



**FLINDERS
UNIVERSITY**

**ADELAIDE
AUSTRALIA**

Adaptive Multi Mode Vibration Control of Dynamically Loaded Flexible Structures

by

Hendra Tjahyadi, *B.Eng.(Electronics), M.Eng.(Instrumentation and
Control)*

School of Informatics and Engineering,
Faculty of Science and Engineering

July , 2006

A thesis presented to the
Flinders University of South Australia
in total fulfillment of the requirements for the degree of
Doctor of Philosophy

Adelaide, South Australia, 2007
© (Hendra Tjahyadi, 2007)

Contents

Abstract	xv
List of Abbreviations	xix
Certification	xx
Acknowledgements	xxi
Publication in Support of the Thesis	xxiii
1 Introduction	1
1.1 Motivation	1
1.2 Research Methodology	5
1.3 Control of Vibration	6
1.4 Active Vibration Control	8
1.4.1 Feedforward and Feedback Control	8
1.4.2 Wave Control and Modal Control	9
1.5 Modal Based Controllers for Multi-mode Vibration Control	10
1.6 Control Methods for Systems with Varying Parameters	13
1.7 Control Methods for Systems with Uncertainties	16

<i>CONTENTS</i>	iii
1.7.1 Robust Control	16
1.7.2 Adaptive Control	17
1.8 Natural Frequency Estimator	20
1.9 Multiple Model Adaptive Control	22
1.10 Aim of the Thesis	23
1.11 Outline of the Thesis	24
1.12 Original Contributions to the Thesis	27
2 Modelling of Flexible Structures	30
2.1 Introduction	30
2.2 Description of Experimental Plant (Experimental Model)	32
2.3 Analytical Model	36
2.3.1 Flexural Vibration of Beams	37
2.3.2 Modal Analysis	41
2.4 Modal Analysis Using ANSYS (Numerical Models)	60
2.5 Simulation Models	61
2.6 Summary	67
3 Multiple Model Resonant Control	74
3.1 Introduction	75
3.2 Structure of a Resonant Controller	76
3.3 Discrete-time Resonant Control	82
3.3.1 Input-output Stability	84
3.3.2 Stability of a Discrete-time Resonant Control System	86

<i>CONTENTS</i>	iv
3.4 Multiple Model Control	88
3.5 Multi-model Multi-mode Resonant Control (M ⁴ RC)	94
3.5.1 Case 1: All the Possible Loading Condition are <i>a priori</i> Known	94
3.5.2 Case 2: Only the upper and Lower Bounds of Operating Region are <i>a priori</i> Known	100
3.6 Simulation Studies	103
3.6.1 Resonant Controller	103
3.6.2 M ⁴ RC	111
3.7 Experimental Studies	118
3.7.1 Resonant Controller	119
3.7.2 M ⁴ RC	126
3.8 Summary	131
4 Natural Frequency Estimator	133
4.1 Introduction	133
4.2 On-line Parameter Estimation Using RLS	141
4.3 Stability of the RLS Algorithm	148
4.4 Factors Which Influence the Accuracy of RLS	150
4.4.1 RLS Characteristics	151
4.4.2 Parameter Tracking Resolution	155
4.4.3 Parameter Perturbation Error	158
4.5 Design of the Natural Frequency Estimator for Flexible Structures	159
4.6 Simulation Studies of the Proposed Natural Frequency Estimator	164

4.6.1	Effects of Prefiltering and T Selection	165
4.6.2	Natural Frequency Estimator for Cantilever Beam Models	168
4.7	Experimental Studies	174
4.8	Summary	178
5	Adaptive Resonant Control	180
5.1	Introduction	180
5.2	Adaptive Resonant Control (ARC)	183
5.3	Multi-model Multi-mode Adaptive Resonant Control (M ⁴ ARC)	187
5.4	Simulation Studies of ARC and M ⁴ ARC	192
5.5	Experimental Studies	199
5.6	Summary	205
6	Summary, Conclusion and Future Work	207
6.1	Summary	207
6.2	Conclusion	215
6.3	Recommendations for Future Work	215
A	Simulation Models	218
B	Passivity	227
C	SimulinkTM Models	229
C.1	Simulations	229
C.2	Experiments	240
	Bibliography	245

