Master's Thesis: Numerical modelling of a railway ballast with a heterogeneous medium

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Submitted to the College of Science and Engineering in partial fulfilment of the requirements for the degree of Master of Engineering (Civil) at Flinders University – Adelaide Australia.

Declaration

I certify that this work does not incorporate without acknowledgement any material previously submitted for a degree or diploma in any university; and that to the best of my knowledge and belief it does not contain any material previously published or written by another person except where due reference is made in the text.

Dalton Rieck – 27/05/2019

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Abstract

The railway industry is another public and private infrastructure component that is looking to utilise new ways to improve and increase capacity, with the aim of improving transportation conditions for consumers into the future. As a result, many studies are being conducted in modelling various engineering components.

In this particular context, transport engineers are looking to improve the overall capacity of high-speed train lines. This will provide faster speed and higher capacity alternatives in line with the technological growth and development of these trains. Recent tests have demonstrated that the capacity exists for trains to exceed present limits, but that the system would not yet support such a capacity. When in close proximity to train lines, as is common in highly urbanised environments throughout the world that have significant train transportation demands, nearby infrastructure can become significantly damaged by wave propagation as this capacity and demand slowly increases in line with population growth.

Studies are beginning to examine the various components of a train-line under stress, and how we can come to better conclusions regarding the behaviour of line components in both the long and short term. The model that I have conducted testing with has been developed in an attempt to gain a greater understanding of ballast behaviour in particular, and how the impacts of wave propagation can be mitigated.

The ballast behaviour will be analysed through the implementation of a 3D spectral element model (SEM) with a perfectly matched layer (PML), allowing for simulation of ballast layer properties and loading conditions. The idea was to monitor different sensors to understand how the heterogeneous stress field fluctuates, and the impact this has on heterogeneous modelling cases compared to homogeneous modelling cases. The SEM model has been constructed with numerical underpinning, and has been tested via supercomputing clusters accessed remotely in Paris.

It will assist future construction methods and provide grounding for greater potential forecasting of damaging impact to infrastructure by improving our understanding of capacity requirements, and allowing greater conceptualisation of how we can design to those requirements.

1 Introduction

1.1 Background

Railway systems have become one of the most advanced and fast developing modes of transport across the past few decades. Recent technological advancements have enabled large growth in both the capacity and velocity of railway systems. Trains have become faster and heavier in order to cater for an increase in demand, with traffic infrastructure in densely populated areas steadily increasing (Connelly et al. 2013). Relatively low air pollution per passenger compared with road vehicles, and higher maximum speeds have made railways competitive with air and road transport, particularly throughout Europe (Krylov et al. 2000). A significant trend in this context is the increasing speed and weight of railway trains, which impose heavier loads on the tracks. These factors have created a growing concern regarding the problems associated with ground vibrations generated from railway traffic, and the environmental effects of noise and vibrations on the surrounding environment (Hall 2002).

The noise and vibrational impact on the environment can be important for all types of trains. Heavy freight coaches, for example, produce a low frequency impact that is carried large distances, while high speed trains generate high amplitude excitations. With time and the repeated passage of trains, the ballast layer settles down. This induces a levelling defect of the track, and may force the train companies to locally reduce the velocity of the passing trains for the comfort and security of passengers. These defects are therefore regularly controlled, and costly maintenance operations are organised (Correa et al. 2017). As a result, many studies have been conducted on the nature of ground vibrations surrounding railway systems, in an attempt to best understand and mitigate potential issues.

The same technological advancements which have facilitated the growth of these systems has also started to create new avenues and possibilities for analysing ground vibrations, with the aim of reducing potential problems and predicting levels with greater accuracy (Hall 2002). Obtaining a greater understanding of the physical and mechanical characteristics of the rail substructure, in particular the ballast layer, would improve efficiency and decrease maintenance and infrastructure costs (Indraratna 2006). Rather than allow civil engineering challenges to prevent the further growth of high speed railway systems, it is important to push our understandings of track design and soil mechanics in order to continue to facilitate this growth and maintain satisfaction of demand (Connolly et al. 2013).

1.2 Overview of related literature

Most studies in the field of vibrational analysis start by concerning themselves with the analysis of wave propagation. The first study to do this in great depth examined what would happen if train speeds exceeded the velocity of the Rayleigh surface waves in supporting soil. Ultimately, this study demonstrated that ground vibration boom would be the net result, and that any future design for a high-speed line would need to take this into account by being aware of the potential ramifications. Some suggestions made by Krylov revolved around the use of the Rayleigh wave velocity as an indication of the maximum train speeds beyond which excessive ground vibrations could be expected (Krylov et al. 2000).

More research was conducted to provide some early context surrounding the problem via the conduction of free field vibrational measurements in the field. Previously, any available data regarding the influence of the train speed on the vibrational amplitude was rare, considering that once the line was constructed, the train ran constantly in commercial operation. However, a railway company organised homologation testing on a new high-speed track between Brussels and Paris before its inauguration, providing data that would serve to validate numerical prediction models for high-speed train induced vibrations into the future. Measurements were taken at speeds varying between 160 and 330 km/h, and from a range of distances, 4 to 72m. This data complements the above study in terms of allowing for progressive validation of the same central issue (Degrande & Schillemans 2001).

At this point many studies started to branch into examination of mitigation/isolation systems relative to the vibration problem. This was usually done via finite element (FE) modelling. The first paper of this nature reviewed as part of this study used an FE model to evaluate the potential efficacy in the vibration damping of trenches possessing different geometric dimensions and in-filled material (Zoccoli et al. 2014). Another study used a combined FE and boundary element (BE) model, applied for the analysis of a track over a half-space and subject to a moving harmonic source. Again, various methods were considered and compared, looking at the capacity for a trench filled with a relatively soft material (Andersen & Nielsen 2005). Another mitigation approach reviewed the use of mats and the impact on the vibrations transmitted, finding that a considerable reduction of the free field vibrations could be achieved, but that it could also lead to higher displacements of the rail than those observed for the non-isolated scenario (Costa et al. 2012). There is room for further exploration of these systems into the future, as most studies surrounding mitigation systems to this point have been relatively ineffectual and inconclusive in terms of resolute solution.

Physical modelling and representation of a large-scale ballast structure for experimentation has also typically proven difficult, owing to the fact that such a large model would be required in order to obtain accurate solutions. It has however, served as an approach for a number of studies. One study looked to perform a specific analysis of the physical and mechanical properties of ballast that affect performance, followed by large scale testing centred around cylindrical tri-axial testing. The test findings suggested that deformations of fresh and recycled ballast vary non-linearly with the number of load cycles, and that with the insertion of geosynthetic materials, the extent of degradation and settlement were reduced. This information is important to consider when seeking an understanding of "typical" ballast behaviour experimentally (Indranatra 2006). Another experimental study presented a reduced scale experiment with three sleepers in order to better understand the dynamic behaviour of railway tracks, to identify the evolution of the mechanical properties and the settlement versus the number of cycles. This study found that settlement was found to be a function of the sleepers and observes that the increase of settlement per cycle was very high (Al Shaer et al. 2008).

Extensive use of analytical modelling to this point had been wide spread, with low computational need. These approaches, however, were only valid for a narrow range of track conditions. As a result, unrealistic assumptions relating to the excitation source had often been made. Early analytical approaches modelled the track as single beam (typically Euler-Bernoulli), which soon developed to a single layer method (Auersch 2008). Further improvements were then made using two and three layer methods, which could allow for additional features to be modelled. Discretely supported models were also created in order to incorporate the impact of sleepers and models where the subgrade could be considered as an elastic half-space (Connolly et al. 2013).

It was soon determined that for higher accuracy estimates, numerical modelling techniques were required. Recent technological advancements enabled this to become a possibility, with large simulations being able to be undertaken with technology that was unavailable for early numerical researchers (Connolly et al. 2013). As per the analytical approaches, these could also be divided into different categorisations of dimensions and layers, and the multi-body vehicle approaches were used to create more realistic representations of excitations from railway systems.

A variety of numerical approaches were used as part of one review I analysed, including constant axle loads and multibody loads. Discussion presented an outline of the challenge in selecting an appropriate vehicle model that could be accurate while minimising computational requirements. It found that it was difficult to make direct comparisons between their accuracy with such a wide range available. It also found that to model vehicle characteristics using non-linear theory, time domain analysis is more suitable, however if the system can be assumed linear then frequency domain analysis is sufficient (Kouroussis et al. 2014).

Ballast modelling has come under significant review with the growth of new numerical approaches. Many have started to examine the suitability of the discrete element method (DEM), due to its ability to simulate a large body of individual ballast particles. This is required as these particles are normally large and can’t be modelled as a continuum material. This approach allows for the examination of the unique nature of ballast particles, however it does require a large computational output. Research has been aimed at reducing this demand (Connelly et al. 2013).

One such approach to modelling ballasted railway tracks was introduced as part of another study, which proposed a move from homogeneous to heterogeneous, via the induction of a continuum model. The study randomly modelled Young's Modulus as a fluctuating parameter, with statistics being defined on discrete simulations of granular samples. Previous models hadn't taken into account the complex and dynamic behaviours of the ballast structure. The outcome for this study was indefinite, and the study proposed that the heterogeneity should be investigated in greater depth. Although the initial results were of benefit and encouraging, they proposed that several aspects could yet be further improved (Correa et al. 2017).

1.3 Terms and scope

The focus of this thesis is to conduct further testing on the dynamical behaviour of a randomly-fluctuating heterogeneous continuum model of a railway ballast as part of a high-speed train line system. This includes examining the numerical analysis of a randomised Young's Modulus medium, and its application to a numerical model of a railway ballast layer.

Testing was conducted under the supervision of a team within the MSSMAT (Mechanics of Soils, Structures and Materials) department of the University of CentraleSupelec in Paris, France. The model used for testing had previously been developed and applied in a number of studies, however was still in a very experimental phase of development and application. As a result, further testing and analysis on the model was to be performed across a two-month period under the project team.

The aim of new research was to monitor different sensors placed within the model, to understand how and if the stress field fluctuates compared to homogeneous models. The team was primarily interested in ensuring that the heterogeneity remained when the model was reconstructed and reapplied in application, as well as examining the impact this heterogeneity had on a numerical simulation of a passing train. There was added benefit in having the model reconstructed and tested by a student with no prior experience on the project, as it would ensure similar broader use for those new to what is a highly specialised field. The main intention of further analysis using this model is that it will enable greater understanding of ballast behaviour into the future, particularly with the introduction of the heterogeneity, which may open up further avenues of exploration for design. The tests and analysis performed, as well as the description of the construction process, compliment a literature review of existing approaches to understanding wave propagation characteristics, with a specific focus on train systems and railway ballasts.

