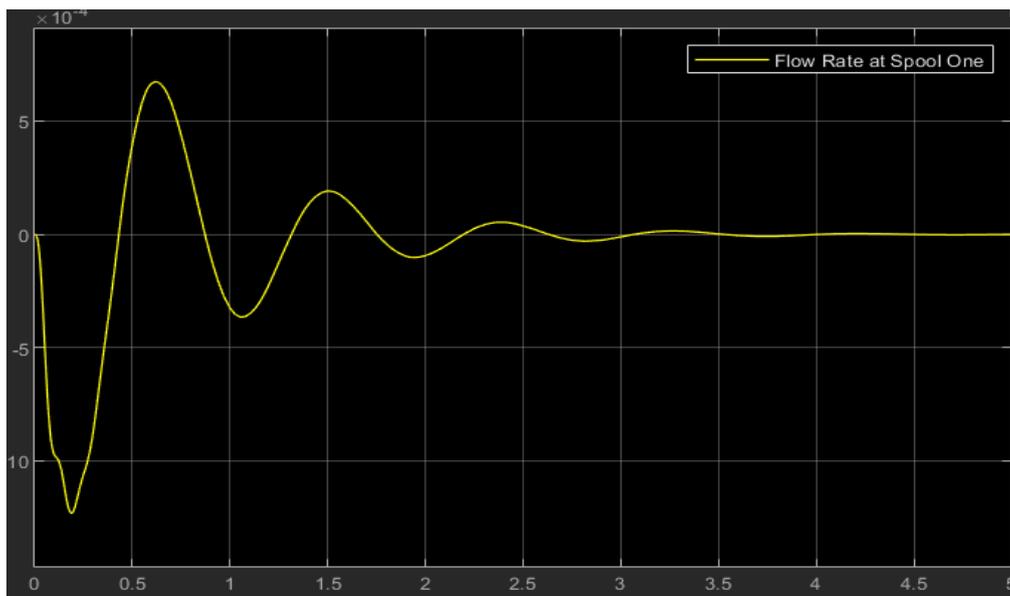


Figure 50-Flow Rate at Spool One

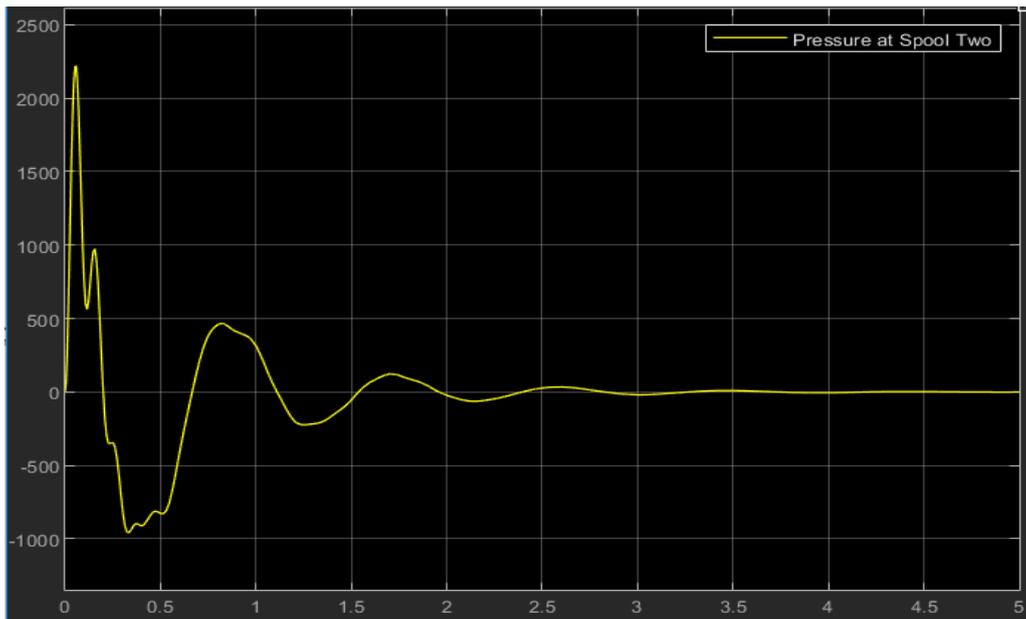


Figure 51- Pressure at Spool Two

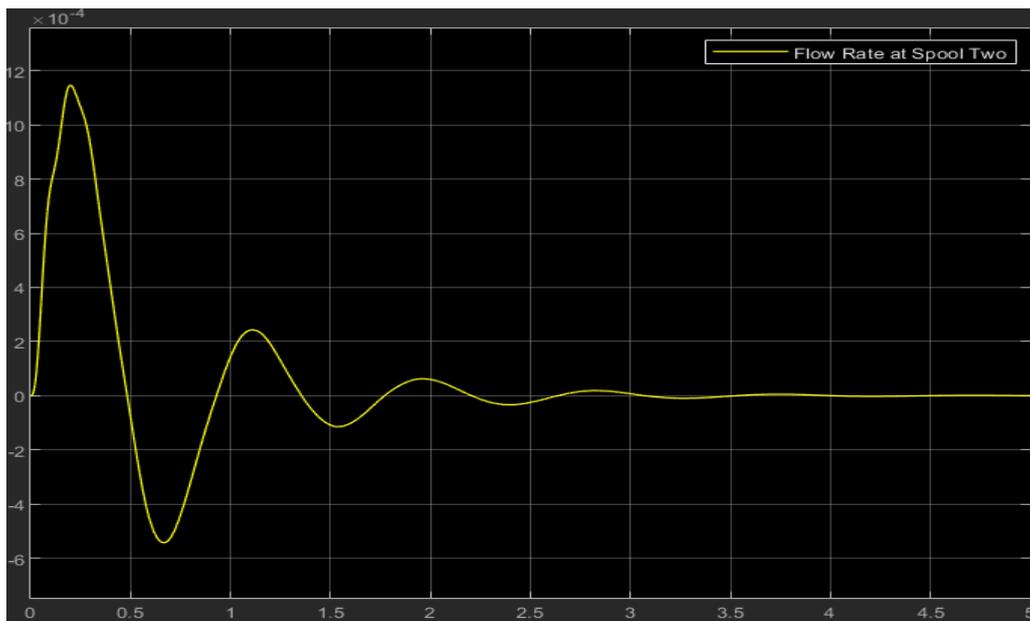


Figure 52- Flow Rate at Spool Two

Figure 49, 50, 51 and 52 show the pressure and flow rate of the spool valve when the flow is followed the path to the *cylinder chamber A and B*, respectively. Figures 51 and 52 indicated that if spool valve followed the left side position then the pressure (2200 Pa) and flow rate (1.2 litres/second) are increasing at some point according to the requirement of suspension input



and went back to normal. While the figure 49 and 50 show the opposite trend for pressure (3000 Pa) and flow rate (15 litres/second) is on the negative side of the graph and went back to normal after 4 seconds.