List of Figures

1.1	The twisted roadway of the Tacoma Narrows bridge before its failure.	2
1.2	Block diagram of gain scheduling control	14
1.3	Block diagram of the Self Tuning Regulator scheme	18
1.4	Block diagram of Model Reference Adaptive System scheme	19
2.1	Plant with a collocated sensor-actuator pair	34
2.2	Loading model configurations	35
2.3	Change of frequency responses for various loading conditions	37
2.4	A beam in flexural vibration	39
2.5	A small element of the beam	39
2.6	A cantilever beam	47
2.7	A cantilever beam with n attached masses	51
2.8	Frequency responses of simulation models	63
2.9	Alternating pole-zero pattern of flexible structures with collocated sensor and actuator	64
2.10	Comparative responses of the three-mode model and the ten-mode model of Model 1 with white noise input	66
2.11	Comparative responses of the three-mode model and the ten-mode model of Model 1 with pulse signal input	67

2.12	Comparative responses of the three-mode model and the ten-mode model of Model 4 with white noise input	68
2.13	Comparative responses of the three-mode model and the ten-mode model of Model 4 with pulse signal input	69
2.14	Comparative responses of real plant and simulation model for Model 1	69
2.15	Comparative responses of real plant and simulation model for Model 2	70
2.16	Comparative responses of real plant and simulation model for Model 3	70
2.17	Comparative responses of real plant and simulation model for Model 4	71
2.18	Mode shapes for Model 1	71
2.19	Mode shapes for Model 2	72
2.20	Mode shapes for Model 3	72
2.21	Mode shapes for Model 4	73
3.1	Frequency response of a flexible cantilever beam	75
3.2	Flexible structure control system	77
3.3	Block diagram of resonant control	77
3.4	Frequency response of a dual mode resonant controller	78
3.5	Closed-loop responses for variations in a plant natural frequencies	81
3.6	Closed-loop responses for variations in a plant damping factors . .	81
3.7	A canonical feedback system	85
3.8	Multiple model control method using weighting function scheme .	90

3.9	Multiple model control method using supervisor scheme	91
3.10	Filter bank system for the m^{th} mode	96
3.11	Switching system for the m^{th} mode	97
3.12	Block diagram of M ⁴ RC for controlling M modes	98
3.13	Comparative responses of BPF with resonant controller structure with Butterworth BPF	99
3.14	Frequency response of resonant controller with $\zeta_c = 0.01$ and $k_d =$ 10	102
3.15	Model array in the M ⁴ RC model bank	102
3.16	Response of Model 1 and the corresponding control signal	106
3.17	Response of Model 2 and the corresponding control signal	106
3.18	Response of Model 3 and the corresponding control signal	107
3.19	Response of Model 4 and the corresponding control signal	107
3.20	Frequency response of Model 1	108
3.21	Frequency response of Model 2	108
3.22	Frequency response of Model 3	109
3.23	Frequency response of Model 4	109
3.24	Frequency response of Model 1 with only the 2^{nd} mode controller active	110
3.25	Open-loop system response for the $1 \rightarrow 3 \rightarrow 4$ model sequence	111
3.26	Closed-loop system response for the $1 \rightarrow 3 \rightarrow 4$ model sequence	112
3.27	Schematic diagram of the implemented M ⁴ RC	113
3.28	Schematic diagram of the multi-model control with MMSE super- visor scheme	114

3.29	M ⁴ RC switching behaviour for the 1 → 3 → 4 model sequence . . .	115
3.30	Closed-loop multiple model resonant control responses for the 1 → 3 → 4 model sequence	115
3.31	M ⁴ RC switching behaviour for the 1 → 2 → 4 model sequence . . .	116
3.32	Control signals generated by the multiple model resonant control for the 1 → 2 → 4 model sequence	117
3.33	Closed-loop responses of multiple model resonant control for the 1 → 2 → 4 model sequence	117
3.34	The experimental set-up	119
3.35	Response of Model 1 and the corresponding control signal	120
3.36	Response of Model 2 and the corresponding control signal	121
3.37	Response of Model 3 and the corresponding control signal	121
3.38	Response of Model 4 and the corresponding control signal	122
3.39	Frequency response of Model 1	122
3.40	Frequency response of Model 2	123
3.41	Frequency response of Model 3	123
3.42	Frequency response of Model 4	124
3.43	Frequency response of Model 1 with only the 2 nd mode controller active	124
3.44	Open-loop system response for the 1 → 3 → 4 model sequence . . .	125
3.45	Closed-loop system response for the 1 → 3 → 4 model sequence with the controller designed based on Model 3	126
3.46	M ⁴ RC closed-loop response for the 1 → 3 → 4 model sequence . . .	127
3.47	M ⁴ RC switching behaviour for the 1 → 3 → 4 model sequence . . .	128

