The Influence of Students' and Teachers' Mathematics-related Beliefs on Students' Mathematics Performance

by

Xuan Cuong Dang

Thesis
Submitted to Flinders University
for the degree of

Doctor of Philosophy

College of Education, Psychology, and Social Work
January 2021

ABSTRACT

The influence of students' and teachers' mathematics-related beliefs on students' mathematics performance was investigated in a sample of 620 ninth grade students and 46 mathematics teachers from Vietnam. The beliefs of the students were investigated using a 76-item, 4-point Likert scale *Mathematics-Related Beliefs Questionnaire for Students (MBQ-S)*, while a 61-item *Mathematics-Related Beliefs Questionnaire for Teachers (MBQ-T)* was used to investigate the beliefs of the teachers. The questionnaires included items investigating beliefs about the certainty of mathematics knowledge, the usefulness of mathematics, mathematical problem-solving, the role of effort, mathematics ability, self-efficacy, and mathematics teaching. Students' mathematics performance was determined by using students' scores in the district mathematics exam as well as by analysing their performance in two specially designed mathematics tests: one curriculum-based, and the other based on the OECD PISA assessment. The latter test was intended to investigate the transferability of mathematics knowledge to real-world contexts.

Confirmatory Factor Analysis was applied to assess each students' belief construct in the process of obtaining the best measurement model, followed by structural equation modelling to investigate the influence of beliefs on the students' performance. The best structural model revealed a complex pattern of interrelationships amongst beliefs, with beliefs about the nature of mathematics influencing beliefs about mathematics ability. Beliefs about mathematics ability were direct positive predictors of self-efficacy beliefs in mathematics which, in turn, were direct positive predictors of mathematics performance in all three tests.

Some of the most notable findings were the following: beliefs that mathematics knowledge is certain and unchangeable were direct positive predictors of beliefs that mathematics problem-solving is about memorising procedures. In contrast, beliefs that mathematics knowledge is useful were direct positive predictors of beliefs that mathematics problem-solving is about understanding procedures. Beliefs that problem-solving is about understanding procedures were direct positive predictors of beliefs about mathematics ability. The results also showed that students' perception of their teacher practices influenced their beliefs that effort is important to improve mathematics learning, which, in turn, were direct positive predictors of beliefs about mathematics ability.

The results revealed that the Vietnamese students tended to agree with statements indicating that mathematics is useful and that effort is important to improve mathematics learning. They had high self-efficacy and positive perceptions of their teachers' practices. An investigation of the differences between low and high performers in the test based on the OECD PISA exam revealed statistically significant differences. The high mathematics performers were more likely to agree with statements indicating that mathematics knowledge is not certain and absolute, that mathematics is useful, that problem solving is more about understanding than memorisation and statements indicating high self-efficacy in mathematics.

The data showed that the Vietnamese teachers held inconsistent beliefs about the certainty of mathematics knowledge, mathematical problem solving, students' mathematics ability, and mathematics teaching. Multilevel structural equation modelling showed that teachers self-efficacy beliefs were direct positive predictors of their students' mathematics performance. The model also showed that teachers' beliefs that mathematics knowledge is malleable were direct positive predictors of their beliefs that mathematical problem-solving is about understanding procedures. In turn, these beliefs directly predicted teachers' beliefs in the constructive teaching of mathematics.

The research revealed a meaningful pattern of complex relationships between the mathematics-related beliefs of students and their teachers on students' mathematics performance, enhancing and clarifying previous research findings with both theoretical and practical significance. The research also provided valuable information for Vietnamese stakeholders about mathematics learning and teaching in Vietnam.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

DECLARATION

I certify that this thesis:

1. does not incorporate without acknowledgment any material previously submitted for a degree or diploma in any university; and
2. to the best of my knowledge and belief, does not contain any material previously published or written by another person except where due reference is made in the text.