The choice was made to include the monitoring of the randomised medium and its impact on the stress fields within the ballast layer, as well as provide examples of testing performed on the model comparing heterogeneous and homogeneous cases. The testing provides an opportunity to demonstrate the experimental advancements currently being made within the field, and to showcase the tasks learned and performed as part of a working placement with the MSSMAT team. The inclusion of a review of existing studies relative to the subject area of wave propagation also provides an opportunity to demonstrate the context of the study and to highlight the infancy of numerical modelling within the field, especially in application to railway ballasts.

The MSSMAT team took an approach that avoided me simplistically testing the model, as they were partially interested in understanding how easy the model would be for somebody with no previous experience to use. This meant that within the two-month period working under supervision, that I was tasked with compiling the model and applying the random medium. This involved a great deal of reading about the theoretical underpinnings behind the conception of the model, determining which source codes and parameters would be required to collectively produce the output, and effectively collecting and applying them to produce an output.

The major reason that this approach was taken was to provide a clear understanding of the work involved with constructing such a complex model, demonstrating that the testing is just a small part of the overall process, and to learn about the theory surrounding the model rather than just applying it. As well, it was a great way to regain familiarity with Linux coding, as it had been a lengthy period of time since I had performed any coding. As a result, the scope of the thesis has more of a research focus, and is limited in terms of potential avenues to explore via testing due to the time constraints such an approach applied.

1.4 Outline of current situation

The railway industry is a public and private infrastructure component that is looking to utilise new ways to improve and increase capacity, with the aim of improving transportation conditions for consumers into the future. As a result, many studies are being conducted in modelling various engineering components.

In this particular context, transport engineers are looking to improve the overall capacity of high-speed train lines. This will provide faster speed and higher capacity alternatives in line with the technological growth and development of these trains. Recent tests have demonstrated that the capacity exists for trains to exceed present limits, but that the system would not yet support such a capacity. When in close proximity to train lines, as is common in highly urbanised environments throughout the world that have significant train transportation demands, nearby infrastructure can become significantly damaged by wave propagation as this capacity and demand slowly increases in line with population growth (Commonwealth of Australia 2017).

Many studies are beginning to examine the various components of a train-line under stress, and how we can come to better conclusions regarding their behaviour in both the long and short term. The model that I have conducted testing with has been developed in an attempt to gain a greater understanding of ballast behaviour in particular, and how the impacts of wave propagation can be mitigated.

1.5 Existing solutions

While I will describe the existing solutions in greater depth in the literature review, below is an evaluation of some of the most commonly used existing numerical model applications, as taken from the literature (Connelly et al. 2014). It summarises some of the key advantages and disadvantages to each approach, which helped to highlight and identify a knowledge gap (Table 1).

Modelling approach	**Advantages**	**Disadvantages**
Elastodynamic wave propagation	• Analytical expressions for a homogenous half-space excited by a stationary point load. • Analytical approach to include a moving train/track model.	• Many assumptions must be made to reduce model complexity • Challenges have made numerical techniques more common
Finite difference time domain method (FDTD)	• Low computational effort and high-performance ABC's • Relatively straightforward to use	• Reduced performance in modelling domains with complex geometries and free surfaces • Difficult to simulate railway track components
Finite element modelling (FEM)	• Complex geometries can be modelled	• Needs an absorbing boundary condition (ABC) to prevent reflections from edges of the domain
Coupled finite element – boundary element modelling	• Complex geometries can be modelled using FE and large offsets modelled using BE	• Large computational response required

2.5D methods	• Reduces computer run time • Efficient for calculating the soil response for invariant track geometries	• Difficult to model stress distribution associated with ballast tracks
Pipe-in-pipe (PiP) methods	• Efficiently calculate underground vibration	• Relatively untested as opposed to other approaches
Empirical approaches	• Used easily and quickly for initial scoping studies	• Lack of in-depth analysis provided compared to more detailed approaches

Table 1: Advantages and disadvantages of various approaches (Connelly et al. 2014)

It was beneficial working individually for a supervisory team who could help direct the review toward literature that would benefit working knowledge of the overall issue, and in the process of seeking to identify a knowledge gap, this was no different.

Previous work conducted by members of the MSSMAT team had highlighted a real need for a model that fit between a homogeneous continuum approach, and discontinuous models. The homogeneous approach to modelling the medium was found to be unable to reproduce the complex behaviour of a realistic track, whereas the discontinuous approach was very difficult to simulate at the necessary scale.

As a result, a heterogeneous continuum model was proposed to fill the knowledge gap. It involved the modelling of Young's Modulus as a randomly-fluctuating parameter. This was found to have a significant impact on the wave field produced by the passage of a train on a typical ballasted railway track, and as a result it was proposed that this should be investigated further and in more depth (Correa et al. 2017).

1.6 Importance of the proposed research

An increase in urbanisation has resulted in urban areas becoming more densely populated with increasingly complex urban real estate. Similarly, this causes railway track designs to become more complex in nature (e.g. increased number of switches/crossings). These complexities may start to make transfer function testing more challenging, particularly for underground lines. Therefore, there is a clear opportunity for numerical modelling to play a more prominent role in the prediction of ground vibrations, and in further developing the capacity of these systems in the future (Connolly et al. 2015).

It was found that although much of the current research landscape is focused on numerical modelling and also the development of passive vibration abatement solutions, these approaches are yet to gain full acceptance in practice. In research, numerical models are used to benchmark the effect of changing different model properties on vibration levels, however this sensitivity approach was rarely adopted in the reports. Therefore, there are potential opportunities for either commercial consultants to embrace these developments to provide more in-depth solutions, or for research to become better aligned with the more fundamental forms of vibration prediction (Connelly et al. 2015).

1.7 Research problem

The research problem that the study has been aimed at contributing to resolving is to increase the safety margin at the ballast design stage of a high-speed train line. Maintenance plans for ballast layers are just one of many components of a train line requiring review, as a result of the increasing capacities that have come with technological and mechanical development. Ballast layers effectively undergo a higher number of loading cycles, which are not kept into consideration whenever a homogeneous layer is considered. Previous work has not treated the non-linear ballast behaviour, but shakedown analyses can now be run into the future, based on heterogeneous stress fields considering wave-trapping effects.

This thesis report will not target a performance of a shakedown analysis of the model, which was not feasible due to the large run time of the model and time constraints dealt with, as well as my relative inexperience with the model. Instead, it will contribute to a resolution of the research problem by examining the heterogeneous continuum stress fields in the model developed by the MSSMAT team, via the implementation of plots taken within the model stress fields to demonstrate the presence of the heterogeneity. It will also treat the research problem by providing a review of the current methods of modelling the problem, with an intention of highlighting potential avenues for future development and growth.

1.8 Research aims

My overall research aim is to provide grounds for further research within this field in Australia. I am hoping that the concepts discussed throughout this report can and will be used as the basis for further development and exploration of relative ideas into the future, particularly as the importance of the railway industry continues to grow within Australia across the next decade (Commonwealth of Australia 2017).

I am aiming to contribute to the continued development of the numerical model that I've performed testing with, by working in conjunction with the project team to further extend the analysis of their existing model. This will be achieved via exploring the stress field fluctuation of the heterogeneous model, and a review of the current status of experimental analysis in the field to date.

The aim is that this thesis will contribute to the continued progression of movement away from experimental lab-based testing that proves time consuming and costly, to a computational and numerical method that will enable ease of accurate computation and allow for in-depth visualisation of the impacts.

1.9 Experiment methodology

The broad philosophical underpinning that has been applied to my research is a combination of both quantitative and qualitative approaches. This thesis has used both approaches in the stages of both data and knowledge acquisition at various stages throughout the process.

It uses numerical data obtained from testing (quantitative), while also using categorical comparison relative to previous types of testing (qualitative) to obtain a full understanding of the characteristics of wave propagation in high-speed train lines. An in-depth discussion of the methodology behind my study will be provided in a later section.

1.10 Outline

To conclude the introduction, this thesis will consist of a literature review detailing the work that has been undertaken in the field to this point and how it has steadily progressed over the years, followed by a description of the theoretical methodology applied to the study. It will then move to demonstrate specific details of my experiment, present the results of the testing, and then discuss the outcomes before providing some concluding remarks regarding the importance of the work undertaken.

2 Literature Review

The literature reviewed as part of this process has directed me to construct a working thesis title, "Numerical modelling of a railway ballast with a heterogeneous medium". This research has provided me with an overview as to the historical and theoretical standpoints and approaches, their significance, and also an understanding as to why more testing is so crucial in developing our understanding and capacity of this specific type of infrastructure. I will now move to discuss the current state of knowledge within the field, highlighting more of the influential literature below, and demonstrate how this interacts with my proposed area of study.

A review will reference some of the major talking points relative to the literature, and how this has served to provide me with a greater insight in regard to those. The subjects discussed will be some of the core components of the issue at hand, in particular; wave propagation, vibration generation, numerical modelling input parameters, track modelling approaches and ground modelling approaches.

2.1 Vibration generation

The contact point between train rail and wheel produces quasi-static and dynamic components. The quasi-static component comes from the weight of the train, whereas the dynamic component comes from the train speed. The primary cause of vibration stems from the dynamic excitations and is influenced by stiffness, track connection quality and support conditions. Due to the adoption of new technologies, improvements in the ability to maintain track and rail systems have led to this type of excitation becoming less influential (Connelly et al. 2014).

Trains that operate regularly at high speeds produce larger amplitude vibrations. One of the challenges or problems that this has the potential to cause is when the speed is equal to the wave speed in the soil below. It is at this point that magnification of vibration can occur and cause significant levels of damage (Connelly et al. 2014). Any model for prediction of ground-borne vibration must include at least three main components. These three main components, which may themselves include many parts are the source, propagation path and receiver (Bahrekazami 2004). It is important to understand these three links, and how they interact with one another and influence the vibration situation, in order to predict and mitigate potential net negative effects (Leilei 2012).

FIGURE 1 HAS BEEN REMOVED DUE TO COPYRIGHT RESTRICTIONS

Figure 1: The transmission process of train inducted ground vibrations (Hall 2002)

The source is usually summarised into two major categories: naturally produced, and man-made. For situations regarding the excitations near to a high-speed train line, the vibration caused from the passing train is man-made, with the ground vibration being generated by the interaction of the moving train with the track which lies on the underlying soil (Leilei 2012). The propagation path can be broken down as follows. Vibration is generated in the track, and the waves induced by vibration propagate to the surrounding buildings and equipment foundations through the media. Finally, the waves are received by the surrounding buildings and foundations (Figure 1). From these foundations, the vibrations then propagate to other parts of the building. It is for this primary reason that measurement of the ground vibration has become so essential. If a greater understanding of the behaviour and magnitude can be obtained, it will allow for the avoidance of damaging net effects (Leilei 2012).

