

LOGISTIC EQUATIONS WITH DIFFUSION, DELAY AND IMPULSES

by

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October 28, 2005

A thesis presented to the
Flinders University of South Australia
in fulfillment of the requirements for the degree of
Doctor of Philosophy

Adelaide, South Australia, 2005

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Summary

This thesis contains a discussion of logistic equations with diffusion, impulses and time delays both discrete and continuous type. The boundary conditions used in these problems are Dirichlet, Neumann and Robin boundary conditions. Both single and multi species logistic equations are investigated. The impulse times employed here are the fixed ones. Some results on the problems are:

- (1) Single species logistic equation with diffusion, impulses, discrete delay, Dirichlet and Robin boundary conditions.
 - Existence and uniqueness of solution:
 - Dirichlet boundary case(Corollary 3.1).
 - Robin boundary case(Corollary 3.2).
 - Conditions for the existence of zero attractor (Theorem 3.9).
 - Conditions for the existence of positive attractor (Theorem 3.11).
- (2) Logistic equation with diffusion, impulses, continuous delay and Neumann boundary condition.
 - Single species: existence and uniqueness of solution (Theorem 4.1).
 - Single species: conditions for the existence of zero attractors (Theorem 4.2).
 - Single species: conditions for the existence of positive attractor (Theorem 4.3).
 - Multi species: conditions for the existence of positive attractor (Theorem 4.4).

This thesis is organised as follows: in Chapter 2, the background of these problems is presented. Chapter 3 is concerned with the existence and uniqueness of solution, zero and positive attractor of logistic equations with diffusion,

impulses, discrete time delay, and Dirichlet and Robin boundary conditions. We discuss the existence and uniqueness of solution of diffusive logistic equations with distributed delay, impulses and Neumann boundary condition, zero and positive attractors in Chapter 4. Some conditions to obtain a positive attractor for multi species logistic equation with diffusion, distributed delay, impulses and Neumann boundary condition are presented in the last section of Chapter 4.

Certification

I certify that this thesis does not incorporate without acknowledgement any material previously submitted for a degree or diploma in any university; and that to the best of my knowledge and belief it does not contain any material previously published or written by another person except where due reference is made in the text.

Jalina Widjaja

I believe that this thesis is properly presented, conforms to the specifications for the thesis and this is of sufficient standard to be, *prima facie*, worthy of examination.

Dr. Murk J. Bottema, Principal Supervisor

Acknowledgement

Praise to Lord Jesus Christ for His Goodness that leads me through this wonderful episode in my life.

I wish to express my most sincere gratitude to the people who supported me during my study in Adelaide, especially to:

- Dr Murk Jan Bottema, my principal supervisor, for his guidance, patience, understanding and kindness;
- Prof. K. Gopalsamy, my co-supervisor, for all the excellent ideas and advices;
- QUE Project Department of Mathematics, Institut Teknologi Bandung for the scholarship;
- Staff of School of Informatics and Engineering for their kind assistance;
- My family for their love, support and prayers;
- My friends for their support and beautiful friendship, amongst them are: Tandra, Webb, French, Kodhyat, Puntoaji, Budiharto, and Hendrijanto family, Murk, Puspa, Grace, Ping, Gobert, Gaynor, Sariyasa, Kenny, Denise, Nella, Rahmi and Hetifah.

I dedicate this thesis to my dear mother and father.