Xuan Cuong Dang

ACKNOWLEDGEMENTS

The completion of this research has been assisted by many people. First and foremost, my deepest thanks go to my principal supervisor, Professor Stella Vosniadou for her patience, invaluable guidance, and continuous encouragement at every stage of my research journey. I wish to express my appreciation to Dr. Mun Yee Lai, my second supervisor, at University of Technology Sydney for her constructive feedback and academic support. My great appreciation goes to Dr. Igusti Darmawan from the University of Adelaide for his sophisticated advice on data analysis techniques. Great thanks go also to the staff at the College of Education, Psychology, and Social Work and Office of Graduate Research, Flinders University for their assistance. Especial thanks go to Emeritus Professor Mike Lawson, Dr. Peny Van Deur, Dr. Mirella Wyra, and Dr. David Jeffries from the Beliefs about Learning and Teaching (BALT) research group at Flinders University for their valuable feedbacks and encouragement during my PhD journey. I wish to express my appreciation to Associate Professor David Curtis for his supervisory and support throughout the first year of my candidature. I also express my appreciation for Dr. Mai Ngo from College of Humanities, Arts and Social Sciences, Flinders University for her advice on the questionnaires and their translation versions. I would like to thank Dr. Bakir Haryanto for his data analysis tutorials. I am grateful to Ms Abby Sesterka who spent her valuable time editing the thesis.

I express my appreciation to my colleagues at the National Institute of Educational Sciences in Vietnam for their support. Especial thanks to Associate Professor Tran Kieu and Dr. Nguyen Thi Thu Hue at the Institute for their valuable advice on the instruments. I would like to express my appreciation to the Department of Education and Training in Nghe An, the principals, mathematics teachers and 9th grade students at selected schools, who willingly participated in the research. I would like to acknowledge the Department of Foreign Affairs and Trading (DFAT) for their support through Australia Awards Scholarship (AAS). Without this scholarship, I have not had this great opportunity to approach a new research environment where I have enriched my research experience with international standards. Especial thanks go to the International Students Services (ISS) at Flinders University for their kind support. Last but not the least, I am sincerely thankful to members of my beloved family who always encourage and support me to finish my research. A special mention goes to my wife and my daughter. Thanks also go to all friends who shared their lives with me here in Adelaide.

ABBREVIATIONS

Abbr.	Meaning
*	p-value < .1
**	p-value < .05
***	p-value < .01
CFA	Confirmatory Factor Analysis
CFI	Comparative Fit Index
CR	Composite reliability
D^2	Mahalanobis Distance
df	Degree of Freedom
EFA	Exploratory Factor Analysis
M	Mean
MI	Modification Index
Mplus	Statistical modelling program
MBQ-S	Mathematics-Related Beliefs Questionnaire for Students
MBQ-T	Mathematics-Related Beliefs Questionnaire for Teachers
N	Number of participants
OECD	Organization for Economic Co-operation and Development
p	p-value
PISA	Programme for International Student Assessment
R^2	Coefficient of Determination
RMSEA	Root Mean Square Error of Approximation
Score	Students' scores in district mathematics exam
SD	Standard Deviation
SEM	Structure Equation Modelling
SPSS	Statistical Package for Social Science
SRMR	Standard Root Mean Square Residual
t	*t*-test
Test1	Mathematics curriculum-based problems test results
Test2	PISA-based problems test results
TLI	Tucker-Lewis Index
WLSMV	Weight Least Square Mean and Variance
χ^2	Chi Square
χ^2/df	Normed Chi-Square
THE CONSTRUCTS OF STUDENTS' MATHEMATICS-RELATED BELIEFS	
BaE	Beliefs about the importance of effort
CMK	Beliefs about the certainty of mathematics knowledge
CMK_N	Beliefs that mathematics knowledge is certain