FIGURE 2 HAS BEEN REMOVED DUE TO COPYRIGHT RESTRICTIONS

Figure 2: Transmission of ground vibration velocity contours (Kouroussis et al. 2014)

2.2 Wave propagation

Vibrations generated at the track interface between the wheel and the rail will propagate into the free field, it does so in the form of waves (Figure 2). Waves are shelved into two separate categories, surface waves or body waves (Leilei 2012). Surface waves travel along a structural surface and decrease in magnitude with depth, whereas the major propagation of body waves occurs underneath the soil surface (Connelly et al. 2014) (Figure 3).

FIGURE 3 HAS BEEN REMOVED DUE TO COPYRIGHT RESTRICTIONS

Figure 3: Propagation of body waves and Rayleigh waves (Das 1992)

Compressional waves (P-waves) propagate longitudinally, and travel faster than any other type of wave. Particles are compressed and dilated in the direction of propagation. Shear waves (S-waves) propagate in a transverse direction, and always travel slower than P-waves. Particles are displaced perpendicular to the direction of wave propagation. The slowest type of wave is a Rayleigh wave (Leilei 2012). Compressional, shear and Rayleigh are the most common types of waves, although it is possible for other types of waves to occur, these are the three major types for this particular scenario. Of these three in this scenario, Rayleigh waves hold the most importance, as they have the potential to transmit more than half of the total excitation energy. This obviously means that they are the most likely type of wave to cause damaging effects in the track and to nearby structures (Connelly et al. 2014) (Figure 4).

FIGURE 4 HAS BEEN REMOVED DUE TO COPYRIGHT RESTRICTIONS

Figure 4: Seismic wave distribution (Connolly et al. 2013)

2.3 Numerical modelling inputs

Numerical modelling and analysis requires the input of certain data for parameters which subsequently directly impact the results. Descriptions of a number of the key inputs that are commonly used are below, with discussion of their importance in building an accurate model.

Density is measured as the mass divided by the unit volume of a material, and the value usually increases with depth. This is owing to the fact that lower soil layers have higher consolidation, making the solid particles more tightly compressed and packed together.

Poisson's ratio is defined as the ratio of expansion to the specific contraction caused by a compressional force used in a singular direction. It rations the degree to which the material expands in the opposing directions. Increases in this ratio are usually attributed to the presence of the water table, most evident for clayey soils that become incompressible when fully saturated. P-wave speed will increase in this instance as it becomes more a representation of the water than soil. Alternatively, the S-wave speed doesn't change in this case as water has no shear strength and thus it becomes more a representation of the soil. This ratio also has a minor impact on Rayleigh wave speed, found typically within the high range of the S-wave velocity (Connelly et al. 2014).

Young's modulus is calculated using the tangent modulus of the initial, linear portion of the stress-strain curve. It defines the stiffness of a particular material. It serves as a crucial parameter in railway vibration propagation, considering stiffness is a major criterion used to determine successful workmanship on site. Shear modulus is strain dependent, and soils will behave non-linearly at large strains. This relates to ground vibration in that soil particle deformation is normally comparatively miniscule to its dimensions (Connelly et al. 2014).

Damping is a measurement of the rate at which energy is dissipated as it passes through a particular material. It is normally caused by the ballast when discussing the track. The relationship with frequency is non-linear, making the modelling of this parameter difficult in the time domain compared to frequency domain. In in-situ soils, the value is higher in the upper layers and decreases with depth, due to the soil particles in this layer being less compacted. This means that the wave loses more energy as it passes through the voids. At higher frequencies, a soil may display larger damping values if saturated (Connelly et al. 2014).

2.4 Track modelling

The source of railway vibration is the interaction between the train and track, and it is therefore crucial for it to be modelled as precisely as possible. If modelled in an inaccurate manner, it will produce errors in output relative not only to the track vibration, but various other parameters as well. There are two major methodologies that have been used to model tracks, analytical and numerical. A moving or stationary load can be applied depending on the objective of the model, and the computations can be conducted in either time or frequency domain (Connelly et al. 2014).

2.4.1 Analytical track modelling

The most common approach to modelling the track derived from the literature was the analytical modelling approach, historically preferred due to being more computationally efficient when compared to the numerical modelling approach. Also, for complex track types, absorbing boundaries can be hard to implement. Without these, reflections at model edges can occur enacting errors in the output (Connelly et al. 2014).

Some of the first analytical models were based on the assumption of a single beam representation of the rail. This was typically Euler-Bernoulli in conjunction with a Winkler foundation, and sought to represent the track or a part of the track with soil profile. Considering the above, this meant that the system was modelled homogenously (Auersch 2008).

Low frequency characteristics were investigated by single layer methods, with in-depth examination of the interaction between different beams and plates with the soil. Ultimately, this investigation demonstrated that model accuracy could be further improved by changing the beam formulation from Euler-Bernoulli to Timoshenko. The Timoshenko adoption allowed for additional degrees of freedom and subsequently, higher accuracy in output (Connelly et al. 2014).

Additional components were then allowed for via the adoption of a two and three layer methods. The ballast could be simulated as well as other components of the system, allowing for a more complete representation. However, implementation of a continuous support resulted in the non-inclusion of sleeper effects, and the impact of the discrete sleepers was uniformly distributed. This approach still proved accurate for some problems, but research was still hoping to bring improvements aimed toward including the sleeper passage frequencies (Connelly et al. 2014).

2.4.2 Finite element track modelling

Although analytical models were successful in terms of predicting 2D track geometry response, they lacked application in terms of including varietal effects, such as multi-body vehicle impacts and track/wheel irregularity. As a result, the track has more commonly been modelled using versatile numerical methods. Of these methods, the finite element method (FEM) has proven one of the most popular and commonly used.

FEM track models, similarly to the analytical approaches, can be divided into many different layers (one, two, and three or more) or two dimensions. Multi-body vehicle modelling approaches allowed for a more complete and representative replication of the frequency excitation from a moving train on the track (Figure 5).

FIGURE 5 HAS BEEN REMOVED DUE TO COPYRIGHT RESTRICTIONS

Figure 5: Multi-body vehicle modelling approach (Connelly et al. 2014)

By upgrading the approach from a basic moving point load, this allowed for a fully integrated train-track system simulation, improving model accuracy and allowing for the exploration of vibration response due to the components (Connelly et al. 2014).

2.5 Ballast modelling

The concept of track vibration cannot be fully explored without a solid conceptualisation of ballast behaviour. Recent studies have been approaching the issue from the perspective of attempting to better understand ballast behaviour. The ballast layer is usually comprised of coarse crushed stone, and is the upper layer of railway track structure. The primary role that it plays is to transmit and repartition static and dynamic loads (Correa et al. 2017). Modelling the ballast accurately is difficult, as granular materials display unique behaviours depending on applied stresses and strains (Correa et al. 2017).

There have been developments in ballast modelling, with the discrete element method (DEM) being one of the most popular. The major reason for its popularity is due to the size of ballast particles, which are typically large, and as a result can't be modelled realistically as continuum material. The DEM approach solves this problem by allowing for the simulation of a large body of individual ballast particles, which can then be used to investigate its qualities. One significant challenge with DEM modelling of the ballast is that the non-linear nature and complex geometries of the individual particles results in a large computational demand. Therefore, a focus in reducing this demand has been prevalent, with many authors seeking to integrate DEM ballast models within an FE track modelling framework (Connelly et al. 2013).

Another numerical implementation that has been used in place of DEM modelling is the non-smooth contact dynamics method (NSCD). This method differs from DEM in that it can deal with multiple relations and velocity variations within a single time step. An example of such a granular model is demonstrated below (Figure 6).

FIGURE 6 HAS BEEN REMOVED DUE TO COPYRIGHT RESTRICTIONS

Figure 6: Example of granular medium simulated with NSCD method (Correa et al. 2017)

It yields an implicit scheme, which remains stable for larger time steps. The results obtained with these granular approaches are able to reproduce the solid to liquid transition and the inelastic deformations of the ballast, as well as the seemingly random patterns of contact forces that can be observed experimentally.

2.6 Ground modelling

As the track serves as the source, the ground serves as the transmission path for waves to propagate toward nearby structures. To best understand the holistic issue, research of literature in this area was conducted as well, finding other studies that had been attempting to develop methods of modelling this transmission path to understand and predict behaviour.

2.6.1 Finite difference time domain method (FDTD)

The finite difference time domain method (FDTD) is an approximation of the seismic wave equation using a central differencing integration scheme. It is a relatively straightforward numerical modelling tool, where the domain is compartmentalised into stress and velocity components and staggered in 2D/3D space. The advantage of this method is that it doesn't require as much computational effort, and it is easy to divide the workload between a computer cluster or separate number of processors (Connelly et al. 2014). Boundary conditions are also implemented, making it easier to use compared to the other numerical methods. Domain size can be reduced which decreases the number of calculations needing to be performed by the model. One weakness of the method is the reduction in quality due to modelling a domain with complex geometries and free surfaces. The coupling between wheel and rail represents a challenge in simulation that hasn't yet been perfected by this type of model (Connelly et al. 2014).

2.6.2 Finite element method (FEM)

The finite element method (FEM) serves as an alternative that allows for more complex geometries to be modelled, combating the weaknesses of the FDTD method by allowing for track components to be modelled explicitly. It allows for the investigation of track defects and changes in stiffness, at least when modelling in the time domain. Despite the benefits, similarly to the FDTD method, for an unbounded domain (for example, a soil) absorbing boundary conditions are needed in order to stop reflections from edges of the domain (Figure 7).

For time domain FEM modelling, different solutions have been proposed, including infinite elements, a combined FEM/thin layer, and sealed boundary FEM. One approach that is being used more commonly is the use of a perfectly matched layer (PML). A PML is a collection of layers with identical material properties to the domain, which serve to stretch the real and imaginary coordinate space (Connelly et al. 2014). Wave amplitudes are dampened more effectively using this approach compared to previous boundary conditions. One weakness is the extremely large computation expense that comes with time domain FEM modelling for 3D wave propagation. Computing at each individual time step can take days, and depending of the scope of the project this may not prove suitable (Connelly et al. 2014).

FIGURE 7 HAS BEEN REMOVED DUE TO COPYRIGHT RESTRICTIONS

Figure 7: Coupled BE/FE approach (Connelly et al. 2014).