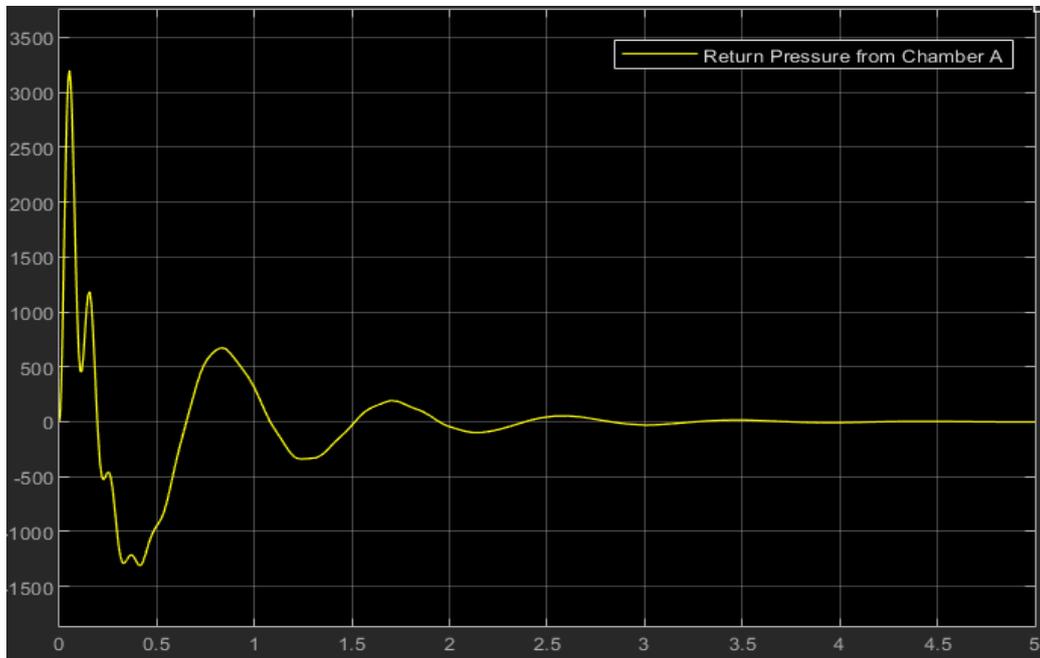


Figure 53- Return Pressure to Sump from Chamber A

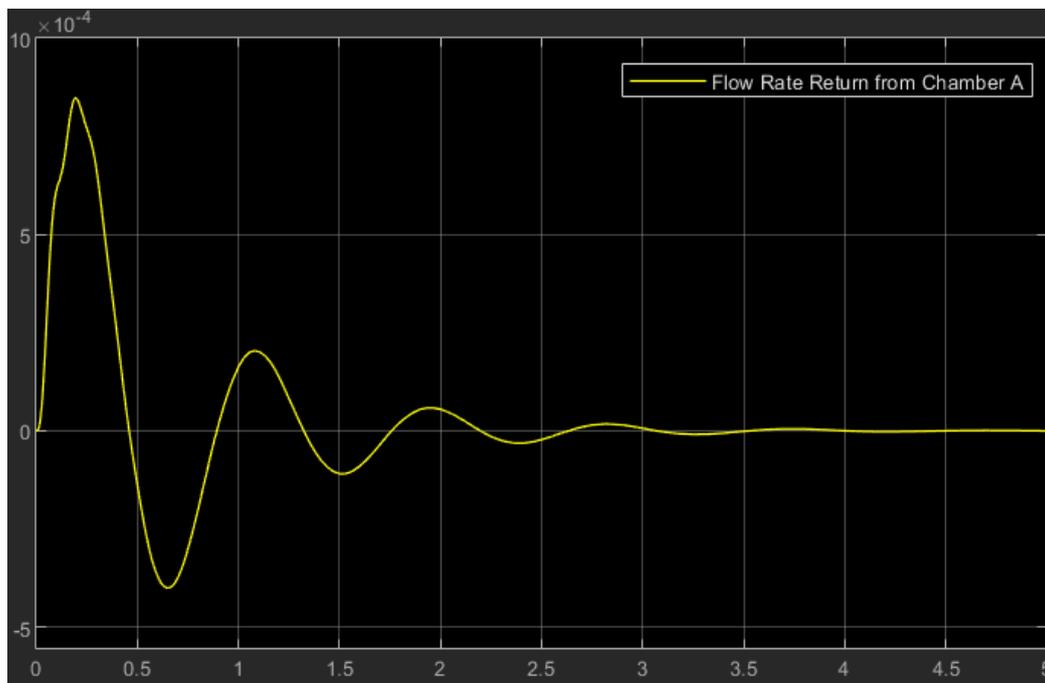
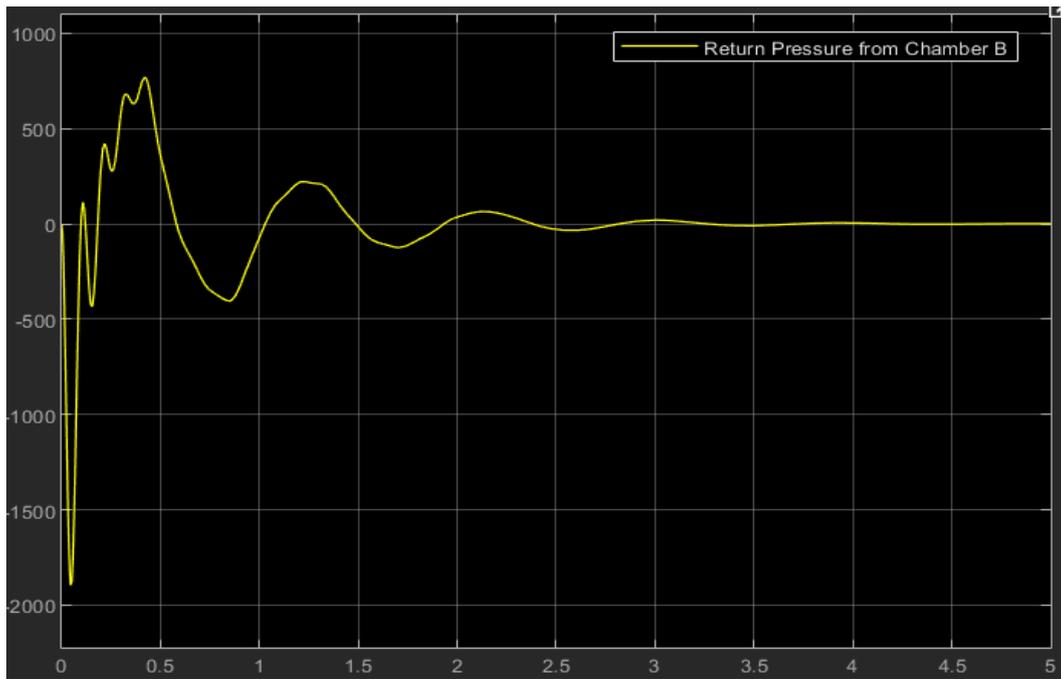
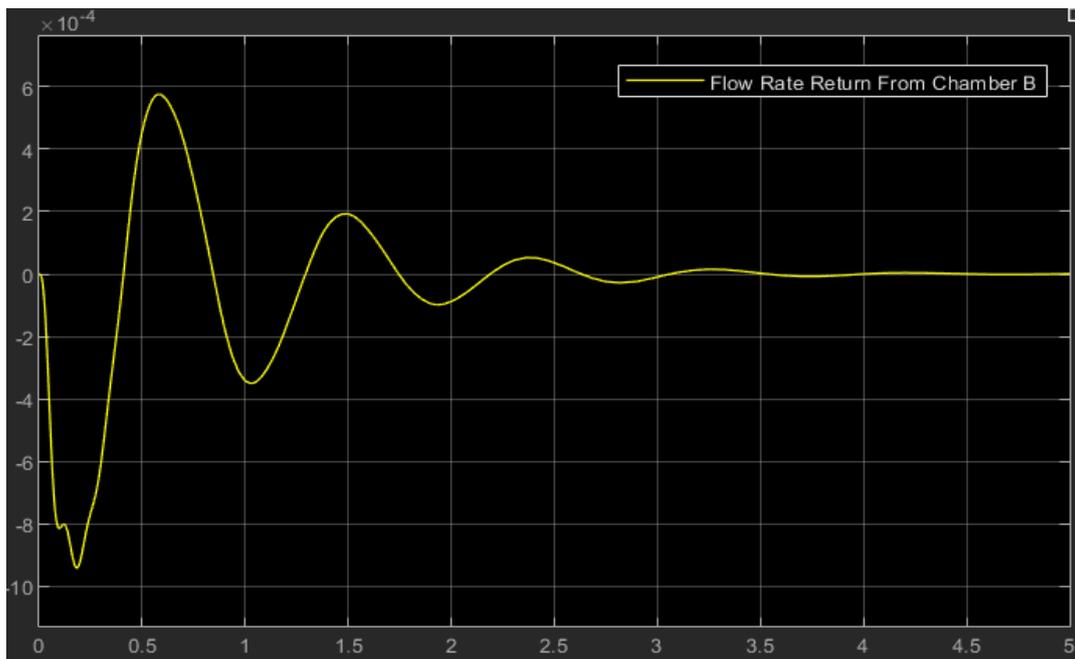


Figure 54- Return Flow Rate to Sump from Chamber A



*Figure 55- Return Pressure to Sump from Chamber B*



*Figure 56- Return Flow Rate to Sump from Chamber B*

The pressure and flow rate for return to sump form the chamber A and B are showing in figure 53, 54 and 55, 56, respectively are shown the return to sump behaviour of the flowrate and pressure.