CMK_P	Beliefs that mathematics knowledge is malleable
MA	Beliefs that mathematics ability can be changed
MPS	Beliefs about mathematical problem-solving
MPS_N	Beliefs that mathematical problem-solving is about memorising procedures and formulas
MPS_P	Beliefs that mathematical problem-solving is about understanding procedures
Per	Students' perception of teachers' practices
SelfE	Students’ self-efficacy beliefs
UoM	Beliefs that mathematics is useful
THE CONSTRUCTS OF TEACHERS' MATHEMATICS-RELATED BELIEFS	
BaEt	Beliefs about the role of student effort in their learning
CMKt	Beliefs about the certainty of mathematics knowledge
CMKt_N	Beliefs that mathematics knowledge is certain
CMKt_P	Beliefs that mathematics knowledge is malleable
MAt	Beliefs about students’ mathematics ability
MAt_N	Beliefs that students’ mathematics ability is innate
MAt_P	Beliefs that students’ mathematics ability can be changed
MPSt	Beliefs about mathematical problem-solving
MPSt_N	Beliefs that mathematical problem-solving is about memorising procedures
MPSt_P	Beliefs that mathematical problem-solving is about understanding procedures
SelfEt	Teachers’ self-efficacy beliefs
SelfEt_N	Teachers’ low self-efficacy beliefs
SelfEt_P	Teachers’ high self-efficacy beliefs
Teach	Beliefs about mathematics teaching
Teach_N	Beliefs in transmissive teaching of mathematics
Teach_P	Beliefs in constructive teaching of mathematics
UoMt	Beliefs about the usefulness of mathematics
UoMt_N	Beliefs that mathematics is not useful
UoMt_P	Beliefs that mathematics is useful

CHAPTER 1 INTRODUCTION

1.1 Statement of the problem

Over the last forty years, researchers in the field of mathematics education have started to investigate the beliefs, attitudes, emotions, values of students and their teachers, and how these factors may influence the academic performance of students in mathematics (Hannula, 2012; Hannula, Leder, Morselli, Vollstedt, & Zhang, 2019; McLeod & Adams, 1989). In such research, students' and teachers' mathematics-related beliefs have been given particular importance (Goldin, Rösken, & Törner, 2009; G. C. Leder, E. Pehkonen, & G. Törner, 2002; Rott, Törner, Peters-Dasdemir, & Möller, 2018). Several kinds of mathematics-related beliefs have been investigated, such as epistemological beliefs about mathematics (e.g., whether mathematics knowledge is fixed and certain or malleable), beliefs about the nature of mathematics (e.g., whether mathematics is useful and interesting or not), beliefs about mathematics teaching and learning (e.g., beliefs that mathematical problem-solving is about memorisation or understanding), and beliefs about self as a mathematics learner (e.g., whether mathematics ability is innate or can be changed).

Research has shown that such mathematics-related beliefs are related to each other and can have important effects on students' academic performance. For example, beliefs about the nature of mathematics have been found to influence beliefs about mathematics learning, especially beliefs about self as a mathematics learner (Kloosterman, 1996; McLeod, 1992). Schommer-Aikins (2004) and her colleagues have suggested that epistemological beliefs may influence beliefs about learning and may indirectly influence academic performance through learning beliefs (Schommer-Aikins, Bird, & Bakken, 2010). Recently, Depaepe et al. (2016) also argued that epistemological beliefs in mathematics are associated with other types of beliefs about mathematics teaching and learning.

Teachers' mathematics-related beliefs have also been grouped in three categories: beliefs about the nature of mathematics, beliefs about mathematics teaching, and beliefs about mathematics learning (Cooney, 2003; Cross, 2009; Ernest, 1989a; Thompson, 1992). There is a large body of literature that investigates the relationships between teachers' beliefs and their practices as well as students' learning outcomes (Ambrose, Clement, Philipp, & Chauvot, 2004; Buehl & Beck, 2015; Cross, 2015; Mosvold & Fauskanger, 2014; Pajares, 1992).