2.7 Mitigation of ground vibration

Mitigation of ground vibration has been a point of reference for a number of studies, with an aim at reducing the issue by implementation of form of structure. Studies conducted recommended a variety of mitigation measures, with ballast mats and other special track forms being most common (Costa et al. 2012). Other active solutions have also been proposed with a tendency to focus on the source rather than the receiver, with the only passive solutions being building modifications (Talbot & Hunt 2003).

Soil vibration mitigation via the implementation of a trench structure has also started to be popularly reviewed and simulated. They are located at distance from the track and are typically filled using low density material, in order to increase the reflection coefficient between the soil and trench and increase its effectiveness. Numerical modelling has been conducted using this scenario in a lot of depth, and it has been found that the ideal trench depth (if used) should be roughly half the length of the largest Rayleigh wavelength (Connelly et al. 2014).

2.8 Critical velocity effects

The development of increased speed capacities for modern trains means that the speeds often approach critical wave velocities in the system, which leads to increases in structural vibration, and potentially large rail deflections. There are two major critical wave velocities in the track-ground system: the velocity of the Rayleigh surface wave in the ground, and the minimum phase velocity of bending waves propagating in the track (Kouroussis et al. 2014).

The latter velocity is referred to as the track critical velocity. These velocities are both commonly exceeded by modern high-speed trains, and the likelihood is even higher in the case of soft soils, where critical velocities become very low. Studies have reviewed and discussed the effects of transient rail deflections on associated ground vibrations in the cases of train speeds approaching and exceeding Rayleigh wave and track critical velocities (Krylov et al. 2000). It is difficult to analyse these effects due to the complicated nature of the interaction between the track and the ground. This interaction produces many different Rayleigh waves, speeds and frequencies).

Trains exceeding the Rayleigh wave velocity can potentially give rise to the creation of a Mach cone, and track deflections can become quite large (El Kacimi et al. 2013). This can result in large scale shear strains, which causes the system to act in a non-linear fashion. As a result, non-linear theory is used to model material behaviour, which serves as a real challenge for modelling critical velocity effects and increases model complexity (Krylov 2015).

2.9 Heterogeneous continuum modelling

As part of my review an examination of one particular model of a ballast was conducted, which is aiming to help mitigate the issues that have been raised throughout this review, by improving the understanding of the mechanical behaviour of the track system.

A heterogeneous continuum model was used in this study, with Young's Modulus being modelled as a randomly-fluctuating parameter with statistics defined on simulations of granular samples with realistic shapes. The research found that the stress distributions from the heterogeneous model matched the equivalent stresses in simulations, and the authors proposed further use of the model to investigate the impact on the wave field produced by the passage of a train on a typical ballasted railway track further (Correa et al. 2015). An example of such a model that was developed is as below (Figure 8).

FIGURE 8 HAS BEEN REMOVED DUE TO COPYRIGHT RESTRICTIONS

Figure 8: Example of wave propagation analysis (Correa et al. 2017)

There is no notion of particles hiding within this model. It is a granular model, without particle geometry at the microscale. This is because testing was conducted using the spectral element method, opposed to the discrete element method. The major reason for this is that using the spectral continuum approach does not require a switch to the more cumbersome discrete approach. Although the discrete approach is accurate in terms of micromechanics, it is not as competitive in terms of saving time compared to the spectral approach.

Past authors (Correa et al. 2017) have adopted the discrete approach in order to determine the efficiency of both approaches to this problem. On the macroscale, the two different model types are the same in terms of output, however on the microscale the discrete approach has been known to be more accurate, as this approach is able to cater for particle geometry. Given the large applicability and decreased run-time advantages of the approach, the team was most interested in using and testing the spectral continuum approach on the macroscale.

Weaknesses of the model are that it fails to take into account situations when grains rearrange, rendering it inappropriate for fatigue simulation. It also considers locally averaged stress and strain, and localised stress states are required for fatigue simulation at the interface with the sleepers. The identification process was also performed in statics, and more realistic and predictive simulations would require the inverse problem to be performed on dynamic measurements (Correa et al. 2017).

Another study conducted by this author involved the construction of large scale randomly fluctuating continuum models of ballasted railway tracks. The numerical simulation of dispersion curves in railway tracks is usually performed homogeneously. However, homogeneous continuum models fail to reproduce complex dynamical behaviours, and as a result a heterogeneous model was proposed by the authors. These models allowed for construction of dispersion curves for the ballasted layers. The heterogeneity was shown to create relatively slow waves at medium frequencies and a filter effect in the higher frequencies (Correa et al. 2017).

Because of the geometry of the ballast and its heterogeneity, the author proposed that it was impossible to solve the dynamical equations analytically, and that moving forward computer simulations would be therefore an irreplaceable and unavoidable tool to understand the dynamical behaviour of the soil-ballast system (Correa et al. 2017).

2.10 Conclusion

The analysis of the literature to this point has provided some interesting points for discussion as pointed out. It can be demonstrated that railway vibration assessments are becoming increasingly common, and considering the strong infrastructure investment predicted in the coming years (particularly light rail transit in urban area, and also high-speed rail), this trend does appear set to continue. Furthermore, the predicted increase in global urbanisation will likely result in more people/structures being affected by railway vibrations, thus further supporting this.

This increasing urbanisation has also resulted in urban areas becoming more densely populated with increasingly complex urban real estate. Similarly, this causes railway track designs to become more complex in nature (e.g. increased number of switches/crossings). These complexities may start to make transfer function testing more challenging, particularly for underground lines. Therefore, there is a clear opportunity for numerical modelling to play a more prominent role in the prediction of ground vibrations (Connolly et al. 2015).

It was found that although much of the current research land-scape is focused on numerical modelling and also the development of passive vibration abatement solutions, these approaches are yet to gain full acceptance in practise. In research, numerical models are used to benchmark the effect of changing different model properties on vibration levels, however this sensitivity approach was rarely adopted in the reports. Therefore, there are potential opportunities for either commercial consultants to embrace these developments to provide more in-depth solutions, or for research to become better aligned with the more fundamental forms of vibration prediction (Connelly et al. 2015).

It is my aim to further extend the body of research relative to numerical modelling, in the hopes that we can extend the depth and breadth of knowledge into Australia.

3 Theoretical methodology

3.1 Overview

Quantitative data concerns numeric variables and are measures of values or counts, usually expressed in the form of numbers. Qualitative data concerns categorical variables, and is a measurement of “type” usually represented by a name, symbol or number code. Data collected with a numeric variable will always be quantitative, and data collected about a categorical variable will always be qualitative. The data provides different outcomes, and are often used together to get a full picture of a scenario (Australian Bureau of Statistics 2013).

The broad philosophical underpinning that has been applied to my thesis is an approach that serves as a combination of both quantitative and qualitative approaches. The numerical data obtained from my testing (quantitative) has been compared with testing conducted in past studies (qualitative), to obtain a full and thorough understanding of the characteristics of the subject.

In order to reference the model that experiments and testing was performed with, the randomised continuum heterogeneous model of a granular medium that was applied to the model first needs to be referred to, and the theoretical underpinning behind it. The next section of this report will detail and describe the theoretical background involved in the creation of the randomised continuum heterogeneous medium. It will then describe how this medium had previously been applied in testing in conjunction with the numerical model of the railway ballast, before finally moving to detail my own experimental process with the stochastic medium and numerical ballast model in depth.

3.2 Stochastic medium of Young’s Modulus

This randomised representation of Young’s Modulus was originally created and developed by members of the MSSMAT team that supervised my time at CentraleSupelec. As they had created both the medium and the model, they provided me with literature that helped to detail the theoretical background behind the development of the medium, including the decisions surrounding realisation of the field and how it could be applied to a numerical model of a railway ballast. Below is a theoretical background of the development of the medium, with the application to the model to follow.

3.2.1 Correlation model

To create the correlation model, assumptions were made that the material be idealised as a two-phase medium: void matrix and impenetrable spheres. This provides an appropriate order of magnitude of the scale of fluctuations in space.

The correlation structure represents dense packing of impenetrable spherical particles of diameter d, with volume ratio Ø. This function depends only on Ø and d and shows the impenetrability condition and cosine decaying behaviour, more pronounced with volume ratio. This model is based on the Meyer representation of the canonical function, by which the 2-point matrix probability function $S_2(r)$ can be written as a sum of a finite number of terms. Formally, this relation reads, using also the Ornstein-Zernike relation (1):

(1)

$$S_2(r) = 1 - nV_2(r) + n^2\mathcal{F}^{-1}\,[(\widetilde{m}(k)^2\left(\frac{\tilde{c}(k)}{1-n\tilde{c}(k)}\right)]$$

where $\mathcal{F}^{-1}$ denotes the inverse Fourier transform of a function of the k variable, n is the number density of the particles and $V_2(r)$ is the volume of the union of two spheres whose centers are separated by r, equal to (2):

(2)

$$V_2\left(r < \underline{d}\right) = 4\pi/3(1 + \frac{3}{4}\frac{2r}{\underline{d}} - \frac{1}{16}(\frac{2r}{\underline{d}})^3)$$

for $r > d$ and $V_2(r > d) = 8\pi/3$ otherwise. The Fourier transform of the indicator function of the particle (equal to 1 inside the particle, and 0 outside) $\widetilde{m}(k)$ is equal to (3):

(3)

$$\widetilde{m}(k) = \frac{4\pi}{k}(\frac{sink}{k^2} - \frac{cosk}{k})$$

and, using the Percus-Yevick approximation, the Fourier transform of the direct correlation function is given by (4):

(4)

$$\tilde{c}(k) = -\frac{4\pi}{k^3}\{\lambda_1[\sin(2k) - 2kcos(2k)] + \frac{3\eta\lambda_2}{k}[4ksin(2k) + (2 - 4k^2)\cos(2k) - 2] + \frac{\eta\lambda_1}{2k^3}[(-2k^4 + 6k^2 - 3)\cos(2k) + (4k^3 - 6k)\sin(2k) + 3]\}$$

with,

$\eta = \frac{4\pi n}{3}, \lambda_1 = (1 + 2\eta)^2 / (1 - \eta)^4$ and $\lambda_2 = -\left(1 + \frac{\eta}{2}\right)^2 / (1 - \eta)^4$

Correlation function for an impenetrable sphere packing with constant diameter d, with volume ratio Ø. Four cases were presented here: d = 3.9cm and Ø = 0.5682 (solid line); d = 3.9cm and Ø = 0.2 (dashed line); d = 6cm and Ø = 0.5682 (dotted line); d = 6cm and Ø = 0.2 (dash-dotted line). Finally, the sought correlation model is produced (Figure 9), which is a normalised autocovariance, is directly related to the 2-point matrix probability function through the relation (5):

(5)

$$\mathcal{R}(r) = S_2(r) - (1 - \eta)^2 / \eta(1 - \eta)$$

The reason for normalising is to remove the discontinuity at the origin, in order to obtain a continuous model of Young's Modulus.