3.48	M ⁴ RC closed-loop response for the 1 → 2 → 4 model sequence . . .	129
3.49	M ⁴ RC switching behaviour for the 1 → 2 → 4 model sequence . . .	130
3.50	M ⁴ RC closed-loop response for the 1 → 4 model sequence	130
3.51	M ⁴ RC switching behaviour for the 1 → 4 model sequence	131
4.1	Block diagram of an on-line parameter estimator	142
4.2	Recursive structure of parameter adaptation algorithm	142
4.3	Equivalent feedback representation of RLS	150
4.4	Frequency responses of $\hat{A}(e^{j\omega})$ for different order	154
4.5	Effect of sampling period on pole location	157
4.6	Estimation result without prefiltering	166
4.7	Estimation result with prefiltering using single LPF	167
4.8	Estimation result with prefiltering using BPFs	167
4.9	Estimation result with prefiltering using BPFs and different sam- pling period	168
4.10	Schematic diagram for natural frequency estimator	169
4.11	Estimation results for Model 1	171
4.12	Magnified steady-state results for Model 1 estimation	171
4.13	Estimation results for Model 4	172
4.14	Magnified steady-state results for Model 4 estimation	172
4.15	Estimation results for the 1 → 3 → 4 load sequence	173
4.16	Estimation result for the 1 → 2 → 4 load sequence	173
4.17	Estimation results for the 1 → 3 → 4 load sequence	175
4.18	Estimation results for the 1 → 2 → 4 load sequence	176

4.19	Magnified steady-state results for Model 1 estimation	176
4.20	Magnified steady-state results for Model 2 estimation	177
4.21	Magnified steady-state results for Model 3 estimation	177
4.22	Magnified steady-state results for Model 4 estimation	178
5.1	Block diagram of the MMAC	182
5.2	Block diagram of the ARC for controlling three modes.	183
5.3	Block diagram of the M ⁴ ARC for controlling three modes	188
5.4	Frequency response of resonant controller with $\zeta_c = 0.05$ and $k_d =$ 10	190
5.5	Model array in the M ⁴ ARC model bank	191
5.6	Simulation responses of the (a) M ⁴ RC, (b) ARC and (c) M ⁴ ARC for the 1 → 2 → 4 model sequence	195
5.7	M ⁴ ARC switching behaviour for the 1 → 2 → 4 model sequence .	195
5.8	Simulation responses of the (a) M ⁴ RC, (b) ARC and (c) M ⁴ ARC for the 1 → 3 → 4 model sequence	197
5.9	Simulation responses of the (a) M ⁴ RC, (b) ARC and (c) M ⁴ ARC for the 1 → 4 → 1 model sequence	197
5.10	M ⁴ ARC switching behaviour for the 1 → 3 → 4 model sequence .	198
5.11	M ⁴ ARC switching behaviour for the 1 → 4 → 1 model sequence .	199
5.12	Responses of the (a) M ⁴ RC, (b) ARC and (c) M ⁴ ARC for the 1 → 2 → 4 model sequence	201
5.13	M ⁴ ARC switching behaviour for the 1 → 2 → 4 model sequence .	201
5.14	Responses of the (a) M ⁴ RC, (b) ARC and (c) M ⁴ ARC for the 1 → 3 → 4 model sequence	203

5.15 Responses of the (a) M ⁴ RC, (b) ARC and (c) M ⁴ ARC for the 1 → 4 model sequence	203
5.16 M ⁴ ARC switching behaviour for the 1 → 3 → 4 model sequence	204
5.17 M ⁴ ARC switching behaviour for the 1 → 4 model sequence	204
C.1 Schematic diagram of <i>adap_sim</i> model	232
C.2 Detail schematic diagram of the frequency estimator block	233
C.3 Detail schematic diagram of the controller block	234
C.4 Schematic diagram of <i>m4rc_sim</i> model	236
C.5 Schematic diagram of filter bank and switching system block	237
C.6 Schematic diagram of <i>narendra_sim</i> model	238
C.7 Schematic diagram of <i>m4arc_sim</i> model	239
C.8 Detail of filter bank and switching systems block of <i>m4arc_sim</i> model	240
C.9 Schematic diagram of <i>adapt_exp</i> model	242