Although different aspects of mathematics-related beliefs have been investigated, most studies, in both Western and Asian countries, have focused on defining the constructs and measuring mathematics-related beliefs, with only a few studies having investigated mathematics-related beliefs as a factor influencing mathematics performance. In comparison with other cultural groups, the high academic performance in mathematics among Asian students shown in results of international assessment programs (e.g., PISA and TIMSS) has drawn great attention from researchers in the field of mathematics education around the world. However, research on students' beliefs is especially scarce in Asian countries where some aspects of mathematics-related beliefs, such as epistemological beliefs about mathematics, have hardly been studied and where little is known about the effects of these beliefs on students' mathematics performance. Moreover, there is no research on this topic in Vietnam. Although Vietnam is an Asian country, it differs from other Asian countries both culturally and in terms of its education system.

Vietnam is one of the countries in the East Asia and Pacific regions. There are 54 different ethnic groups, where Kinh is the majority (82.6%). The Gross National Income per capita in 2019 was only USD $2,540 in comparison to an average of over $11,700 for its region of East Asia and Pacific (World Bank, 2020a), and nearly USD $40,000 in Europe (World Bank, 2020b). The general education system provides schooling from kindergarten (4-5 years old) to 12th grade (18 years old). In 2018, the Vietnamese government issued a new general curriculum which was developed using a competence-based approach. Mathematics is one of the core subjects in this education system. The practical situation of mathematics teaching showed that teachers have used transmissive teaching approaches where they have focused on teaching mathematics concepts and theories, rather than constructive teaching approaches, where they have focused on mathematical problem-solving in real-world situations (T. T. Nguyen et al., 2020; T. T. Nguyen, Trinh, & Tran, 2019). However, Vietnamese students have shown high scores from the international assessment in mathematics such as PISA. Therefore, investigating mathematics-related beliefs of Vietnamese teachers and students and their influence on students' mathematics performance can add a significant contribution to the research literature.

The purpose of present research is to examine, in greater detail than previous research, a wide range of possible mathematics-related beliefs of students and teachers in order to understand how these beliefs influence the mathematics performance of students from Vietnam. More specifically, the research sought to answer the following three questions:

Research Question 1: What are the emergent profiles of Vietnamese students' and teachers' mathematics-related beliefs? And how do these profiles differ among female and male students, and among low and high mathematics performers?

Research Question 2: How do students' mathematics-related beliefs influence their mathematics performance?

Research Question 3: How do teachers' mathematics-related beliefs influence their students' mathematics performance?

1.2 Research design and methodology

The research was implemented through six phases, as shown in Figure 1.1. In the first phase, published works related to mathematics-related beliefs and their relationships to mathematics performance were reviewed. Based on the results of the literature search, three research questions and hypotheses were proposed to examine chosen constructs of students' and teachers' mathematics-related beliefs as well as to investigate the relationships among these beliefs and students' mathematics performance.