FIGURE 9 HAS BEEN REMOVED DUE TO COPYRIGHT RESTRICTIONS

Figure 9: Correlation function for an impenetrable sphere (Correa et al. 2017)

3.2.2 First order marginal law for Young's Modulus

The first-order marginal law reflects the fact that Young's Modulus is a positive quantity. Two of these were considered in the development and selection:

1. Log-normal law
2. Gamma law

Both models are parameterised by an average μ_E and variance σ_E^2 . The probability density function of the log-normal distribution is (6):

(6)

$$\mathbb{P}(E) = 1/E_s\sqrt{2\pi}\exp(-\frac{(lnE - m)^2}{2s^2})$$

where $s^2 = \ln(1 + \sigma_E^2/\mu_E^2)$ and $m = \ln(\mu_E) - s^2/2$ are the variance and mean of the underlying Gaussian distribution. The probability density function of Gamma distribution is (7):

(7)

$$\mathbb{P}(E) = \frac{E^{k-1}}{\Gamma(k)\theta^k}\exp(-\frac{E}{\theta})$$

where $\theta = \sigma_E^2/\mu_E$ and $k = \mu_E^2/\sigma_E^2$ are the shape parameters of the Gamma distribution, and $\Gamma\,(k) = \int_0^{+\infty} t^{k-1}\exp(-t)\,dt$ is the Gamma function.

An example of these two densities (for $\mu_E = 1\frac{N}{m^2}$ and $\sigma_E^2 = 10\mu_E^2$) is plotted in Figure 10. They mainly differ in the tails, that is to say for the description of very small and very large values. Note also that, as desired for a positive parameter, the support of both function is limited to $\mathbb{R}^+$ (Correa et al. 2017).

FIGURE 10 HAS BEEN REMOVED DUE TO COPYRIGHT RESTRICTIONS

Figure 10: Probability density function for the log normal distribution (dashed line) and Gamma distribution (solid line) (Correa et al. 2017).

3.2.3 Realisation of the stochastic field

Realisation of the stochastic field is required for a numerical simulation. Therefore, the team devised two separate ways that generation of samples could be produced: either in the space or in the spectral domain.

The spectral domain approach involved summing a large number of functions oscillating with random phases and with amplitudes designed to match the desired correlation model (8):

(8)

$$E(x) = 2\sum_{j=1}^{N}\sqrt{\sigma_E^2\mathcal{R}(k_j)\Delta k}\,\zeta_j\cos(k_j x + \phi_j)$$

where ζ_j are independent unit centred Gaussian random variables, the $\emptyset_j$ are independent random variables, uniform over $[0,2\pi]$, *N* in the number of terms in the sum, Δk is the discretisation step in wave-number space, and $\mathcal{R}(k_j)$ is the normalised power spectral density function in wave-number space (the Fourier transform of the normalised auto-covariance). This algorithm generates random fields that are asymptotically Gaussian and asymptotically ergodic, so that a Rosenblatt transformation is then applied to obtain the desired first-order marginal density (Correa et al. 2017).

This technique allows for efficient use of the Fast Fourier Transform (FFT) algorithm. However, when simulating realisations over large clusters of supercomputers, as required in the simulations conducted as part of my testing, the complexity of the FFT is too large. Therefore, consideration is given to a particular implementation of the spectral representation method, which considers independent realisations over each processor and overlaps these to merge the continuity and correlation over the global domain.

The reliability assessment of this model has been detailed throughout another report by the project team (Correa et al. 2017). This assessment was covered in the generation of the correlation models which will also be briefly discussed below. Stress distributions were obtained using the continuum model and the discrete element method, and these distributions were compared in the report to experimental results. The comparison was accurate and favourable, and thus validated the confidence levels for further use.

3.2.4 Identification process

A general identification process was also introduced in assisting with the parameter identification of the randomised medium (Correa et al. 2017).

The steps of the general identification process are summarised below:

1. An initial value of the variance is chosen
2. A realisation of heterogeneous Young's Modulus is generated
3. The distribution of stresses $\rho\ (\sigma_{zz})$ in the Finite Element model (described in Section 4.1.2) is computed
4. The L^2 distance between that distribution and the reference is computed as below (9):

(9)

$$L_r = \int (p(\sigma_{zz}) - p_r(\sigma_{zz}))^2 d\sigma_{zz}$$

5. Depending on the distance obtained, and using a simplex algorithm, new values of the variance are proposed, and an iterative process is considered on steps 1-5.
6. If convergence is not obtained, a new value of the variance is proposed and a new iteration starts at Step 2, else the algorithm exits.

Overall, the most accurate reproduction of the stress distribution in a granular sample was previously obtained by modelling the Young's Modulus as a random field with Gamma first-order marginal density and granular correlation model. The variance depends on the averaging volume for the equivalent stresses and has been identified as close to $\frac{\sigma_E^2}{E^2} \approx 11$ for $V = (4cm)^3$.

3.3 Application to numerical model of railway ballast

As the stochastic model of Young's Modulus defined in the above section had been created, it was then subsequently proposed to be applied to a numerical model of a high-speed train line railway ballast. The major aim of seeking to conduct tests on a model such as this was to analyse the impact of the heterogeneity it models on a dynamic simulation of the passage of a train. As I have also tested with the same model for my own experimental use, below is a theoretical background of the numerical model of the ballast, and a description as to how the solver that produces the final results works.

3.3.1 Numerical model of the ballast

The numerical model developed by the team is a one-way track segment with a length of 38m. The ballast has a height of 48cm and width ranging from 3.9m at the top to 5m at the bottom. Beneath, the soil is numerically modelled on a width of 20m and a depth of 5m. The concrete sleepers have dimensions 20 x 30 x 200m3, and are separated by d = 0.6m, there are embedded in the soil on a height of 10cm. This simple set up is representative of a realistic ballasted railway track (Correa et al. 2017).

In the simulated domain, the wave field is the solution of the wave equation (10):

(10)

$$\nabla \cdot \sigma - \rho u = 0$$

where the material is assumed isotropic and linear, so that the strain is $\varepsilon = \frac{\nabla u + \nabla u^T}{2}$ and the stress is $\sigma = \lambda(x) Tr\varepsilon \, \mathbb{I} + 2\mu\varepsilon$, with λ and μ the Lame parameters, and $\mathbb{I}$ is the identity second-order tensor. The density is denoted *p*. The Lame parameters are related to the wave velocities through $V_p = \sqrt{(\lambda + 2\mu)/p}$ and $V_s = \sqrt{\mu/p}$. The soil is assumed homogeneous in all simulations, with V_s= 180 m/s, V_p= 350 m/s, and *p* = 1900 kg/m^3. The concrete sleepers are assumed homogeneous in all simulations, with V_s= 2500 m/s, V_p= 4500 m/s, and *p* = 2400 kg/m^3. In the simulations where the ballast is assumed homogeneous, V_s= 150 m/s, V_p= 380 m/s, and *p* = 1900 kg/m^3 In the simulations where the ballast is assumed heterogeneous, the random model defined in the above section has been applied, and the average values are taken equal to those of the homogeneous case.

The vertical loading was also considered, and models the influence of a train-rail system on the sleepers. It works to take into account the elasticity of the rail by distributing the point loads of the bogie of a train onto concurrent sleepers. The movement of the train is factored in by moving the position of the point loads at the appropriate velocity. The loading on each of the sleepers is given by (11):

(11)

$$F_i(t) = \frac{QY}{2}[C^{(v_0(t-\delta_{1)-a)})^2/d^2} + C^{(v_0(t-\delta_{1)-a-L)})^2/d^2}]$$

where Q is the load magnitude, $L = 3\text{m}$ is the wheelbase, and $\text{a} = 5\text{d} = 3\text{m}$ is the critical distance, beyond which the load is assumed to vanish, $V_0 = 100m/s$ is the train velocity and $C = 0.61$ and $Y = 0.41$ are constants dependent on the soil-ballast combination. The latter values are obtained from the experimental values in literature reviewed previously in my report (Al Shaer et al. 2008). In order to produce a "moving load" each sleeper is associated with a delay δ_i.

Soil property values were provided to the laboratory teams at CentraleSupelec by SNCF (Societe Nationale des Chemins de Fer francais), or "French National Railway Company" as translated into English. These values were provided as part of the existing partnership between CentraleSupelec and SNCF in the field of study, and were based on data collected experimentally by the organisation.

3.3.2 Description of the solver

In order to produce the solution, the model needs to be placed through a solver. The solver used by the project team when previously testing this model was based on the Spectral Element Method (SEM). It is a high-order FEM that uses a Gauss-Lobatto-Legendre (GLL) quadrature rule for integration and Lagrange polynomials based on the quadrature nodes. Inserting the polynomial functions and quadrature rules into the variational form of (12):

$$MV = F_{ext} - F_{int}(U) \tag{12}$$

where:

- U and V are vectors of the displacement and velocity at the nodes, respectively,
- M is the mass matrix
- F_{ext} and F_{int} are vectors of influence of the passing train and the internal forces, respectively.

As it uses high-order polynomials, the method is exponentially precise: increasing polynomial order leads to an exponential decline of the error. This is called spectral precision and is the major reason the SEM method is so named. Using under-integration, the mass matrix develops naturally diagonal, which permits to use an explicit second-order finite difference scheme in time. Although the stability condition requires the use of very small stages, the construction of the solution at each time step is easy because the inversion of the mass matrix is immediate (Correa et al. 2017).

4 Experimental details

4.1 Overview

The supervisory team were primarily interested in adding sensors to the existing model (Figure 11), to monitor how and if the stress field fluctuated in the stochastic medium. It was important to ensure that the heterogeneity remained when the model was reconstructed and reapplied in application, and that it accurately demonstrated randomised variation. The primary role of experimentation was to analyse the fields across a random number of different sensors within the stochastic medium, as well as examine the differences between testing with the heterogeneity in place and without. There was an added benefit in having the model reconstructed and tested by a student with no previous experience working with such complex computation, in that another of the aims of research as identified by the supervisory team was to ensure broader use across what is a highly specialised field of knowledge.

FIGURE 11 HAS BEEN REMOVED DUE TO COPYRIGHT RESTRICTIONS

Figure 11: Screenshot of additional sensors (white) added to model

4.2 Testing structure

The first two weeks of the experimental process consisted of being supplied literature by the MSSMAT team, as they wanted to provide the overall context in which the model developed would be tested. It served to provide the background behind the research problem and allowed conceptualisation of what this research was hoping to achieve. With only basic prior knowledge and understanding of such a specific area of geotechnical engineering, it assisted in providing some perspective regarding what could be accomplished with the experimental testing.