### 8.1.1.3 Cylinder

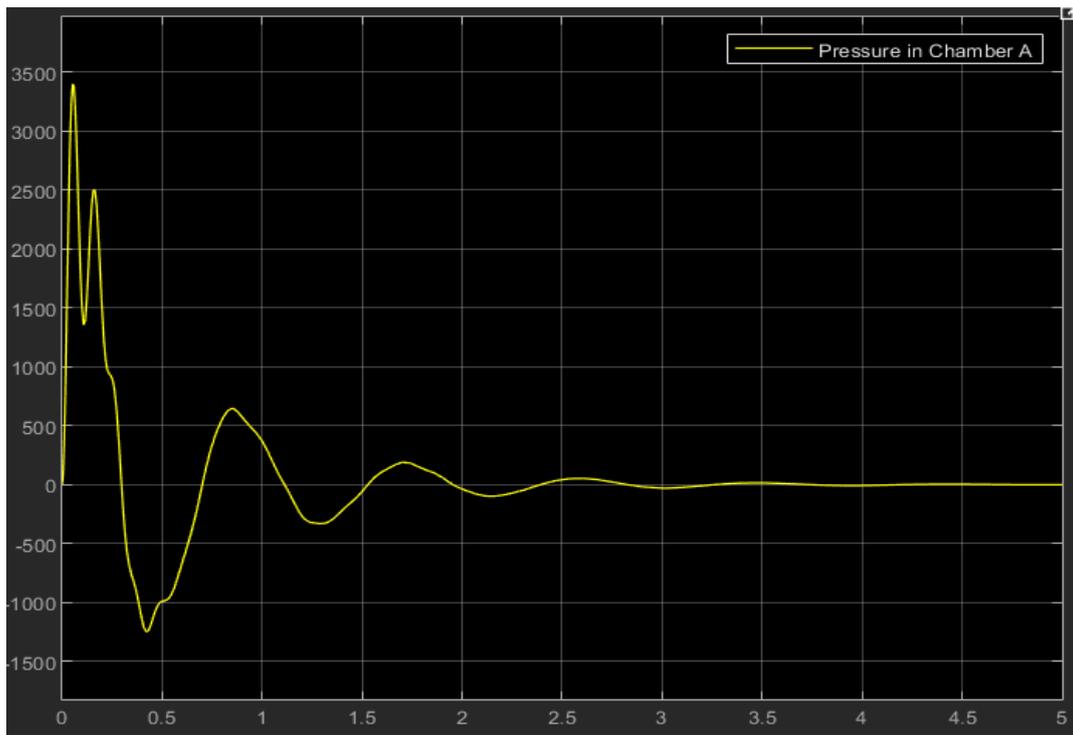


Figure 57- Pressure in Chamber A

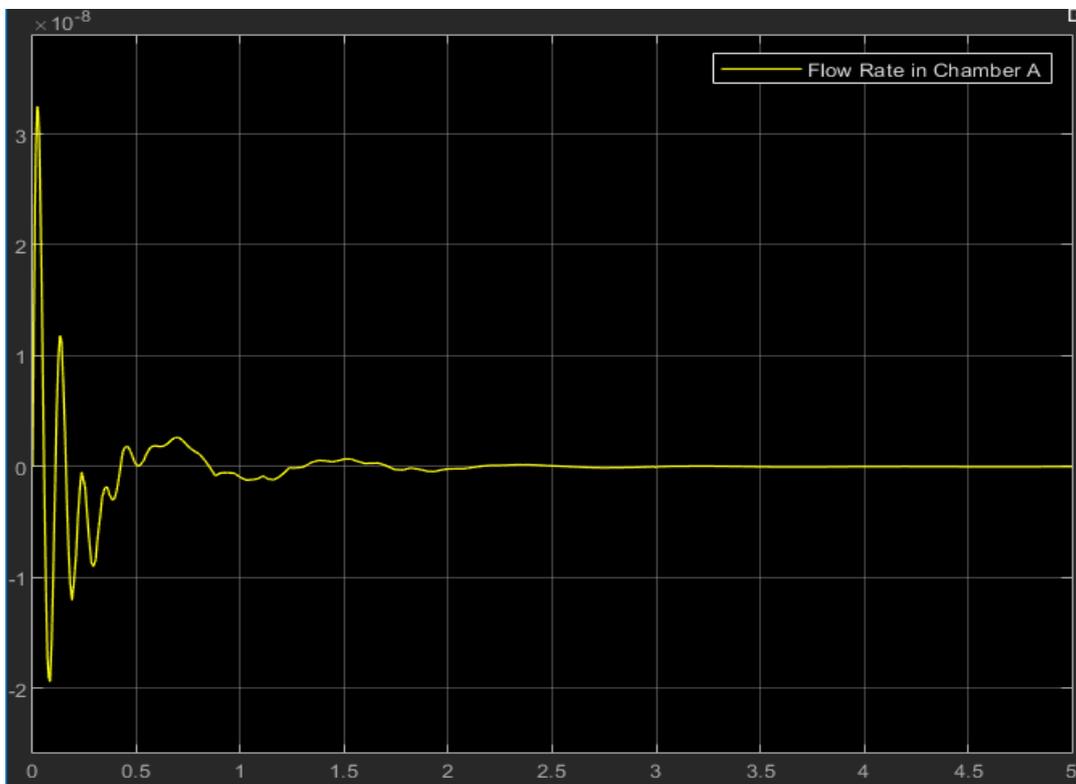


Figure 58- Flow Rate in Chamber A

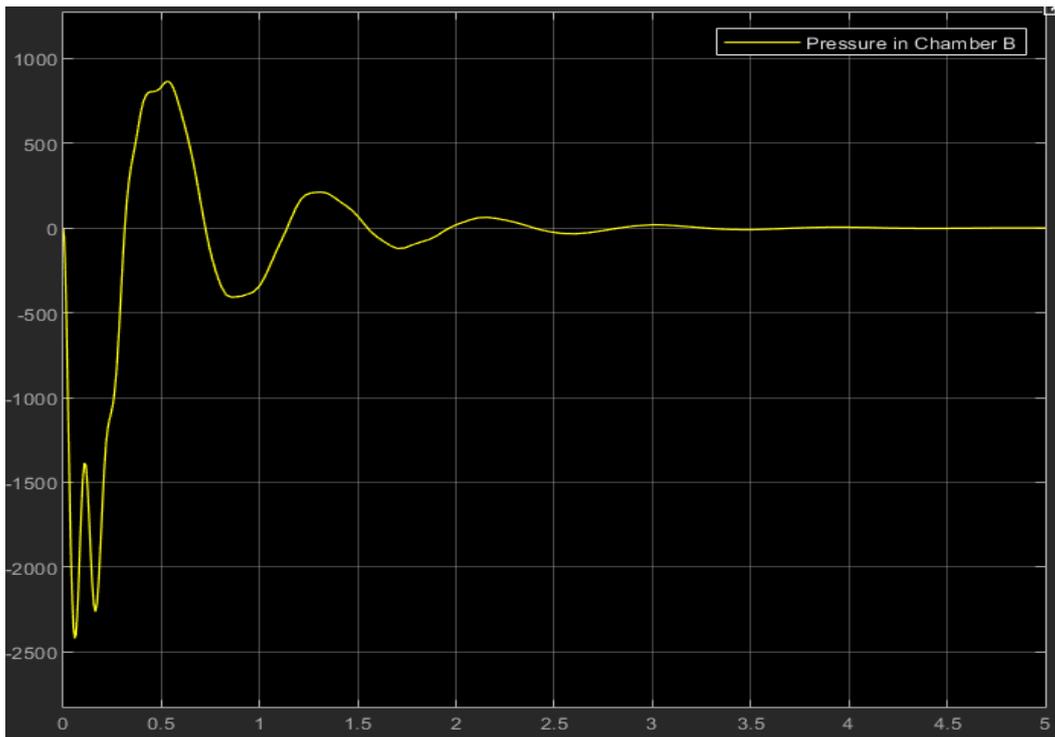


Figure 59- Pressure in Chamber B

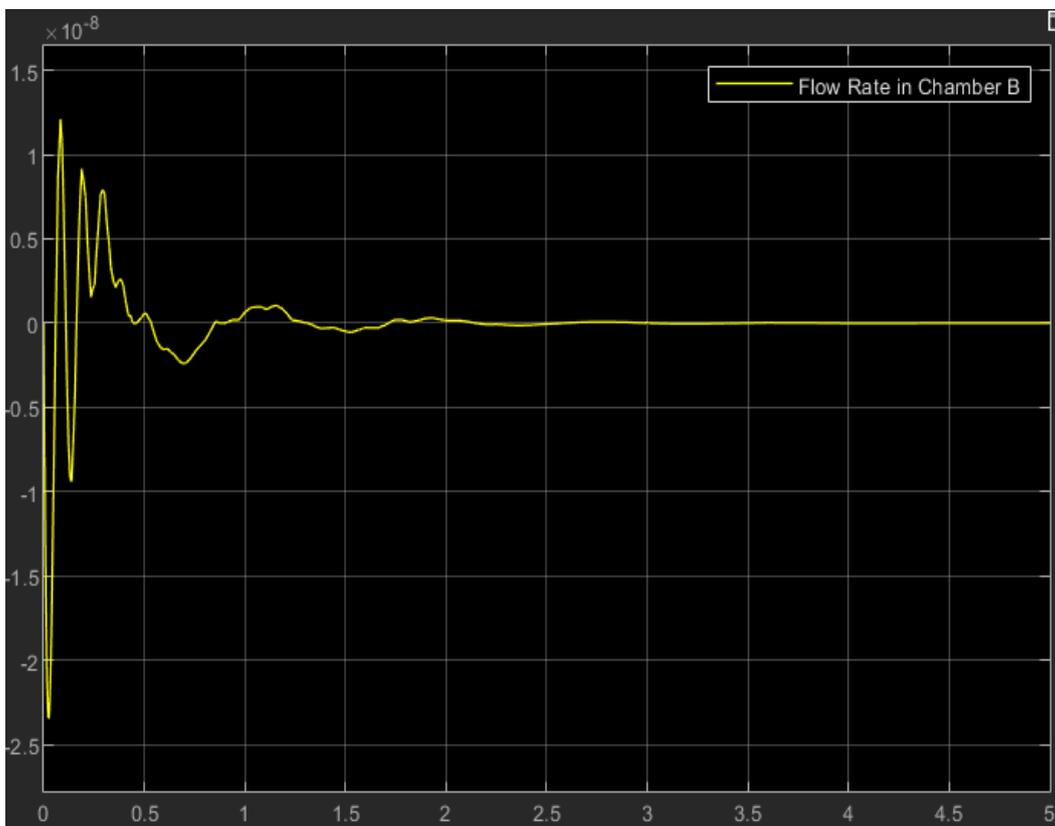
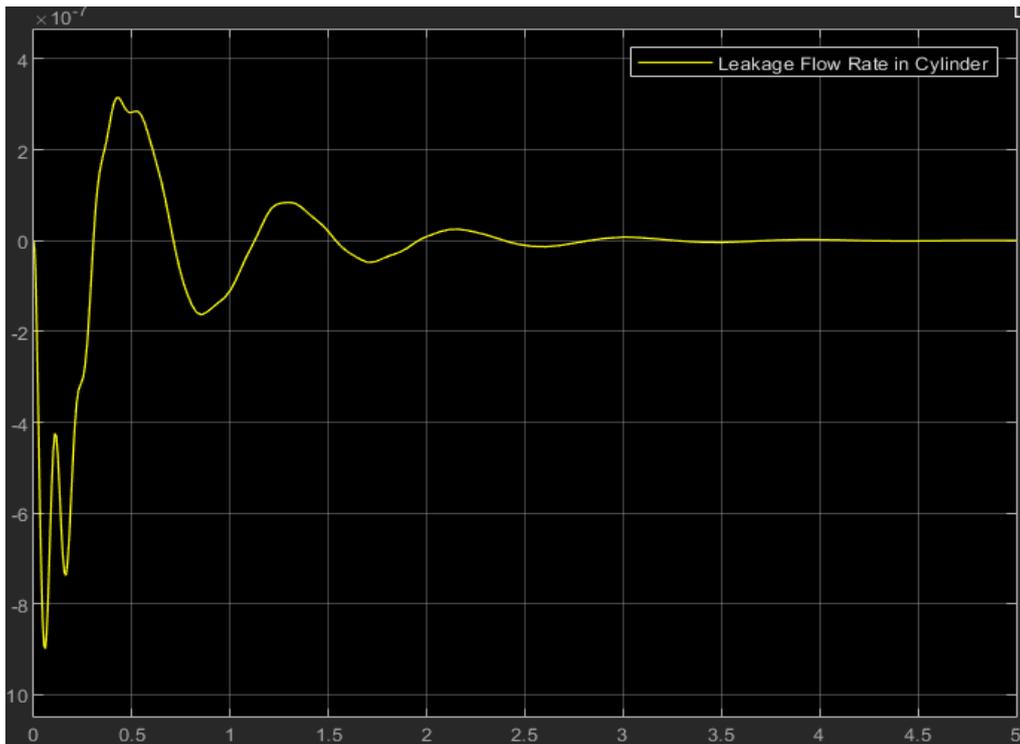


Figure 60- Flow Rate in Chamber B

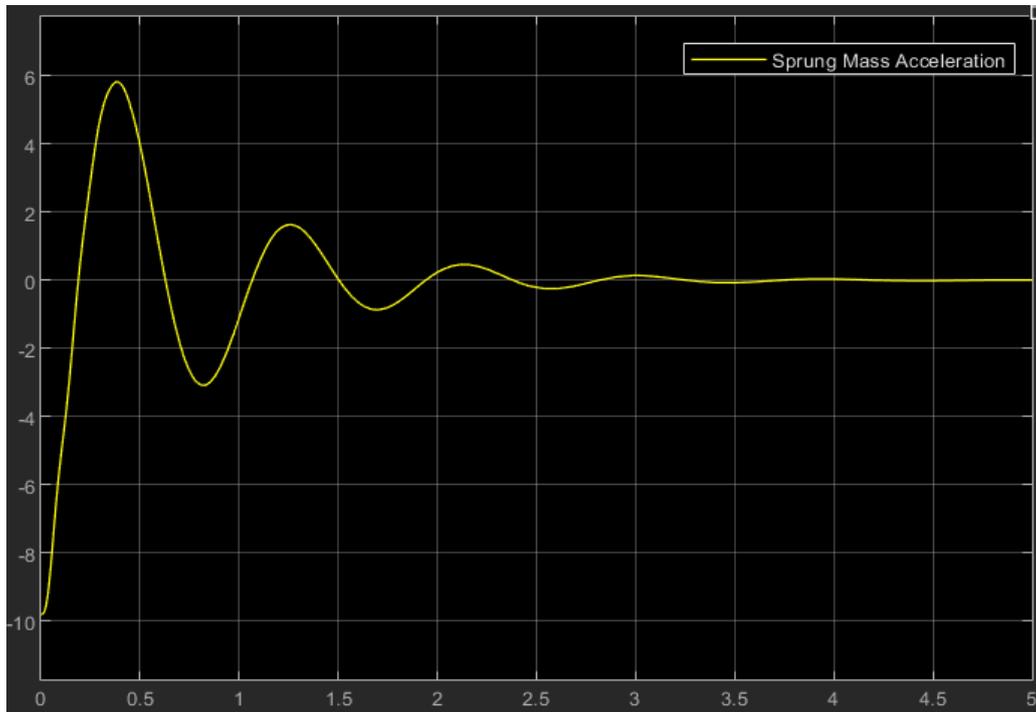


*Figure 61- Flow Rate Due to Inside Leakage in the Cylinder*

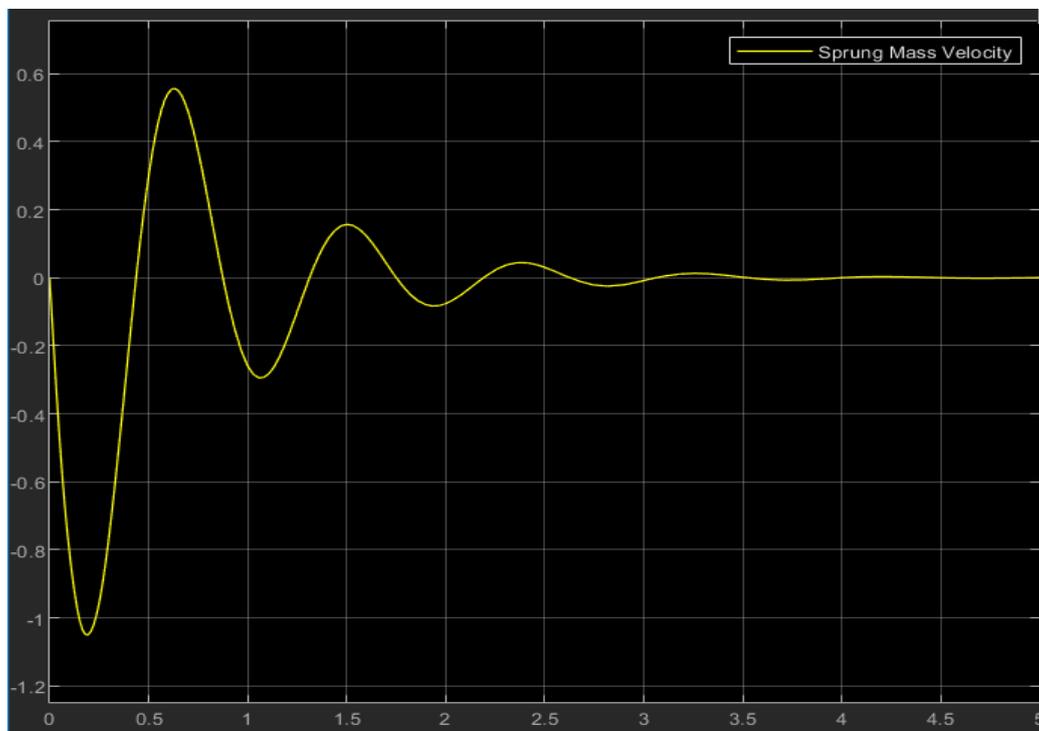
As can be seen in figure 57 and 58 of pressure and flow rate for chamber A, at some point both are increasing and then go back to the zero. While figure 59 and 60 of the chamber B shows the opposite trend where both are on the negative side than go up to some point and back to zero. For both chambers, A and B graphs show the fluctuations in the pressure and flowrate when the load is increased on suspension. Also, there is a huge amount of pressure inside both the chamber and there are nominal flow rates. Figure 61 shows the leakage flow rate inside the cylinder where it can be seen that it is very nominal (-0.0008 litre/second). It is first on the downward side of the graph which means leakage flow rate travelling from chamber A to B and after 4.5 seconds it is back to the normal position when no load suspension.