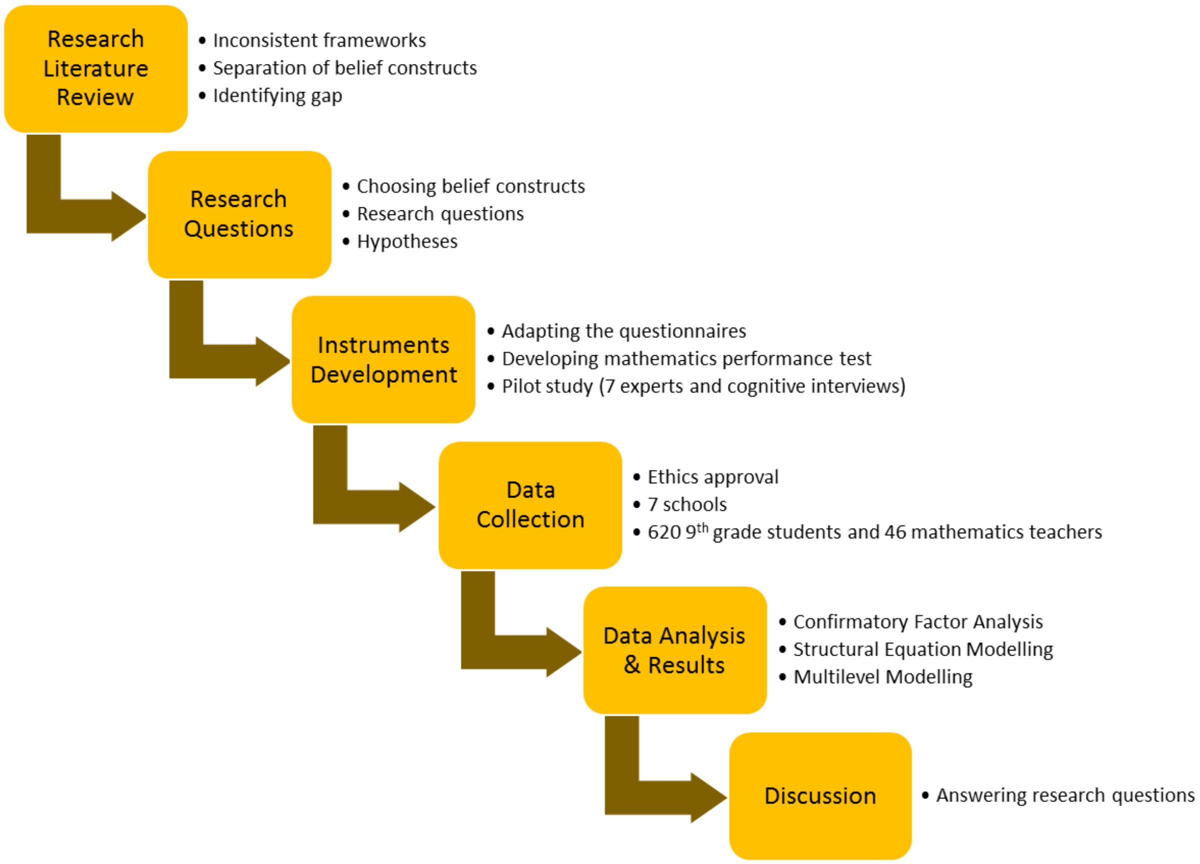


Figure 1.1: The research process of the present research

In the third phase, student and teacher questionnaires were developed to investigate their mathematics-related beliefs. A mathematics performance test was developed to assess students' ability, not only in solving problems based on the taught curriculum, but also problems that were unfamiliar to them and based more on real-world problems, such as the kinds of problems used in the OECD PISA exam. The questionnaires were validated by international and national experts as well as through cognitive interviews with Vietnamese students and teachers. The final instruments were used to collect data from 620 ninth grade students and 46 mathematics teachers in Vietnam. Confirmatory Factor Analysis, Structural Education Modelling, and Multilevel Modelling were employed for analysis. The resulting measurement models and structural models were evaluated carefully before reporting the main results. In the final phase, findings of the present research were discussed in relation to current research literature to address three research questions.

1.3 Significance of the research

The research constructed and validated two questionnaires to investigate students' and teachers' mathematics-related beliefs. The findings from the questionnaire which investigated the beliefs of students showed that the Vietnamese students held a number of beliefs that have been found to be positive predictors of mathematics performance. For example, they believed that mathematics is useful, that effort is important to improve mathematics learning, and that mathematics ability is malleable. They also had high self-efficacy and positive perceptions of teachers' practices. An advantage and innovation of the questionnaires is that they included items that represented beliefs that were both positive and negative predictors of students' performance. The results showed that the mathematics teachers tended to simultaneously agree both with positive and negative beliefs, especially in the areas that tested beliefs about the certainty of mathematics knowledge, mathematical problem-solving, students' mathematics ability, and the nature of mathematics teaching.

Prior research has investigated the relationships among some special belief constructs and mathematics performance in relative isolation with other potential belief constructs. The present research stands as one of few studies that have used structural equation modelling to examine the interrelations among many different belief constructs that represented different categories of mathematics-related beliefs (i.e., epistemological beliefs, beliefs about nature of mathematics, beliefs about mathematics teaching and learning, and beliefs about the self) and students' mathematics performance.