After familiarising myself with the literature surrounding the problem, then began the process of familiarisation with the remote supercomputing cluster database, and learning how to navigate within that space. Once able to understand the location within the system and the overall structure of the cluster, experimentation with the mesh via generating my own versions of an existing geometry began. After a period of experimentation with the mesh geometry, progress was made in compiling the source code and inputting parameters to prepare the SEM calculations required, finally culminating the experimental process by running the different test simulations to be presented in the following results section. A general breakdown of the testing structure is below (Table 2). The section below will deal with some of the key experimental stages encountered during the study, and detail how the final results were obtained which will be presented in a later section.

Task	Duration
Literature Review	Week 1-2
Mesh experimentation/realisation	Week 2-4
Code formatting	Week 4-6
Code assembly	Week 6-7
Modelling outputs	Week 7-8

Table 2: Experimental testing structure

4.3 Supercomputing cluster

The computation was run on a mesocentre jointly shared and operated by two institutions in Paris – CentraleSupelec and ENS Paris-Saclay. The name of the mesocentre was Fusion, and it was referred to by the project team as the Moulon Mesocentre, in reference to the location (Moulon) of both labs in Paris. The mesocentre was originally created in order for the organisations to pool their computing resources and set up a common support team.

The major objectives of computing using the shared cluster were twofold:

1. Economies of scale, extensibility of the system and eco-responsible pooling.
2. To allow the experimentation, the adjustment and then the consolidation of the establishment of shared computing resources between institutions, with a particular attention given to (i) the organisation of an efficient and proximity support structure and (ii) the setting up of a joint scientific animation based on the exchange of skills and know-how between the different disciplines.

The mesocentre is intended to increase its computing capabilities and open itself to researches from other institutions as well as industrial companies. Benefiting from the environment of the Paris-Saclay University, the mesocentre aims to create a space for scientific exchanges and to contribute to the emergence of high-performance computing strategies and projects between different actors of the Paris-Saclay University (CentraleSupelec n.d.).

It would be through this supercomputing cluster that the ability to perform the SEM calculation on the model would be provided, to analyse the heterogeneity as applied to the railway ballast.

4.4 Simulation generation process

In the first two weeks, reservations were expressed to the project team that it was taking a long time to understand and grasp the basics of the system set up within the cluster. However, these fears were soon put to ease as they informed that familiarity with the system regularly took a period of one month. The learning curve being experienced with the code and system was deemed normal. Building and compiling the model as new via a new personal database within the system took almost half the entirety of the working period (one month).

With a vast array of modules, codes, source files and programs available within the cluster, it was important to generate an awareness of where the project was located within the system and the overall level of progress at any given time. It really enabled conceptualisation of the project location within the system at a given point in time, and helped in allowing effective navigation of the shared cluster space.

The step-by-step process has been included below.

1. Create geo file in Gmsh
2. Create a new folder within the shared computing space
3. Copy files from sources into new folder
4. Generate the mesh using the geo file previously created in Gmsh
5. Run pymesh code to generate the h5 file
6. Run prepro code to generate the complete mesh
7. Run SEM code with the completed mesh
8. Visualise the output files and parameters using Paraview

I made sure to clarify with one of the supervisors regarding the suitability of these stages to the experimental testing process, to determine whether I had omitted any crucial stages. The project team gave the step-by-step the tick of approval, clarifying that it was a complete way of looking at the different stages of submission.

4.5 Mesh experimentation and realisation

In order to actually perform the SEM calculation, I needed a structure to run it on first. As a result, a basic geometry of the structure we were modelling needed to be produced via the generation of a mesh. The program used to generate this mesh was Gmsh, a free 3D finite mesh generator.

A number of scholars have previously investigated the impact of damping as a wave propagation method, including Zoccoli et al. (2014) and Correa et al. (2017). As a result, my initial experimentations with the mesh centred around the potential addition of a damping structure or trench to the existing model. I believed it would be a unique way to use the existing structure, and would be interesting to determine the impact of the structure combined with the heterogeneity that would also be applied. Below is a basic preliminary design of my changes to the existing structure (Figure 12).

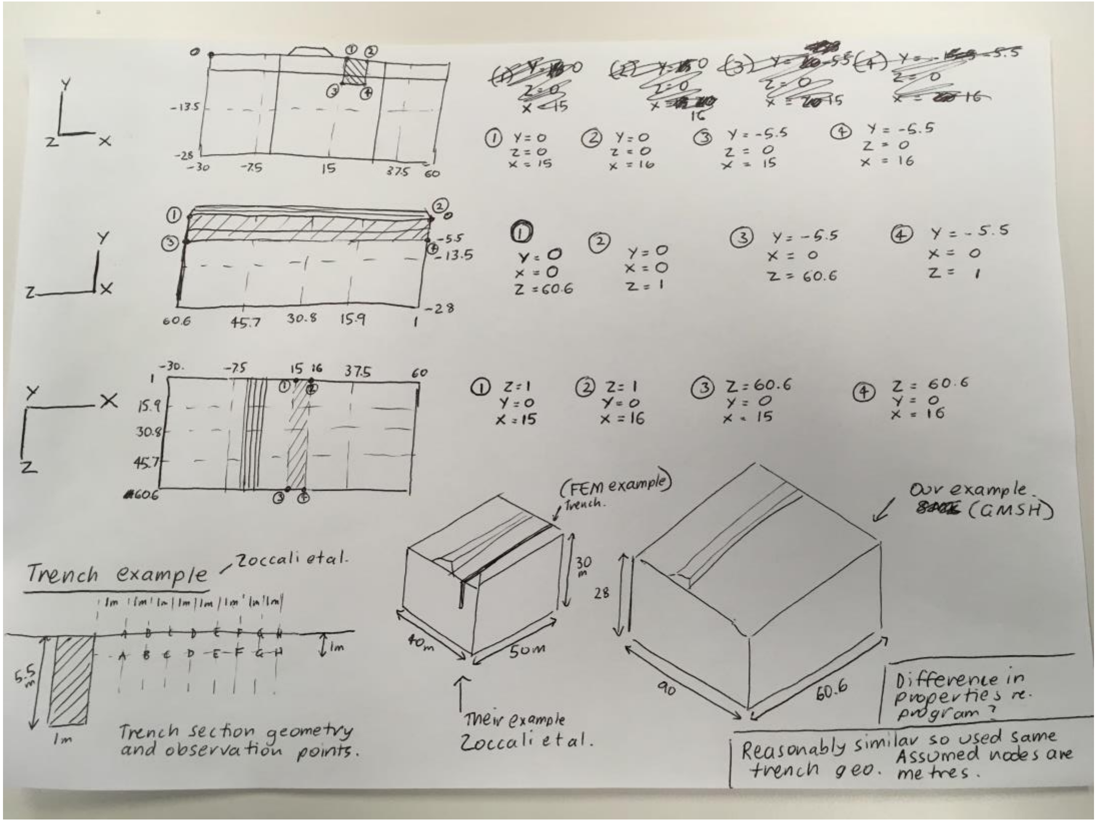

Figure 12: Preliminary design undertaken regarding the geometry of the various points

Experimentation started by looking at the existing model that had been produced and used previously (Correa et al. 2017). This structure was changed and a new design created, with the inclusion of trenches to act as a damping mechanism. The geometry was a little complicated at first, but after adding the specific coordinates to the existing geometry, trenches were added in similar size and shape to those proposed by previous authors (Zoccoli et al. 2014).

The initial theory behind examining the impacts of damping structures was to look for any insertion loss, the ratio between the full spectrum at some observation points and some at the source. I was initially interested in checking the results and the two frequencies to determine whether or not the damping trenches (Figure 13) would produce any variation to the outcome.

FIGURE 13 HAS BEEN REMOVED DUE TO COPYRIGHT RESTRICTION

Figure 13: Mesh geometry with trench adjustment

With the changes applied, the geometry was then meshed, and assessed for the presence of any triangular or tetrahedral shapes. This assessment was necessary as the meshing software would be unable to process these particular shapes. If any were present, the geometry and conditions would need to be adjusted in order to avoid the presence of these shape types. The project team advised that they had no formal process in place to ensure these shape types would be factored out, and unfortunately it seemed to be a matter of trial and error, of adjusting the geometry until the shapes were ultimately factored out. Figure 14 demonstrates the geometry of the mesh that was to be generated. This mesh was able to be reduced down to 256 present triangular shapes, however both myself and the project team were unable to reduce this number further.

FIGURE 14 HAS BEEN REMOVED DUE TO COPYRIGHT RESTRICTIONS

Figure 14: Mesh geometry with trenching

Being unable to immediately reduce the number of incompatible shape types, the concept of performing testing on the model to analyse the impact of damping was soon abandoned due to the previously mentioned time constraints applied to my supervision. The benefit in experimenting with making adjustments to the existing structure meant that I had learned a lot about how the mesh could be manipulated to achieve a different desired outcome, albeit that we unsuccessfully managed to do so on this occasion.

FIGURE 15 HAS BEEN REMOVED DUE TO COPYRIGHT RESTRICTIONS

Figure 15: Mesh geometry used for ballast model

The decision was soon made for me to return to the existing version of the model structure and focus on meshing that, so that I could begin to spend more time modelling and running the code, which would take a significant period of time as it was. As a result, the existing version was soon modelled which can be seen in the image above (Figure 15).

4.6 SEM code formatting and assembly

Once the physical mesh had been created, we were then ready to apply the SEM code to the mesh, as well as generate the randomised medium that would be applied to the model. The code applied is a spreading seismic wave, both 2D and 3D, based on the spectral element method. It takes into account the spread in heterogeneous environments that have particularly complex surface geometry, such as elastic or visco-elastic, isotropic, anisotropic and fluid interfaces (Aubry 2017). The code used was entirely rewritten for the purposes of this study, in order to demonstrate the reusability of the model. However, the inputting format was based on past studies conducted by the team that were verified against experimental data to confirm their efficiency (Correa et al. 2017).

To prepare the launch of the SEM code applied to the mesh, and the generation of the random field, six major files were required. Each of the files has a different function (many requiring different modules and software packages to run) and contributes to the SEM calculation in different ways. I've listed each of these files below, with a brief example as to the relevance of the file to the result, and what they do in producing an output.

1. **Mesh input:** The mesh input file determines the type of mesh that will be generated and the number of processors that will be used in the generation process. The example file below demonstrates that 240 processors have been selected for use,

and that the mesh type selected is #1 (which corresponds to "On the Fly") (Figure 16).