## 8.1.2 For Suspension Part

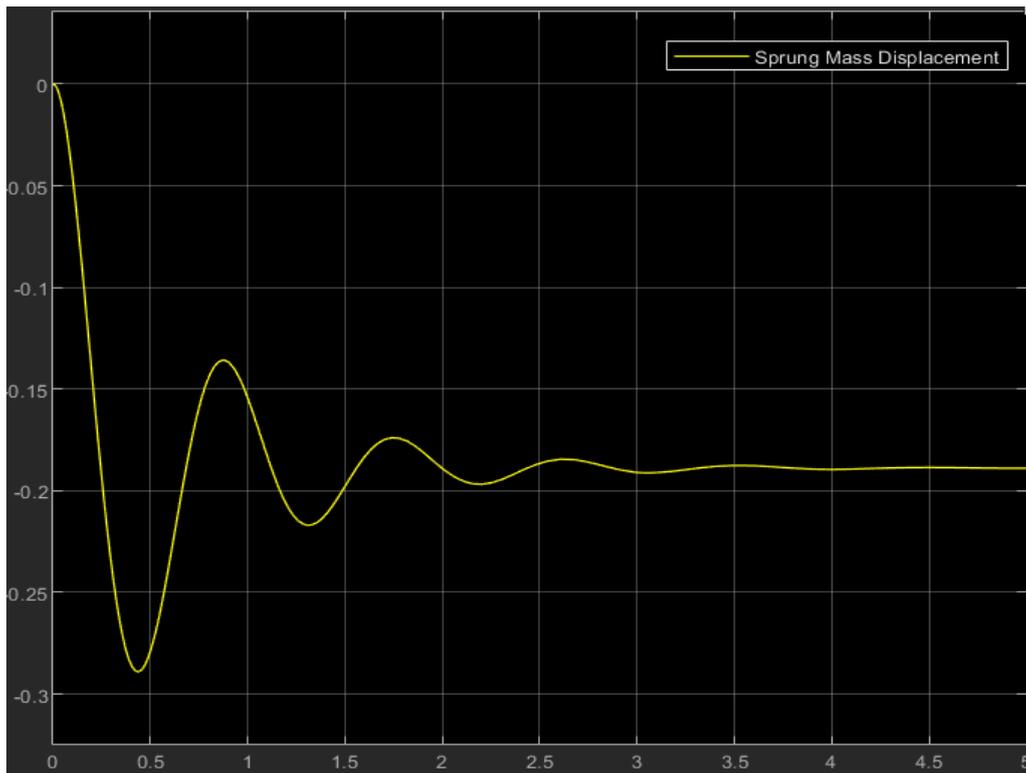
### 8.1.2.1 Sprung Mass



*Figure 62- Sprung Mass Acceleration*



*Figure 63- Sprung Mass Velocity*



*Figure 64- Sprung Mass Displacement*

The step input has been applied as a *source of flow (Sf)* considered 60 km/hr velocity is coming from the road. Figure 62, 63 and 64 shows the acceleration, velocity and displacement behaviour of the sprung mass, respectively. As from the figure 62, the acceleration of the sprung mass is travelling between  $6 \text{ m/sec}^2$  at upward and  $10 \text{ m/sec}^2$  at downward and after 4.5 seconds sprung mass come to the steady position. The sprung mass velocity is showing in figure 63 shows that it starts to move downward ( $-10 \text{ m/sec}$ ) and suspension force pushes back to the normal position after 4 seconds. While there is some error in displacement graph (figure 64) of sprung mass where it is not coming to the normal position after some interval and continuously going downward till  $-0.3$  meters and resting on the  $-0.2$  meters. This error can be fixed by design the proper controller because to provide the correct amount of the actuation force required to sense the position of the sprung and un-sprung mass.



### 8.1.2.2 Unsprung Mass

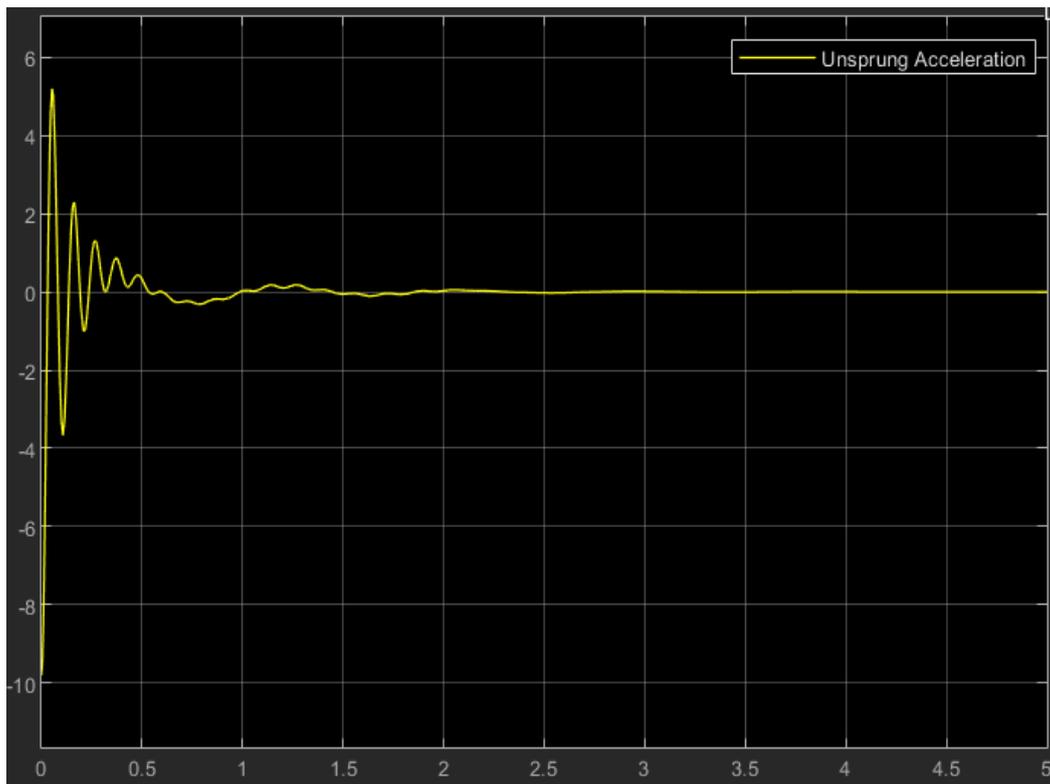


Figure 65- Unsprung Mass Acceleration

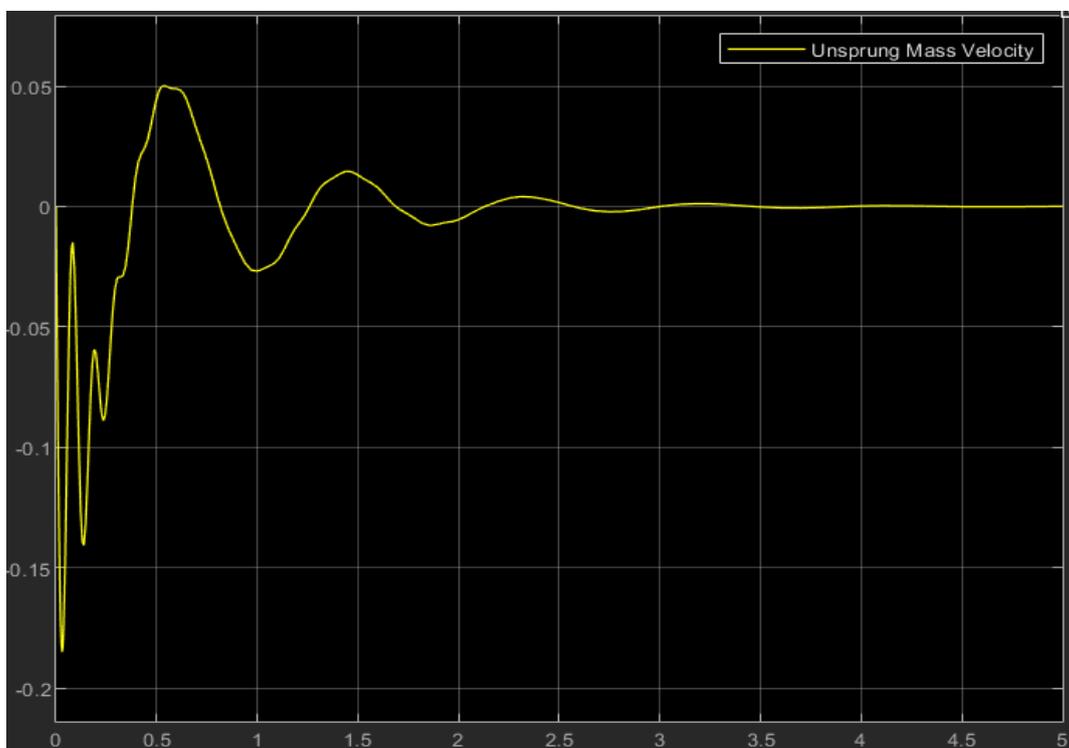
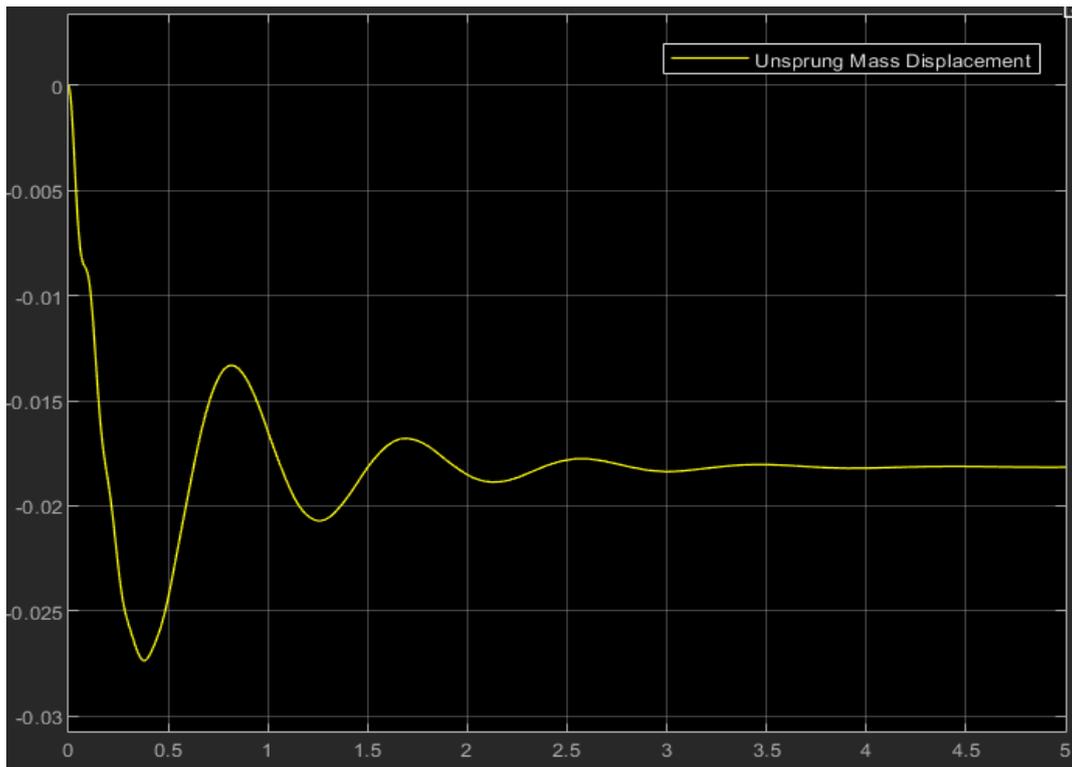