List of Tables

2.1	Properties of the beam	33
2.2	Model parameters	34
2.3	The first three natural frequencies of the experimental models	36
2.4	The first three frequency equation's roots of the models	58
2.5	Comparison of natural frequencies obtained from experimental and analytical results	59
2.6	Comparison of natural frequency results from ANSYS with results from analytical method	61
3.1	Models' parameters	80
3.2	Plant and controller configurations for resonant controller simulation study	104
3.3	Attenuation level for the range of models	105
3.4	Plant and controller configurations for M ⁴ RC simulation study	113
3.5	Attenuation level for the range of models	120
5.1	Plant and controller configuration for three different simulation study cases	193
5.2	Maximum overshoot percentage and settling time of ARC and M ⁴ ARC for different loading changes	196

5.3	Maximum overshoot percentage and settling time of ARC and M ⁴ ARC for different loading changes	202
A.1	The first ten natural frequencies, eigenfunctions and damping ratios of Model 1	220
A.2	The first ten natural frequencies, eigenfunctions and damping ratios of Model 2	220
A.3	The first ten natural frequencies, eigenfunctions and damping ratios of Model 3	221
A.4	The first ten natural frequencies, eigenfunctions and damping ratios of Model 4	221
A.5	The transfer function coefficients and pole positions of Model 1	223
A.6	The transfer function coefficients and pole positions of Model 2	224
A.7	The transfer function coefficients and pole positions of Model 3	225
A.8	The transfer function coefficients and pole positions of Model 4	226
C.1	C S-functions that are used for simulation and experimental implementations	230
C.2	Simulink model for simulations.	231
C.3	Simulink model for experimental implementation	241

Abstract

In this thesis, three control methodologies are proposed for suppressing multi-mode vibration in flexible structures. Controllers developed using these methods are designed to (i) be able to cope with large and sudden changes in the system's parameters, (ii) be robust to unmodelled dynamics, and (iii) have a fast transient response. In addition, the controllers are designed to employ a minimum number of sensor-actuator pairs, and yet pose a minimum computational demand so as to allow real-time implementation.

A cantilever beam with magnetically clamped loads is designed and constructed as the research vehicle for evaluation of the proposed controllers. Using this set-up, sudden and large dynamic variations of the beam loading can be tested, and the corresponding changes in the plant's parameters can be observed. Modal testing reveals that the first three modes of the plant are the most significant and need to be suppressed. It is also identified that the first and third modes are spaced more than a decade apart in frequency. The latter characteristic increases the difficulty of effectively controlling all three modes simultaneously using one controller. To overcome this problem, the resonant control method is chosen as the basis for the control methodologies discussed in this thesis.

The key advantage of resonant control is that it can be tuned to provide specific attenuation only at and immediately close to the resonant frequency of concern. Consequently, it does not cause control spillover to other modes owing to unmodeled dynamics. Because of these properties, a resonant controller

can be configured to form a parallel structure with the objective of targeting and cancelling multiple modes individually. This is possible regardless of the mode spacing. In addition, resonant control requires only a minimum number of collocated sensor-actuator pairs for multi-mode vibration cancellation. All these characteristics make resonant control a suitable candidate for multi-mode vibration cancellation of flexible structures.

Since a resonant controller provides negligible attenuation away from the natural frequencies that it has been specifically designed for, it is very sensitive to changes of a system's natural frequencies and becomes ineffective when these mode frequencies change. Hence, for the case of a dynamically loaded structure with consequent variations in mode frequencies, the resonant control method must be modified to allow tracking of system parameter changes. This consideration forms the theme of this thesis, which is to allow adaptive multi-mode vibration control of dynamically-loaded flexible structures. Three controller design methodologies based on the resonant control principle are consequently proposed and evaluated.