FIGURE 16 HAS BEEN REMOVED DUE TO COPYRIGHT RESTRICTIONS

Figure 16: Mesh input file

2. **Material specification:** The material specification file specifies the basic geometry of the mesh, and contains information regarding the inclusion of PML's and how many mesh layers will be present and required to construct the model. An example of this file is in the screenshot below (Figure 17).

FIGURE 17 HAS BEEN REMOVED DUE TO COPYRIGHT RESTRICTIONS

Figure 17: Material specification file

3. **Material input:** The material input file is a file that is compiled by the mesher at the time that the mesh is generated and run. It describes the properties of materials and the directions of the PML. It contains the number of media, type of medium, the P-wave velocities and attenuations, the density, and the order of the element (Figure 18).

FIGURE 18 HAS BEEN REMOVED DUE TO COPYRIGHT RESTRICTIONS

Figure 18: Material input file

4. **Input specification:** The input specification file describes the loading conditions, specifies the simulation time step, and identifies specific areas of the mesh to save in the file result. It also contains parameters of temporal integration, physical time of calculation, as well as description of the source or sources, descriptions of the sensory outputs, and description of snapshot outputs (Figure 19).

FIGURE 19 HAS BEEN REMOVED DUE TO COPYRIGHT RESTRICTIONS

Figure 19: Input specification file

5. **Random specification:** The random specification file describes the randomised heterogeneous Young's Modulus layer (Figure 20) to be applied to the modelling conditions as denoted in the previous files.

FIGURE 20 HAS BEEN REMOVED DUE TO COPYRIGHT RESTRICTIONS

Figure 20: Random specification file

4.7 Modelling outputs

The eventual computations were extremely large-scale in run time. It would take nearly one week for a modelling time step of 1.5 seconds in total. Considering the large-scale of the output, however, it was deemed by the team to be the fastest and most efficient method of performing calculations of this nature. This was in agreement with my own research into other methods of solution uncovered as part of the literature review.

The large-scale run time meant that any errors were quite difficult to properly detect until after the calculation had run. While the calculation would not produce any errors in terms of output, if a parameter had been input incorrectly, it ensured that the resulting outputs would be completely incorrect. This meant that correct parameter input was essential, and needed to be double and triple checked prior to running the calculation.

Across the testing period of two months, a number of simulations were conducted. My first three tests of the model were all complete failures; primarily due to source programs linked to the input codes being entered incorrectly. These issues were also combined with the remote server connection which could inexplicably crash from time to time, as well as posed issues in running calculations back in Australia. My fourth test was a successful model, and I was able to use this complete computation to obtain the desired results.

5 Results

5.1 Stress field fluctuations in the heterogeneous medium

The graphs presented (Figures 21-25) represent the G values (shear modulus) shown on the y axis, and the time step shown on the x axis. As can be demonstrated by the results, instead of similar and constant values across the material we can see that there are instead variations. These variations demonstrate and highlight the presence of the heterogeneity.

Figure 21: Point 1 analysis - Stress deviation (xz)

Figure 22: Point 13 analysis - Stress deviation (xz)

Figure 23: Point 63 analysis - Stress deviation (xz)

Figure 24: Point 85 analysis - Stress deviation (xz)

Figure 25: Point 100 analysis - Stress deviation (xz)

5.2 Impact on the ballast model

With the impact of the heterogeneity on the stress field being verified, the images below (Figures 26-27) demonstrate the visualisation of the density and randomisation occurring within the ballast layer due to the applied medium. They serve as a visual representation of the heterogeneity that causes the fluctuations in variation across the stress fields above.

It should be noted that there is no discrepancy between the two figures above. They are included only as they represent the heterogeneity in the ballast layer across different angles, to best highlight the heterogeneity opposed to the rest of the track model. The constant, standardised nature of the other portion of the track model are visible, as is the randomised, granular vision of the ballast portion.

FIGURE 26 HAS BEEN REMOVED DUE TO COPYRIGHT RESTRICTIONS

Figure 26: Density representation for heterogeneous case

FIGURE 27 HAS BEEN REMOVED DUE TO COPYRIGHT RESTRICTIONS

Figure 27: Density representation for heterogeneous case

With the randomisation being verified and visible, it was time to contrast the differences in propagation between a homogeneous and heterogeneous case. The results below demonstrate the potential for the heterogeneous case to demonstrate a "wave-trapping" mechanism in the ballast (Figures 28-33). The parameter being modelled for the following comparison images is displacement. Figures 28 and 29 serve as the first comparison, 30 and 31 the second, and 32 and 33 the third. Figures 28, 30 and 32 are all the homogenous testing cases, with Figures 29, 31 and 33 the testing cases with the heterogeneous layer applied. The time step of 1.5 seconds is represented in simulation by a number of iterations ranging from 0000 to 0029; to present a dispersion across the simulation I've selected three different iterations for comparison at random – 0007, 0019 and 0028.

FIGURE 28 HAS BEEN REMOVED DUE TO COPYRIGHT RESTRICTIONS

Figure 28: Homogeneous model of ballast iteration 0007

FIGURE 29 HAS BEEN REMOVED DUE TO COPYRIGHT RESTRICTIONS

Figure 29: Heterogeneous model of ballast iteration 0007

FIGURE 30 HAS BEEN REMOVED DUE TO COPYRIGHT RESTRICTIONS

Figure 30: Homogeneous model of ballast iteration 0019

FIGURE 31 HAS BEEN REMOVED DUE TO COPYRIGHT RESTRICTIONS

Figure 31: Heterogeneous model of ballast iteration 0019

FIGURE 32 HAS BEEN REMOVED DUE TO COPYRIGHT RESTRICTIONS

Figure 32: Homogeneous model of ballast iteration 0028

FIGURE 33 HAS BEEN REMOVED DUE TO COPYRIGHT RESTRICTIONS

Figure 33: Heterogeneous model of ballast iteration 0028

6 Discussion

6.1 Explanation of results

This section will discuss the results obtained from analysing the stress field fluctuations in the heterogeneous medium, as well as the impact this medium had on the ballast model as demonstrated by the parametrical visualisation outputs. The following sections will then explain their overall importance in helping satisfy the research aims, relation to previous studies, and the significance and limitations of the results.

6.1.1 Stress field fluctuations in the heterogeneous medium

The core concept behind an evaluation of the heterogeneous medium was to ideally identify stress field fluctuations within the applied medium. In order to perform this identification, I monitored a number of different sensors within the medium across a number of random points. Across these random points, the stress fields presented in the results section demonstrate that some points have lower shear modulus (G) values and therefore lower stresses (compared to homogeneous medium). Alternatively, there is also the presence of higher shear modulus values. The dispersion presented across the 5 points analysed complies and proves the success of the heterogeneity of the stress field that has been modelled.

The graphs presented (Figures 21-25) represent the G values (shear modulus) shown on the y axis, and the time step shown on the x axis. As can be demonstrated by the results, instead of similar and constant values across the material we can see that there are instead variations. These variations demonstrate and highlight the presence of the heterogeneity. Measures were taken across the randomised medium to highlight the effect the heterogeneity had on the shear modulus value. As can be seen, it demonstrates how the stress field fluctuates across various points in the medium. This served as one of the main results of the testing, in that as long as the stress fields are demonstrating differences, it means that there is heterogeneity applied, and this in turn satisfied the MSSMAT team.

Referring to the images presented, the heterogeneity is best represented in the significant drop in terms of shear modulus value between the significant rise demonstrated in Figure 23, compared with the steep decline represented in Figure 25.

6.1.2 Impact on the ballast model

The impact that this method has on the model can be linked to the major benefits in adopting an SEM approach toward modelling such problems. In this instance, the SEM method provides a larger degree of accuracy compared to standard FE models. It allows for the accurate integration of a wave-length lambda, with less integration points than the standard FEM – e.g. 5 integration points for SEM compared with 10 integration points for FEM. This requirement of less integration points means that the computational costs are reduced further. This is of particular benefit when you consider that the SEM method takes roughly one week of run time to produce an output with a time step of 1.5 seconds.

The heterogeneity applied is represented in visualisation (Figures 26-27). The modelling outputs and results presented (Figure 28-33) demonstrate clear wave trapping within the ballast, which also in turn demonstrate the clear benefit of the heterogeneity in producing a wave trapping mechanism. It works to support the aims of the thesis by clearly pronouncing and demonstrating this wave trapping mechanism. Although the wave trapping demonstrated in my modelling is nowhere near as pronounced as the literature that served as the basis of my work (Correa et al. 2017), this is mainly due to the fact the previous testing pushed the standard deviation of the model to 200 times the average. To undertake the same scale of calculation would have involved an exceptionally low time step (e.g. 1e-9), which would've taken around three months roughly to calculate. For obvious reasons this simply would not have been feasible from a time perspective.

Referring to the images presented, Figures 28 and 29 provide an example of the differences between the heterogeneous and homogeneous testing cases. Figure 28 was modelled homogeneously (via the removal of the material input file), without the heterogeneous medium applied, and although having a much smaller total magnitude in terms of displacement, had a larger total distribution of waves into the ballast. This is interesting when compared to Figure 29, which was modelled with the heterogeneous layer applied.

The comparisons between the two cases demonstrate a clear scattering effect in the homogeneous iterations, opposed to the wave trapping mechanisms demonstrated in the heterogeneous iterations. Figures 28-29 clearly demonstrate that although the higher magnitude was experienced by the heterogeneous case (Figure 29), this example was still able to trap the waves opposed to the scattering effect shown in the homogeneous case at the same iteration (Figure 28).

The magnitudes solidify this point, as although there is a small difference with regard to the peak magnitude of the homogeneous case (0.00033 Pa) and heterogeneous case (0.00046 Pa), it is clear that even with the larger magnitude, less of the waves are spreading away from the ballast. This is the mechanism that the team was primarily interested in continuing to explore.

6.2 How the results support the aims of the thesis

The results support the aims of the thesis by exploring the stress field fluctuations of the heterogeneous medium for validation purposes, as well as examining the impact of this heterogeneity on the ballast model. The results have validated the numerical analysis of the wave field generated, especially in the randomised field applied to Young's Modulus. The numerical importance of the fluctuation in shear modulus value clearly demonstrates the presence of a working heterogeneity, ensuring that the mechanism for random fields works as constructed as new by an inexperienced person with the model. In turn, this fluctuation also serves to apply a wave trapping effect to the ballast as can be demonstrated by the trapping occurring in the model images that followed.