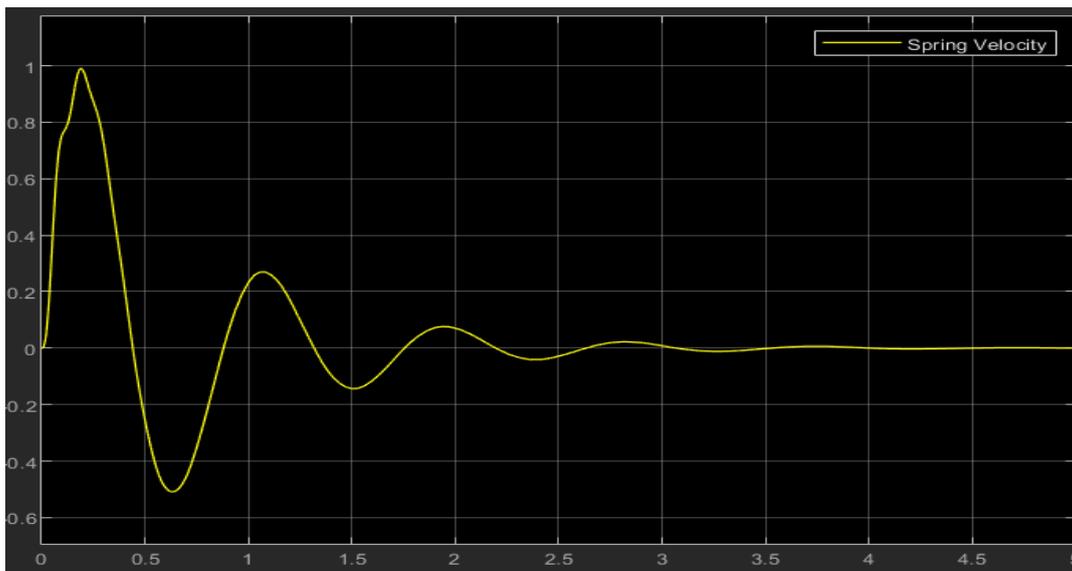
Figure 66- Unsprung Mass Velocity



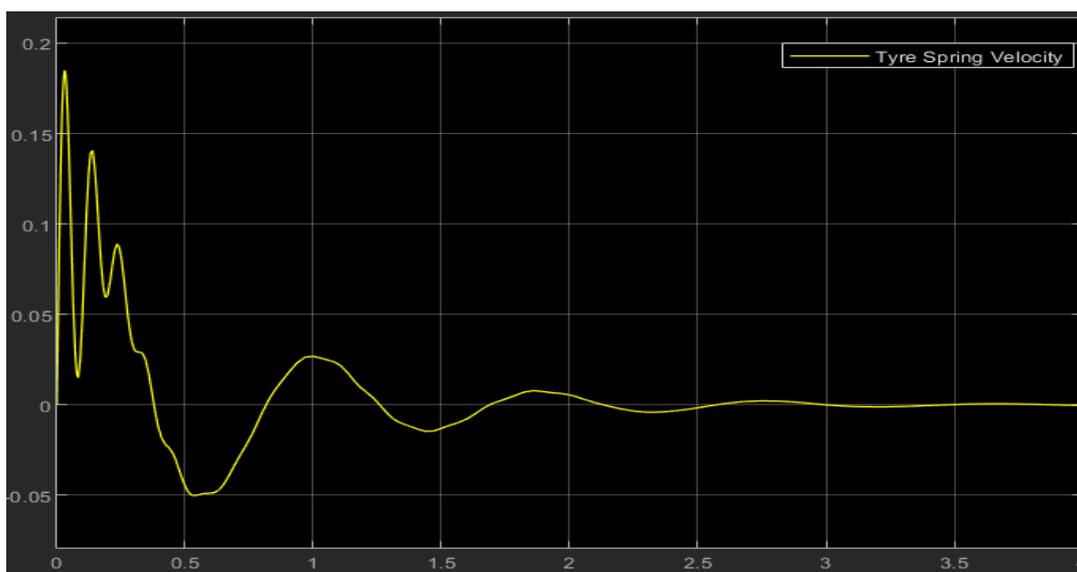
*Figure 67- Unsprung Mass Displacement*

The Unsprung mass acceleration, velocity and displacement behaviour could be seen in figure 65, 66 and 67, respectively. As can be seen in the un-sprung mass acceleration graph (figure 65) the fluctuations in acceleration are happening till the 3.5 seconds then the un-sprung mass acceleration comes to the steady position. Also, it happened very frequently and accelerated between  $5.5 \text{ m/sec}^2$  to  $-10 \text{ m/sec}^2$ . From the un-sprung mass velocity graph (figure 66) its velocity starts from the downward direction (between  $0.05 \text{ m/sec}$  to  $-0.2 \text{ m/second}$ ) where suspension velocity can push it back to the steady position. While the displacement of the un-sprung mass (figure 67) also follows the same trend as the displacement of the sprung mass. Although these errors are present in the graphs can be fixed by use of a proper controller which cannot be designed for this research work.

### 8.1.2.3 Spring and Tyre



*Figure 68-Spring Velocity*



*Figure 69- Tyre Spring Velocity*

Figure 68 and 69 shows the velocity of the suspension spring and tire spring velocity, respectively. The spring is travelling upward till 1 m/sec and comes to the normal position after 3.5 seconds in figure 68. While in figure 69, the tire spring velocity is fluctuated very fast and comes to the normal after some interval. These fast fluctuations happened because the tire is the primary contact with the road and sense more uneven condition of the road.



## *Chapter 9- Conclusion and Future Scope*



## 9.1 Conclusion

To the analysis of any system, the development of the model is the primary aim for the researchers to analyse and make advancements. It is required to check the behaviour first under virtual environment and verify it with the physical prototype. To improve the safety and comfort for any automobile it could be necessary to consider the performance of the suspension system. Therefore, this research is dedicated to the analysis of this system where considered the *Hydraulic Active Suspension System* because its performance is always superior to its passive counterparts. This project aimed to design a mathematical model for the hydraulic active suspension system for a single tyre was achieved. Also, the hydraulic system was modelled and derived the different equations for every component. The derived state-space equations of the system were simulated into the MATLAB/ Simulink environment. Due to time limitations and global pandemic situations, the results of the research could not be proper and would be a lot of possibilities to advance in the future works. Also, if this system could model again with the proper controller (which is out of interest of this research) technique then this system can give the desired output. So, this thesis is provided with the model platform to the control engineers to design the proper controller to get the desired output.

## 9.2 Future Scope

There is always the scope to improve the performance of the *Suspension System* that is also possible for this proposed work as well. It could be done by following suggested works.

1. There are possibilities to advance the results which got in this research.
2. The results could be validated with the result of the physical test rig of *Hydraulic Suspension Active System*.



3. From proper design and implementation of the controller with this hydraulic active suspension system could get the desired output.
4. It could be compared with passive suspension system results.
5. This work can be expanded for whole car suspensions to check the actual behaviour for all four tyres.
6. It could be modelled and analysed with the *Six Degree of Freedom* system. Where yawing, pitching and rolling of any vehicle can be examined.
7. For the hydraulic part, this research for an only *left position* (direction) of the spool valve which can also be reconsidered for the right position or both positions according to the movement of the *Piston*.
8. To control the direction and location of the *Direction Control Valve* controller can be designed.



## Reference

- [1] R. Yerrawar and R. Arakerimath, "Development of methodology for semi active suspension system using MR damper," *Materials Today: Proceedings*, vol. 4, no. 8, pp. 9294-9303, 2017.
- [2] A. C. Mitra, M. Jawarkar, T. Soni, and G. Kiranchand, "Implementation of Taguchi method for robust suspension design," *Procedia Engineering*, vol. 144, pp. 77-84, 2016.
- [3] S. Yang, Y. Lu, and S. Li, "An overview on vehicle dynamics," *International Journal of Dynamics and Control*, vol. 1, no. 4, pp. 385-395, 2013.
- [4] W. Sun, H. Gao, and P. Shi, *Advanced Control for Vehicle Active Suspension Systems*. Springer, 2020.
- [5] K. M. Mahala, P. Gadkari, and A. Deb, "Mathematical models for designing vehicles for ride comfort," in *ICORD 09: Proceedings of the 2nd International Conference on Research into Design, Bangalore, India 07.-09.01. 2009*, 2009.
- [6] R. Sharp and D. Crolla, "Road vehicle suspension system design-a review," *Vehicle system dynamics*, vol. 16, no. 3, pp. 167-192, 1987.
- [7] M. F. Soong, R. Ramli, and A. Saifizul, "Between simplicity and accuracy: Effect of adding modeling details on quarter vehicle model accuracy," *PloS one*, vol. 12, no. 6, 2017.
- [8] J. Mohammadpour and C. W. Scherer, *Control of linear parameter varying systems with applications*. Springer Science & Business Media, 2012.
- [9] A. Alleyne and J. K. Hedrick, "Nonlinear control of a quarter car active suspension," in *1992 American Control Conference*, 1992: IEEE, pp. 21-25.
- [10] X.-m. Dong, M. Yu, C.-r. Liao, and W.-m. Chen, "Comparative research on semi-active control strategies for magneto-rheological suspension," *Nonlinear dynamics*, vol. 59, no. 3, pp. 433-453, 2010.