In the first approach, all possible loading conditions are assumed to be *a priori* known. Based on this assumption, a multi-model multi-mode resonant control (M⁴RC) method is proposed. The basis of the M⁴RC approach is that it comprises a bank of known loading models that are designed such that each model gives optimum attenuation for a particular loading condition. Conceptually, each model is implemented as a set of fixed-parameter controllers, one for each mode of concern. In reality, each mode controller is implemented as an adjustable resonant controller that is loaded with the fixed-model parameters of the corresponding mode. The M⁴RC method takes advantage of the highly frequency-sensitive nature of resonant control to allow simple and rapid selection of the optimum controller. Identification of the set of resonant frequencies is implemented using a bank of

band-pass filters that correspond to the mode frequencies of the known models. At each time interval a supervisor scheme determines for each mode which model has the closest frequency to the observed vibration frequency and switches the corresponding model controller output to attenuate the mode. Selection is handled on a mode-by-mode basis, such that for each mode the closest model is selected. The proposed M⁴RC is relatively simple and less computationally complex compared to other multi-model methods reported in the literature. In particular, the M⁴RC uses a simple supervisor scheme and requires only a single controller per mode. Other multi-model methods use more complex supervision schemes and require one controller per model. The M⁴RC method is evaluated through both simulation and experimental studies. The results reveal that the proposed M⁴RC is very effective for controlling multi-mode vibration of a flexible structure with known loading conditions, but is ineffective for unmodeled loading conditions.

In the second approach, the assumption that all loading conditions are *a priori* known is relaxed. An adaptive multi-mode resonant control (ARC) method is proposed to control the flexible structure for all possible (including unknown) loading conditions. On-line estimation of the structure's natural frequencies is used to update the adaptive resonant controller's parameters. The estimation of the natural frequencies is achieved using a parallel set of second-order recursive least-squares estimators, each of which is designed for a specific mode of concern. To optimise the estimation accuracy for each mode frequency, a different sampling rate suitable for that mode is used for the corresponding estimator. Simulation and experiment results show that the proposed adaptive method can achieve better performance, as measured by attenuation level, over its fixed-parameter counterpart for a range of unmodeled dynamics. The results also reveal that, for the same sequences of known loading changes, the transient responses of the ARC are slower than those of the M⁴RC.

In the third approach, a hybrid multi-model and adaptive resonant control is utilized to improve the transient response of the ARC. The proposed multi-model multi-mode adaptive resonant control (M⁴ARC) method is designed as a combination of the M⁴RC and ARC methods. The basis of the proposed method is to use the M⁴RC fixed-parameter model scheme to deal with transient conditions while the ARC adaptive parameter estimator is still in a state of fluctuation. Then, once the estimator has reached the vicinity of its steady-state, the adaptive model is switched in place of the fixed model to achieve optimum control of the unforeseen loading condition. Whenever a loading change is experienced, the simple M⁴RC supervisor scheme is used to identify the closest model and to load the adjustable resonant controllers with the fixed parameters for that model. Meanwhile, the mode estimators developed for the ARC method are used to identify the exact plant parameters for the modes of concern. As soon as these parameters stop rapidly evolving and reach their steady-state, they are loaded into the respective adjustable controllers. The same process is repeated whenever a loading change occurs. Given the simplicity of the M⁴ARC method and its minimal computation demand, it is easily applicable for real-time implementation. Simulation and experiment results show that the proposed M⁴ARC outperforms both the ARC with respect to transient performance, and the M⁴RC with respect to unmodeled loading conditions.

The outcomes of this thesis provide a basis for further development of the theory and application of active control for flexible structures with unforeseen configuration variations. Moreover, the basis for the proposed multi-model adaptive control can be used in other areas of control (not limited to vibration cancellation) where fast dynamic reconfiguration of the controller is necessary to accommodate structural changes and fluctuating external disturbances.

