Abstract

Recently discovered is a natural decomposition of the Lifshitz-Kreĭn spectral shift function (SSF) à la Lebesgue into the sum of absolutely continuous and singular SSF's. The latter part represents the flow of singular spectrum and takes integer values even within the essential spectrum. The singular SSF may be alternatively characterised as the either of the so-called total resonance index or singular μ-invariant. The first of these measures the total number of poles of the sandwiched resolvent, considered as a function of the coupling parameter, which split from the unit interval as the spectral parameter is perturbed off of the real axis, counting the poles that move into the upper half-plane with a positive sign and those that move to the lower half-plane with a negative sign. The second measures the sum of winding numbers of the eigenvalues of the scattering matrix as it is continuously deformed to the identity in two different ways: by shrinking the coupling parameter to 0 and by sending the imaginary part of the spectral parameter to ∞. This document is in part a review of these facts, which were first established by N. Azamov under the assumption of a trace class perturbation, and also generalises their proofs to the case of relatively trace class perturbations, thereby making them applicable for instance to Schrödinger operators with bounded potentials undergoing integrable perturbations.