- [11] A. Agharkakli, G. S. Sabet, and A. Barouz, "Simulation and analysis of passive and active suspension system using quarter car model for different road profile," *International Journal of Engineering Trends and Technology*, vol. 3, no. 5, pp. 636-644, 2012.
- [12] C. Kuber, "Modelling simulation and control of an active suspension system," *International Journal of Mechanical Engineering & Technology (IJMET)*, vol. 5, no. 11, pp. 66-75, 2014.
- [13] A. E.-N. S. Ahmed, A. S. Ali, N. M. Ghazaly, and G. A. El-Jaber, "PID controller of active suspension system for a quarter car model," *International Journal of Advances in Engineering & Technology*, vol. 8, no. 6, p. 899, 2015.
- [14] J. G. De Jalon and E. Bayo, *Kinematic and dynamic simulation of multibody systems: the real-time challenge*. Springer Science & Business Media, 2012.
- [15] P. Duysinx, O. Bruls, P. Fiset, C. Lauwerys, and J. Swevers, "Optimization of mechatronic systems: application to a modern car equipped with a semi-active suspension," in *Proceedings of the 6th World Congress of Structural and Multidisciplinary Optimization (WCSMO6)*, 2005.
- [16] A. R. Schumann and R. J. Anderson, "Optimal control of an active anti roll suspension for an off-road utility vehicle using interconnected hydragas suspension units," *Vehicle System Dynamics*, vol. 37, no. sup1, pp. 145-156, 2002.
- [17] C. Alexandru and P. Alexandru, "A comparative analysis between the vehicles' passive and active suspensions," *International Journal of Mechanics*, vol. 5, no. 4, pp. 371-378, 2011.
- [18] Y. He and J. McPhee, "Multidisciplinary design optimization of mechatronic vehicles with active suspensions," *Journal of Sound and Vibration*, vol. 283, no. 1-2, pp. 217-241, 2005.



- [19] H. A. Hamersma and P. S. Els, "Improving the braking performance of a vehicle with ABS and a semi-active suspension system on a rough road," *Journal of Terramechanics*, vol. 56, pp. 91-101, 2014.
- [20] J. Feng, S. Zheng, and F. Yu, "Bandwidth-limited active suspension controller for an off-road vehicle based on co-simulation technology," *Frontiers of Mechanical Engineering in china*, vol. 3, no. 1, pp. 111-117, 2008.
- [21] X. Yang and P. Xu, "Road load analysis techniques in automotive engineering," *Yung-Li Lee, Mark E. Barkey, Hong-Tae Kang, 'Metal Fatigue Analysis Handbook: Practical problem-solving techniques for computer-aided engineering', Chapter1*, pp. 1-60, 2011.
- [22] D. S. Joo, N. Al-Holou, J. M. Weaver, T. Lahdhiri, and F. Al-Abbas, "Nonlinear modeling of vehicle suspension system," in *Proceedings of the 2000 American Control Conference. ACC (IEEE Cat. No. 00CH36334)*, 2000, vol. 1, no. 6: IEEE, pp. 115-119.
- [23] J. Hurel, A. Mandow, and A. García-Cerezo, "Nonlinear two-dimensional modeling of a McPherson suspension for kinematics and dynamics simulation," in *2012 12th IEEE International Workshop on Advanced Motion Control (AMC)*, 2012: IEEE, pp. 1-6.
- [24] K. V. Reddy, M. Kodati, K. Chatra, and S. Bandyopadhyay, "A comprehensive kinematic analysis of the double wishbone and MacPherson strut suspension systems," *Mechanism and Machine Theory*, vol. 105, pp. 441-470, 2016.
- [25] M. Fallah, R. Bhat, and W. Xie, "New model and simulation of Macpherson suspension system for ride control applications," *Vehicle System Dynamics*, vol. 47, no. 2, pp. 195-220, 2009.
- [26] H. Habibi, K. H. Shirazi, and M. Shishesaz, "Roll steer minimization of McPherson-strut suspension system using genetic algorithm method," *Mechanism and Machine Theory*, vol. 43, no. 1, pp. 57-67, 2008.



- [27] S. Dutta and S.-B. Choi, "A nonlinear kinematic and dynamic modeling of Macpherson suspension systems with a magneto-rheological damper," *Smart Materials and Structures*, vol. 25, no. 3, p. 035003, 2016.
- [28] W. Jian, L. Zhiyuan, and J. Houhua, "Semi-active suspension control with polynomial inverse model: Frequency response analysis and HIL simulation," in *2015 34th Chinese Control Conference (CCC)*, 2015: IEEE, pp. 8177-8182.
- [29] U. Sabino, K. Hudha, and H. Jamaluddin, "Simulation and experimental evaluation on a skyhook policy-based fuzzy logic control for semi-active suspension system," *Int. J. Structural Engineering*, no. 3, pp. 243-272, 2011.
- [30] S. Dridi, I. B. Salem, and L. Amraoui, "Bond Graph modeling of automotive suspension system using a linear actuator," *International Journal of Scientific & Engineering Research*, vol. 8, no. 1, pp. 1837-1844, 2017.
- [31] M. Toufighi, F. Najafi, and S. Sadati, *Modeling and Fault Diagnosis of an Electrohydraulic Actuator System with a Multidisciplinary Approach Using Bond Graph*. INTECH Open Access Publisher, 2010.
- [32] G. Gonzalez-A and J. Madrigal, "Steady State of Passive and Active Suspensions in the Physical Domain," *International Journal of Advanced Computer Science*, vol. 2, no. 4, pp. 139-145, 2012.
- [33] B. Creed, N. Kahawatte, and S. Varnhagen, "Development of a full car vehicle Dynamics model for use In the design of an active suspension control system," *University of California Davis*, 2010.
- [34] J. Deur, V. Ivanovic, F. Assadian, M. Kuang, H. E. Tseng, and D. Hrovat, "Bond graph modeling of automotive transmissions and drivelines," *Mathematical Modelling*, vol. 7, no. Part 1, 2012.



- [35] A. C. Mitra and N. Benerjee, "Ride comfort and Vehicle handling of Quarter Car Model Using SIMULINK and Bond Graph," in *Proceedings of the 1st International and 16th National Conference on Machines and Mechanisms (iNaCoMM2013)*, 2013.
- [36] M. Emami, S. Mostafavi, and P. Asadollahzadeh, "Modeling and simulation of active hydro-pneumatic suspension system through bond graph," *Mechanics*, vol. 17, no. 3, pp. 312-317, 2011.
- [37] H. Adibi-asl and G. Rideout, "Bond graph modeling and simulation of a full car model with active suspension," in *Proceedings of the 20th IASTED International Conference on Modelling and Simulation*, 2009.
- [38] P. Tiwari and G. Mishra, "Simulation of Quarter Car Model," *IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE)*, vol. 11, no. 2, pp. 85-88, 2014.
- [39] Z. X. Deng and F. Lai, "The Modeling of Electromagnetic Linear Actuator for Vehicle Active Suspension System," in *Applied Mechanics and Materials*, 2011, vol. 40: Trans Tech Publ, pp. 127-132.
- [40] M. S. Reddy, P. Vigneshwar, D. RajaSekhar, K. Akhil, and P. L. N. Reddy, "Optimization Study on Quarter Car Suspension System by RSM and Taguchi," in *Proceedings of the International Conference on Signal, Networks, Computing, and Systems*, 2016: Springer, pp. 261-271.
- [41] A. Goodarzi and A. Khajepour, "Vehicle suspension system technology and design," *Synthesis Lectures on Advances in Automotive Technology*, vol. 1, no. 1, pp. i-77, 2017.
- [42] D. Hanafi, "PID controller design for semi-active car suspension based on model from intelligent system identification," in *2010 Second International Conference on Computer Engineering and Applications*, 2010, vol. 2: IEEE, pp. 60-63.
- [43] Y. Elattar, S. Metwalli, and M. Rabie, "Pdf versus pid controller for active vehicle suspension," in *The International Conference on Applied Mechanics and Mechanical*



- Engineering*, 2016, vol. 17, no. 17th International Conference on Applied Mechanics and Mechanical Engineering: Military Technical College, pp. 1-18.
- [44] Q. Zhou, "Research and simulation on new active suspension control system," 2013.
- [45] H. M. Paynter, *Analysis and design of engineering systems*. MIT press, 1961.
- [46] M. J. Abedini Laksar, "Active suspension development based on energy analysis," Memorial University of Newfoundland, 2016.
- [47] R. Merzouki, A. K. Samantaray, P. M. Pathak, and B. O. Bouamama, *Intelligent mechatronic systems: modeling, control and diagnosis*. Springer Science & Business Media, 2012.
- [48] D. C. Karnopp, D. L. Margolis, and R. C. Rosenberg, *System dynamics: modeling, simulation, and control of mechatronic systems*. John Wiley & Sons, 2012.
- [49] G. Priyandoko, M. Mailah, and H. Jamaluddin, "Vehicle active suspension system using skyhook adaptive neuro active force control," *Mechanical systems and signal processing*, vol. 23, no. 3, pp. 855-868, 2009.
- [50] S. Kashem, R. Nagarajah, and M. Ektesabi, *Vehicle suspension systems and electromagnetic dampers*. Springer, 2018.
- [51] Y. Li, Z. Zhu, and G. Chen, "Bond Graph Modeling and Validation of an Energy Regenerative System for Emulsion Pump Tests," *The Scientific World Journal*, vol. 2014, p. 289839, 2014/05/22 2014, doi: 10.1155/2014/289839.
- [52] M. Muvengei and J. Kihui, "Bond graph modeling of inter-actuator interactions in a multi-cylinder hydraulic system," *World Academy of Science, Engineering and Technology*, vol. 74, pp. 32-41, 2011.
- [53] R. Darus and N. I. Enzai, "Modeling and control active suspension system for a quarter car model," in *2010 International Conference on Science and Social Research (CSSR 2010)*, 2010: IEEE, pp. 1203-1206.



- [54] P. Athanasatos, D. Koulocheris, T. Costopoulos, and V. Spitas, "Experimental Verification of Fault Predictions in High Pressure Hydraulic Systems," *Modern Mechanical Engineering*, vol. 2014, 2014.
- [55] D. K. Chaturvedi, *Modeling and Simulation of Systems Using MATLAB and Simulink*. Baton Rouge, UNITED STATES: Taylor & Francis Group, 2009.
- [56] P. J. Gawthrop and G. P. Bevan, "Bond-graph modeling," *IEEE Control Systems Magazine*, vol. 27, no. 2, pp. 24-45, 2007.