List of Abbreviations

ADC	Analog to Digital Converter
AIS	Adaptive Input Shaping
ARC	Adaptive Resonant Control
ARMAX	Auto Regressive Moving Average with eXternal input
ARX	Auto Regressive with eXternal input
BIBO	Bounded Input Bounded Output
BPF	Band Pass Filter
DAC	Digital to Analog Converter
FEM	Finite Element Method
FFT	Fast Fourier Transform
FRF	Frequency Response Function
HPF	High Pass Filter
IIR	Infinite Impulse Response
IMSC	Independent Modal Space Control
LHP	Left Half Plane
LPF	Low Pass Filter
M ⁴ ARC	Multi-Model Multi-Mode Adaptive Resonant Control
M ⁴ RC	Multi-Model Multi-Mode Resonant Control
MIMO	Multi Input Multi Output
MMAC	Multiple Model Adaptive Control
MMC	Multiple Model Control
MMSE	Minimum Mean Squares Error
MRAS	Model Reference Adaptive System
ODE	Ordinary Differential Equation
PDE	Partial Differential Equation
PLL	Phase Locked Loop
PPF	Positive Position Feedback
RLS	Recursive Least Squares
SDOF	Single Degree of Freedom
SISO	Single Input Single Output
STR	Self Tuning Regulator

Certification

I certify that this thesis does not incorporate without acknowledgement any material previously submitted for a degree or diploma in any university; and that to the best of my knowledge and belief it does not contain any material previously published or written by another person except where due reference is made in the text.

As requested under Clause 14 of Appendix D of the *Flinders University Research Higher Degree Student Information Manual* I hereby agree to waive the conditions referred to in Clause 13(b) and (c), and thus

- Flinders University may lend this thesis to other institutions or individuals for the purpose of scholarly research;
- Flinders University may reproduce this thesis by photocopying or by other means, in total or in part, at the request of other institutions or individuals for the purpose of scholarly research.

Adelaide, 26 July 2006

Hendra Tjahyadi

Acknowledgements

It is a real privilege for me to work under the supervision of Associate Prof. Fangpo He and Associate Prof. Karl Sammut. Their valuable inputs and comments always encourage me to work better, think deeper and broader, and to write clearer. Without their constructive criticism and guidance, I believe my thesis will not be in a good shape. To Fangpo, thank you for long hours discussions and debates, and your intriguing parables. To Karl thank you for your encouragement and patience, and for help me dealing with the bureaucracy.

I want to give thanks to all my friends who help me and my family during my study in Adelaide. My deep gratitude goes to the member of Ascot Community Uniting Church for your supports and prayers. To Alex Yates who thoroughly read my thesis draft and give a very helpful recommendations. Thanks for your willingness to help and your availability for long discussions. My special thanks to reverend John Blanskby, who become my mentor and my wonderful friend during my stay in Adelaide. For my friends in Engineering school Ning, May and Tae Hwan thanks for our friendships.

This research has been undertaken with the support of the AusAid scholarship and Flinders University Postgraduate Scholarship that is gratefully acknowledged. My sincere appreciation for Elaine Kane, the AusAid liaison officer in Flinders University, for her helps and supports.

To my wife Mega and my daughters Clarice and Christy to whom I mostly in debt, thank you so much for your constant love, prayer, support, patience, and

encouragement during the preparation of this thesis.

Finally, I want to thank Jesus Christ my saviour, to be with me wherever I go and even carry me when I pass the desert part of my life journey.

Publications in Support of the Thesis

The major contributions presented in this thesis have been peer reviewed and reported in the following refereed publications:

1. Tjahyadi, H., He, F., and Sammut, K. Vibration control of a cantilever beam using multiple model adaptive control. In *Proceedings of the American Control Conference ACC2004* (Boston, Massachusetts, July 2004), pp. 2907-2908.
2. Tjahyadi, H., He, F., and Sammut, K. Vibration control of a cantilever beam using adaptive resonant control. In *Proceedings of the 5th Asian Control Conference ASCC2004* (Melbourne, Australia, July 2004), pp. 1786-1790.
3. Tjahyadi, H., He, F., and Sammut, K. Vibration control of a cantilever beam using multiple model adaptive resonant control. In *Proceedings of IFAC Workshop on Adaptation and Learning in Control and Signal Processing (ALCOSP04)*(Yokohama, Japan, September 2004), pp. 409-413.
4. Tjahyadi, H., He, F., and Sammut, K. Multi-mode vibration control of a flexible cantilever beam using adaptive resonant control. *Smart Materials and Structures*, 15:270-278, 2006.
5. Tjahyadi, H., He, F., and Sammut, K. M⁴ARC: Multi-Model-Multi-Mode Adaptive Resonant Control for Dynamically-Loaded Flexible Beam Struc-

tures. In preparation to be submitted to *IEEE Trans. Control Systems Technology*