The amount of testing that was initially desired changed due to the compressed testing timeframe. Ideally, it would have been beneficial to test the sensitivity of the shear modulus on the final results, and evaluated how these results changed according to the shear modulus. This required a large-scale amount of work that was not possible to include. The shear modulus is a most important parameter on the results and serviceability of the ballast, as it relates directly to the scattering effect of the waves passing through the ballast. This has also been directly linked to the wave propagation effects that can cause damage and degrade the ballast (Correa et al. 2017).

The results support a basis for further application and development of relative approaches toward wave propagation into the future. Testing conducted extends the depth of knowledge in the field and allows for the continued progression toward a computational and numerical method that enables ease of iteration, computation and allows for in-depth visualisation of the impacts. The modelling of the heterogeneous medium and its application to the railway track model, in conjunction with the literature review extends the body of research relative to numerical modelling of wave propagation, particularly in the field of ballast behaviour.

6.3 Relation to theory and previous research

The current research landscape has been focused on developing numerical modelling approaches toward considering the various components of a train-line under stress, but these approaches are still new and yet to gain full acceptance in practice. Professionals are reticent to adopt these models; due mainly to technological and knowledge gaps. As a result, there are opportunities for professionals to start to embrace technological advancements, to provide more in-depth solutions to geotechnical challenges.

The medium and model developed by the MSSMAT team, and used as part of my testing, had this consideration in mind (Correa et al. 2017). The model that I have conducted testing with has been developed in an attempt to gain a greater understanding of ballast behaviour in particular, and how the impacts of wave propagation can be mitigated in simulation via the presence of a heterogeneity. The MSSMAT team was hoping to develop this model further, to ensure broader use within a highly specialised field. My research and testing, involving reconstruction and retesting of the heterogeneous medium applied to the ballast model, has achieved this in part.

6.4 Significance

The results help to support the continued progression of movement away from experimental lab-based studies and physical testing of ballast components to a computational and numerical method that enables ease of iteration, computation and allows for in-depth visualisation of the impacts. They work to further support a proposed model that can be used as a future reference point for the industry based on previous experiments conducted by the MSSMAT team. The testing conducted has helped the team working with the model to continue to develop its applicability into the future.

The testing and analysis performed, as well as the description of the compilation process, compliment a literature review of existing approaches to understanding wave propagation characteristics, with a specific focus on train systems and railway ballasts. It works to contribute to the growth and development of knowledge within the field of research relative to numerical modelling of wave propagation by testing and proving the validity of the proposed heterogeneous field and its impact on the numerical ballast model, to aid and assist further developmental use.

6.5 Limitations

Limitations of the research presented are linked to the short period of time that I was able to work under supervision with such a complex model. My experience in using a model of similar capability was non-existent, and I was also provided little idea of my role or capacity within the project prior to arrival. To add to this, my arrival in Paris was unexpectedly late due to issues outside of my control, which meant that my window for testing was further shortened.

As a result, testing was conducted on the model for a period of two months, as opposed to the initially anticipated three months. This shortened the experimental scope of the project and reduced the time to fully implement exploration outcomes that were of interest. With more time to learn and work with the model, it is expected that some of these outcomes could be fully explored into the future.

6.6 Future direction

Proposed future directions for research relative to my own findings would involve the performance of a shake-down analysis on the model and medium, as well as testing on the effects and impact of various damping structures on its output. These two areas are the most interesting considering what has been uncovered throughout my own research, and further studies in these areas would further benefit the model immensely.

The shake-down analysis would focus on testing the model to the point of failure repeatedly, determining the number of times the model ballast could potentially be used before failure and reconstruction. Shake-down theory is in effect the mechanism that the results are tested against, it means that whenever there is a design for the ballast failure, empirical knowledge exists that enables us to know the upper and lower limits of the design. Therefore, it would be particularly important in producing greater understandings surrounding ballast design recommendations/limitations and the role of the heterogeneity in that.

Meanwhile, the damping structure analysis would further extend the existing body of research surrounding damping structures acting as wave propagation prevention mechanisms. Multiple structure types could be tested alongside different construction methods, within the context of the model, to demonstrate the different impacts each structure can have on wave propagation away from the railway ballast. A comprehensive review of this could be important in adjusting best practice in terms of construction processes, and play a substantial role in better understanding wave propagation in this important context.

Both areas were uncovered as part of an initial review of the subject, and would strongly compliment both the findings of this study as well as the studies previously conducted by the MSSMAT team.

7 Conclusion

In conclusion, the impact of a heterogeneous medium applied to a high-speed train line model has been investigated through a combination of approaches; literature review, numerical analysis and experimental application.

The literature review focused on demonstrating the evolution and suitability of different existing solutions to investigate the impact of modelling types on the central problem. The numerical analysis and experimental application were performed on a numerical model of a railway ballast of a high-speed train line with an applied randomised heterogeneous layer. It contributes to the growth of knowledge within the field in assisting further developmental use of the model, validating the effectiveness of the heterogeneous medium, and confirming the positive impact on wave propagation the heterogeneity has compared to homogenous modelling approaches.

The methodology and theoretical underpinning surrounding the model and medium were explained in detail, as well as the experimental process surrounding the model construction and testing. The results explored the stress field fluctuations of the heterogeneous medium, and examined the impact of the heterogeneity on the ballast model. Validation of the numerical analysis applied to the randomised field demonstrated a working heterogeneity, which in turn applied a wave trapping effect to the ballast. This allows and supports further development of the body of research relative to numerical modelling of wave propagation, and enables ease of iteration, computation and visualisation of the impacts such aspects have on ballast behaviour and durability.

8 References

Auersch, L 2008, ‘Dynamic interaction of various beams with the underlying soil – finite and infinite, half-space and Winkler models’, European Journal of Mechanics –A/Solids, vol. 27, no. 5, pp. 933-958.

Al Shaer, A Duhamel, D Sab, K Foret, G & Schmitt, L 2008, ‘Experimental settlement and dynamic behavior of a portion of ballasted railway track under high speed trains’, Journal of Sound and Vibration, vol. 316, pp. 211-233.

Andersen, L & Nielsen SRK 2005, ‘Reduction of ground vibration by means of barriers or soil improvement along a railway track’, Soil Dynamics and Earthquake Engineering, vol. 25, pp. 701-716.

Aubry, L 2017, ‘SEM documentation’, Department of Mechanics of Soils, Structures and Materials, CentraleSupelec, Paris.

Australian Bureau of Statistics 2013, *Quantitative and qualitative data*, ABS, Canberra, viewed 24 February 2019, <https://www.abs.gov.au/websitedbs/a3121120.nsf/home/statistical+language+-+quantitative+and+qualitative+data>.

Bahrekazemi, M 2004, ‘Train-induced ground vibration and its prediction’, Department of Civil and Architectural Environment, Royal Institute of Technology, Stockholm.

CentraleSupelec n.d., *Moulon mesocentre*, Paris, viewed 18 January 2019, <https://mesocentre.pages.centralesupelec.fr/user_doc/>.

Commonwealth of Australia 2017, *Australia’s rail industry*, Department of the Senate, Canberra.

Connolly, D Giannopoulos, A & Forde MC 2013, 'Numerical modelling of ground borne vibrations from high speed rail lines on embankments', Soil Dynamics and Earthquake Engineering, vol. 46, pp. 13-19.

Connelly, DP Kouroussis, G Laghrouche, O Ho, CL & Forde MC 2014, 'Benchmarking railway vibrations – Track, vehicle, ground and building effects', Construction and Building Materials, vol. 92, pp. 64-81.

Connelly, DP Marecki, GP Kouroussis, G Thalassinakis, I & Woodward, PK 2015, 'The growth of railway ground vibration problems – A review', Science of the Total Environment, vol. 568, pp. 1276-1282.

Correa, LDA Cottereau, R Voivret, C Aguiar, SC Bongini, E & Faure, B 2017, 'Stochastic heterogeneous material modelling for wave propagation in a ballast layer', EPJ Web of Conferences Powders & Grains, 140.

Correa, LDA Quezada, JC Cottereau, R D'Aguiar, SC Voivret, C 2017, 'Randomly-fluctuating heterogeneous continuum model of a ballasted railway track', Computational Mechanics, vol. 60, no. 5, pp. 845-861.

Costa, PA, Calcada, R & Cardoso AS 2012, 'Ballast mats for the reduction of railway traffic vibrations. Numerical study', Soil Dynamics and Earthquake Engineering, vol. 42, pp. 137-150.

Das, B 1992, 'Principles of Soil Dynamics', California State University, Sacramento.

Degrande, G & Schillemans, L 2001, 'Free field vibrations during the passage of a Thaly's high-speed train at variable speed', Journal of Sound and Vibration, vol. 247, no. 1, pp. 131-144.

El Kacimi, A Woodward, PK Laghrouche O & Medero G 2013, ‘Time domain 3D finite element modelling of train-induced vibration at high speed’, Computers and Structures, vol. 118, pp. 66-73.

Hall, L 2002, ‘Simulations and analyses of train-induced ground vibrations in finite element models’, Soil Dynamics and Earthquake Engineering, vol. 23, no. 5, pp. 403-413.

Indranatra, B Khabbaz, H Salim, W & Christie D 2006, ‘Geotechnical properties of ballast and the role of geosynthetics in rail track stabilisation’, Journal of Ground Improvement, vol. 10, no. 3, pp. 91-102.

Kouroussis, G Connolly, DP & Verlinden, O 2014, ‘Railway-induced ground vibration – a review of vehicle effects’, International Journal of Rail Transportation, vol. 2, no. 2, pp. 69-110.

Krylov, V 2015, ‘Focusing of Rayleigh waves generated by high-speed trains under the condition of ground vibration boom’, Department of Aeronautical and Automotive Engineering, Loughborough University, Leicstershire.

Krylov, VV Dawson, AR Heelis ME & Collop AC 2000, ‘Rail movement and ground waves caused by high-speed trains approaching track-soil critical velocities’, Proceedings of International Mechanical Engineers, Part F: Journal of Rail and Rapid Transit, vol. 214, no. 2, pp. 107-116.

Leilei, X 2012, ‘Influence of in-filled trench as wave barrier on ground vibrations’, Department of Civil and Architectural Environment, Royal Institute of Technology, Stockholm.

Talbot, J & Hunt, HEM 2003, ‘A computationally efficient piled-foundation model for studying the effects of ground-borne vibration on buildings’, Part C: Journal of Mechanical Engineering Science, vol. 217, no. 9, pp. 975-989.

Zoccali, P Cantisani, G & Loprencipe, G 2014, ‘Ground-vibrations induced by trains: filled trenches mitigation capacity and length influence’, Construction and Building Materials, vol. 74, pp. 1-8.