# Appendices

## MATLAB Programming

Content removed for privacy reasons

### Volume and Momentum due to Pressure

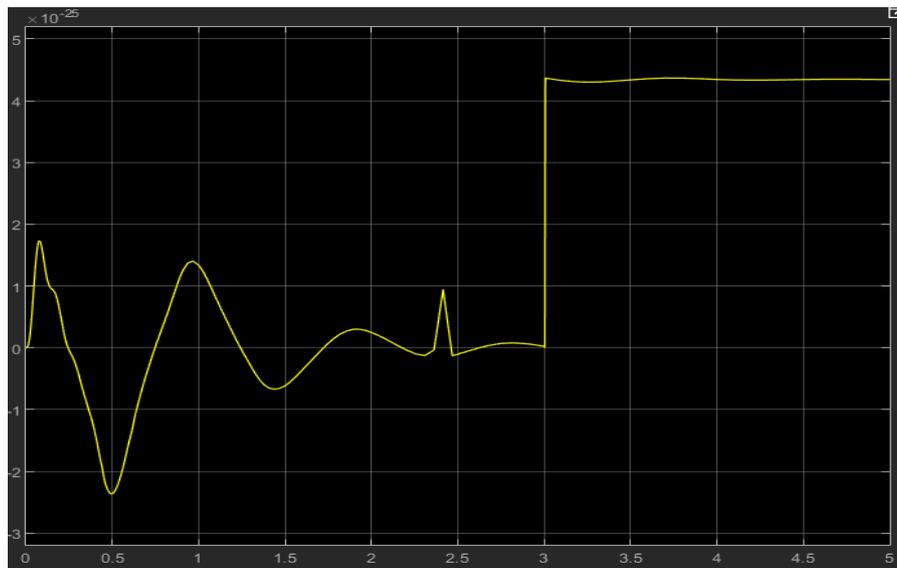
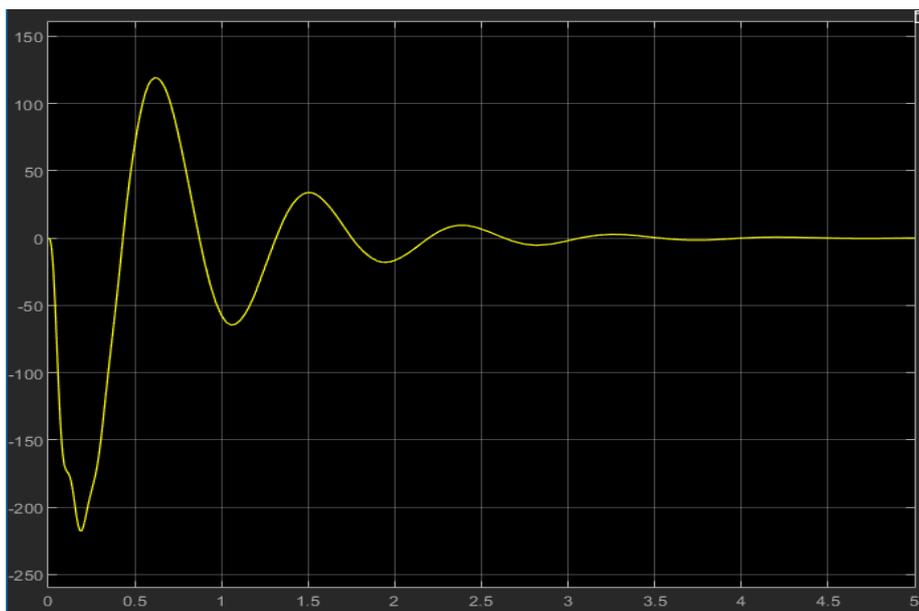
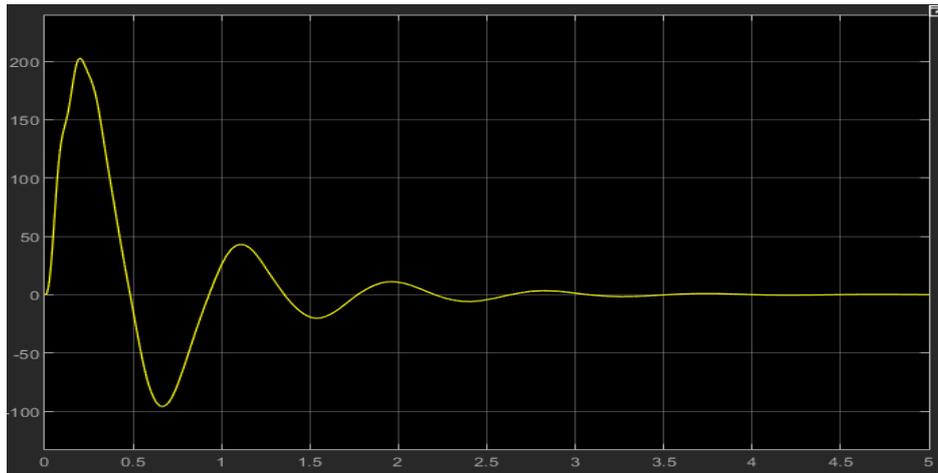


Figure 70- Volume Inside Accumulator

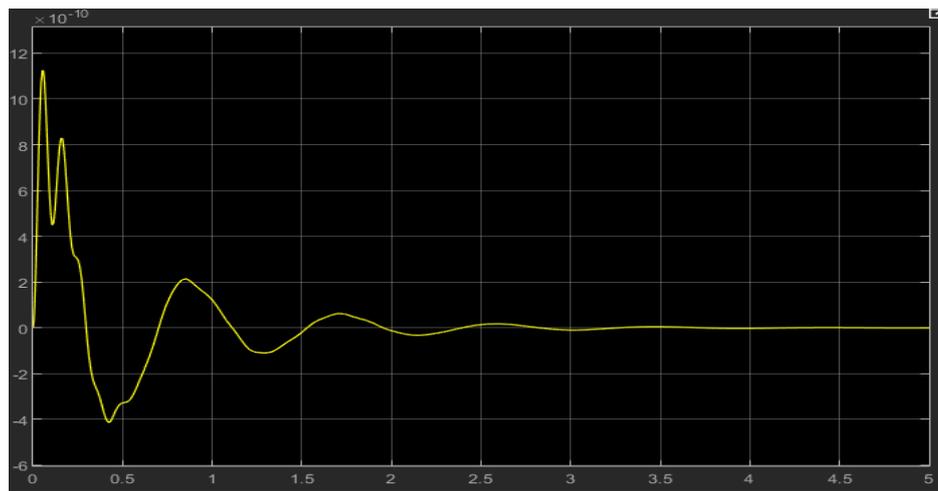




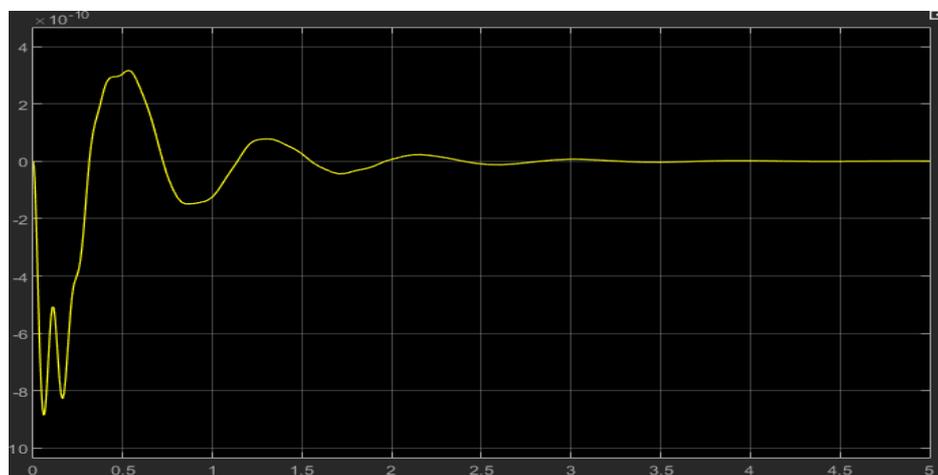
*Figure 71- Momentum Due to Pressure at Spool One*



