Abstract

Although the traveling salesman problem (TSP) has a long research history as a mathematical approach to discover the shortest trip between a set of cities, there is no effective solution for the problem as it is considered an $\mathcal{NP}-hard$ problem of combinatorial optimisation. For example, solving the symmetric traveling salesman problem (STSP) using the fastest computer program, Concorde, for 85,900 vertices (which is the largest instance of TSP which has been solved to provable optimality to date) takes more than 199 CPU days. However, due to the importance of practical applications, methods to find optimal solutions have been developed since the 1950s. Stefan Hougardy and Rasmus T. Schroeder's algorithm for reducing the number of edges in two-dimensional Euclidean instances of TSP is one such method and hastens the STSP process by 11 times in certain test examples. The total runtime for the main part of their algorithm is $O(n^2 \log n)$ for n number of cities. The largest TSP case, 85,900 points, took 2 CPU days to run their algorithm. Concorde needed 16 CPU days to achieve the best outcome for this case. Hougardy and Schroeder's algorithm was presented alongside theoretic graph results showing how they proved that some edges of a TSP instance cannot be part of any optimal TSP trip. This thesis is based on Hougardy and Schroeder's results and shows how unnecessarily edges of a TSP instance can be avoided in any optimal TSP trip. The thesis presents practical results demonstrating how this can be achieved; it also presents a Matlab code for a TSP instance named qatar194. The instance contains 194 nodes and 264 edges after being run with Hougardy and Schroeder's elimination algorithm. This research shows that more edges can be eliminated when $k - opt$ edge exchanges are considered for $k > 3$. Linear programming (LP) is used in combination with subtour elimination constraints (SEC) and comb inequalities. The LP approach is taken by adding extra 28 constraints employing SEC and comb inequalities. The approach allowed the elimination of 58 unnecessarily edges and improved the lower bound to 9,350.6; while the optimal tour length is 9,352. Mixed-integer linear programming (ILP) is then used to find the shortest tour with no unnecessarily edges.