*Figure 72- Momentum Due to Pressure at Spool Two*

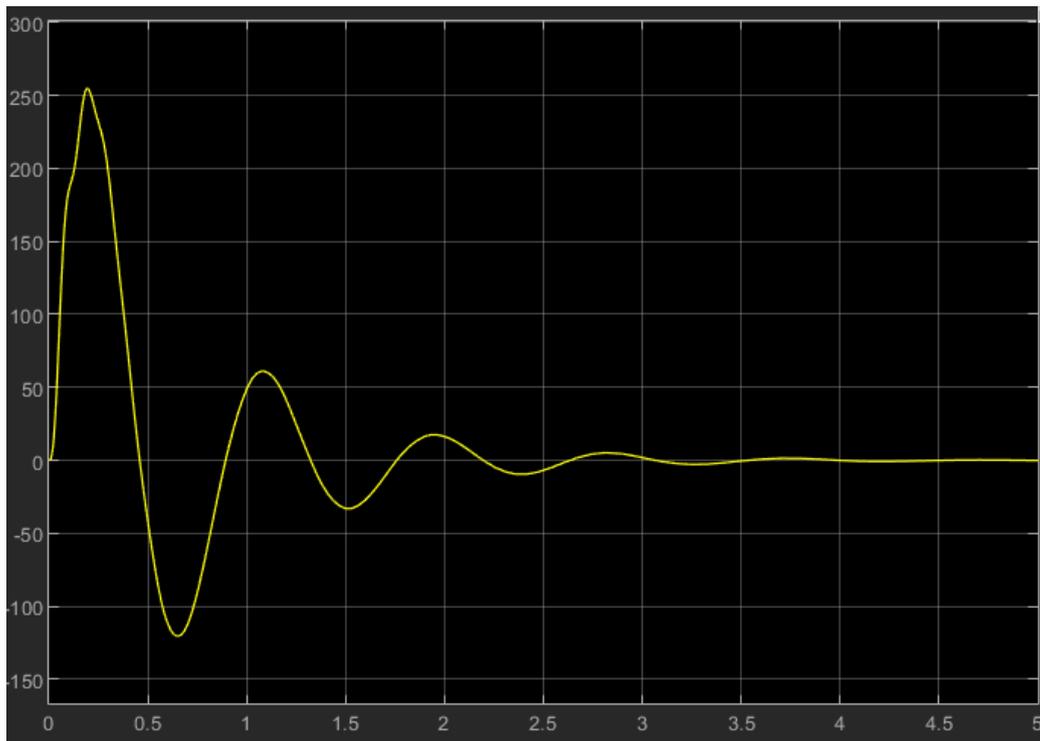


*Figure 73- Volume in Chamber A*

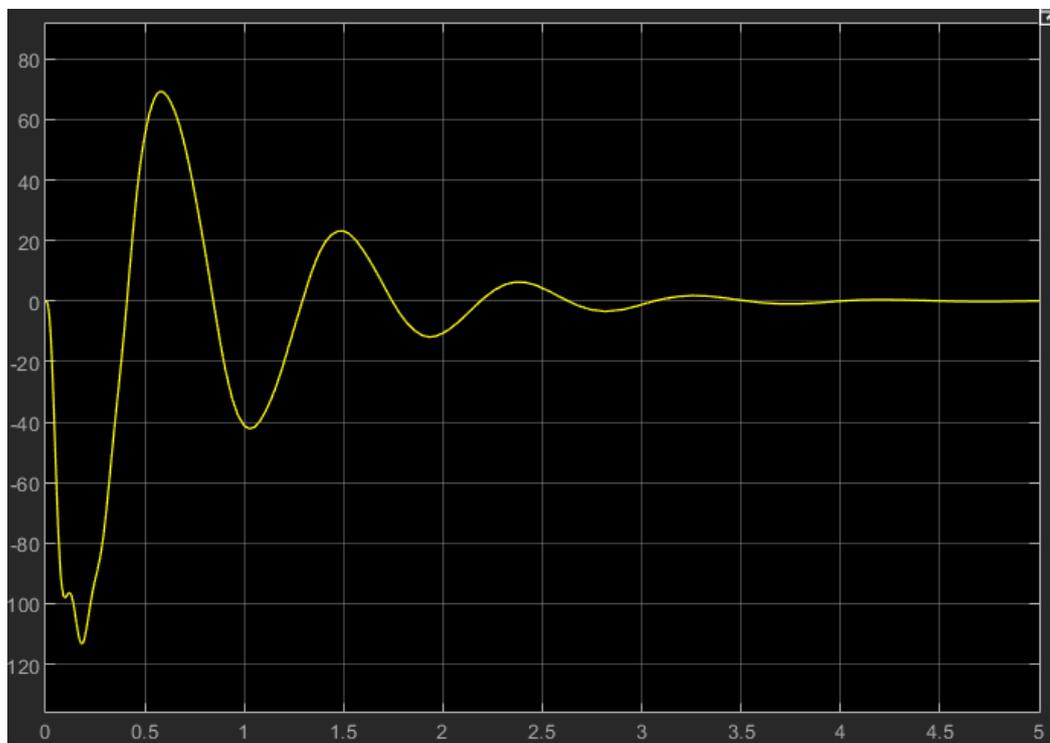




*Figure 74- Volume in Chamber B*



*Figure 75- Momentum Due to Pressure Return to Sump From A*



*Figure 76- Momentum Due to Pressure Return to Sump From B*



### Displacement and Momentum

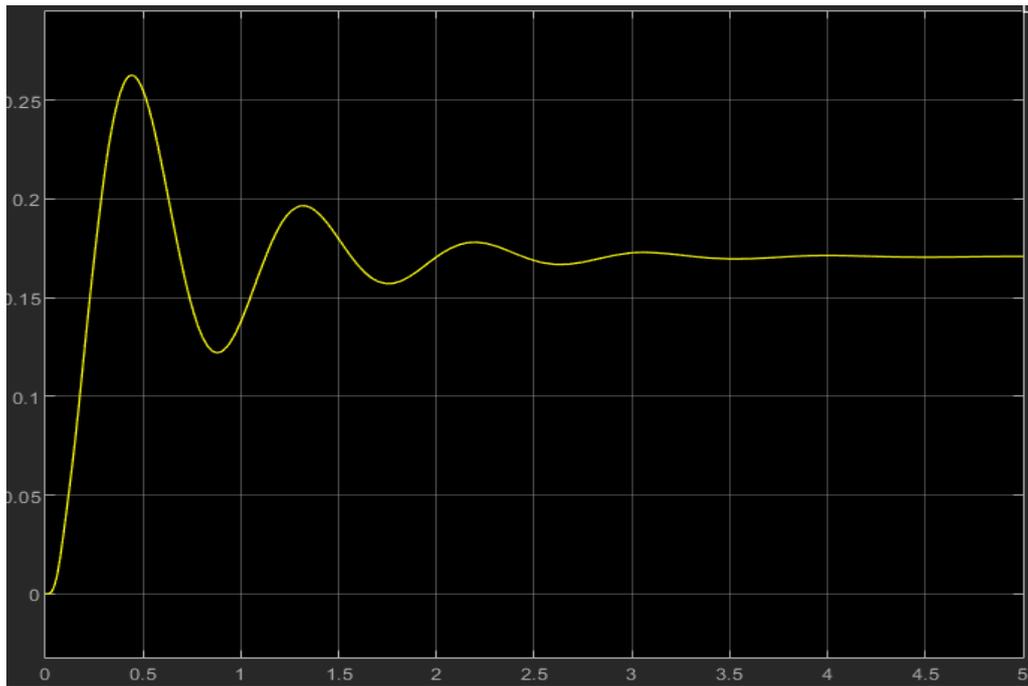


Figure 77- Displacement of the Spring

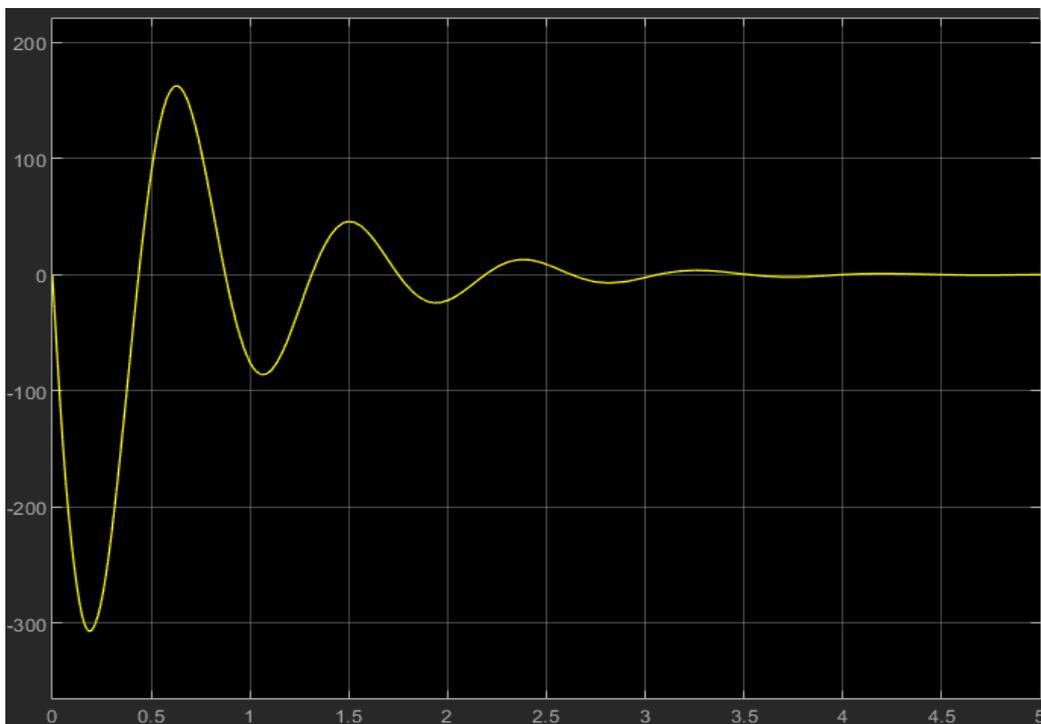
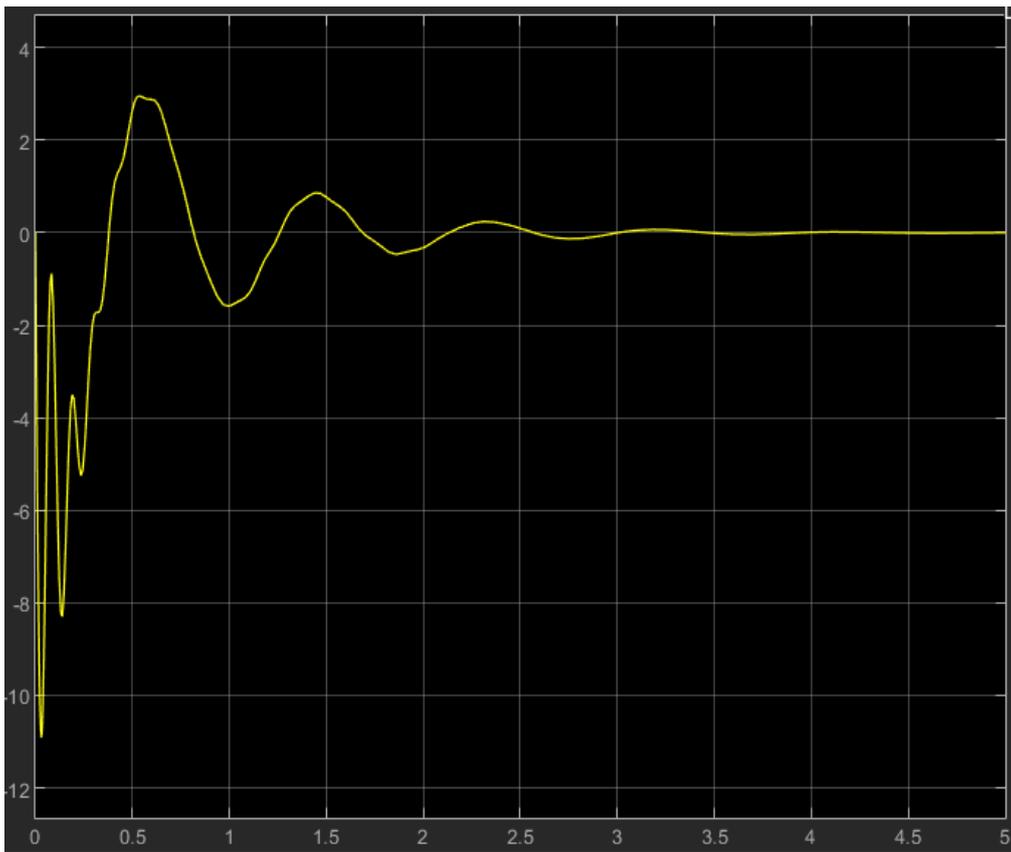
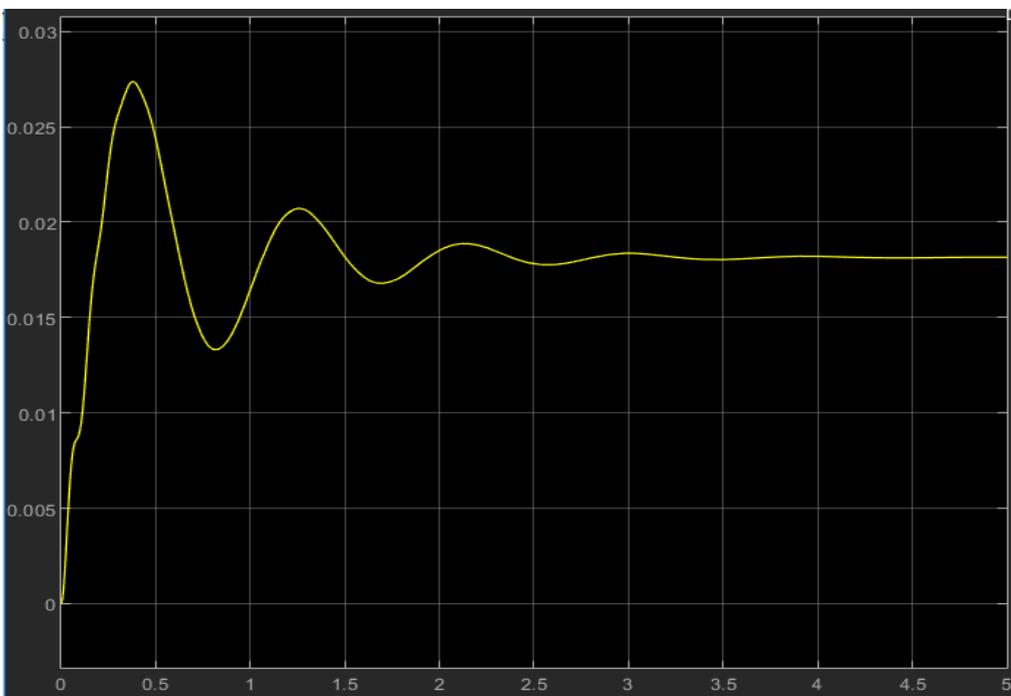


Figure 78- Momentum of Sprung Mass



*Figure 79- Momentum of Unsprung Mass*



*Figure 80- Displacement of the Tire Spring*



*Risk Assessment*

*Content removed for privacy reasons*