The results of the structural equation modelling showed the following associations: (1) students' perceptions of their teachers' practices directly influenced their beliefs that effort is important to improve mathematics learning, beliefs that mathematical problem-solving is about memorising procedures, and beliefs that mathematical problem-solving is about understanding procedures; (2) while beliefs that mathematics knowledge is certain and beliefs that mathematics is useful were positive direct predictors of beliefs that effort is important to improve mathematics learning, the former were directly positive predictors of beliefs that mathematical problem-solving is about memorising procedures, and the latter were positive direct predictors of beliefs that mathematical problem-solving is about understanding procedures; (3) while beliefs about mathematics ability were positively predicted by beliefs that effort is important to improve mathematics learning, and beliefs that mathematical problem-solving is about understanding procedures, these beliefs were negatively predicted by beliefs that mathematical problem-solving is about memorising procedures; (4) beliefs about mathematics ability directly predicted self-efficacy beliefs; and, (5) students' self-efficacy beliefs directly predicted mathematics performance.

In terms of the relationships between teachers' mathematics-related beliefs and students' mathematics performance, the results of the multilevel structural equation modelling showed the following associations: (1) teachers' high self-efficacy beliefs of mathematics teachers directly predicted students' mathematics performance; (2) beliefs that mathematical problem-solving is about understanding procedures directly predicted teachers' beliefs about constructive teaching of mathematics; and, (3) teachers' beliefs that mathematics knowledge is malleable directly predicted their beliefs that mathematical problem-solving is about understanding procedures.

The findings of the research have particular importance within the Vietnamese educational context. In Vietnam, ninth grade students had a high rank in mathematics in some international assessments (e.g., PISA, TIMSS), but the stakeholders have paid little attention to explaining why this was the case, and identifying which factors may be responsible. The findings of the present research provide information about certain factors on mathematics performance that have not been previously investigated. From a more applied perspective, the research provides useful information to teachers and other educational stakeholders for improving the quality of teaching, learning, and instruction, as well as the content of the mathematics curricula. Within the context of Vietnam, the curriculum is in a state of transition from a content-based approach to a competence-based approach. In this transition period, the results of the present research can help educators, researchers, and stakeholders in Vietnam to better understand the beliefs of students and teachers that contribute most to successful mathematics performance, especially in the application of problem-solving skills to everyday problems. Such an understanding can then be used to inform teachers and students and lead to discussions about how beneficial beliefs can be further promoted and beliefs that do not contribute positively can be revised.

1.4 Structure of the present research

This thesis is structured into six chapters to outline the process of the research and present as well as discuss findings. Chapter 2 presents a comprehensive literature review about five main research areas. It begins with a review on general mathematics-related affects to emphasise the central role of beliefs in relation to other affective factors such as attitudes, emotions, and values. Next, different frameworks of mathematics-related beliefs were synthesised to identify inconsistencies. In the third section, research on some specific belief constructs belonging to different categories such as epistemological beliefs about mathematics, beliefs about nature of mathematics, beliefs about mathematics teaching and learning, and beliefs about the self as a mathematics learner were analysed. The differences in students' beliefs were also highlighted. The fourth section addresses the research on teachers' mathematics-related beliefs and how they influence students' mathematics performance. The final section in this chapter presents some relevant research within Vietnamese contexts. Chapter 3 presents an overview of the present research and justifies the reasons for selecting specific belief constructs for investigation. The hypotheses of the research are also presented. In Chapter 4, details about the participants, the procedures, and the process of instrument development are described. This chapter also addresses the statistical procedures of the present research. Chapter 5 presents the entire data analysis process and the results of the main research, then Chapter 6 discusses and reflects upon the main findings as well as draws the conclusions for the present research. This chapter will answer the three research questions as outlined previously and synthesises the entire journey of the present research, discuss the limitations of the present research, as well as propose some directions for future research.

CHAPTER 2 LITERATURE REVIEW

This chapter reviews the research literature regarding mathematics-related beliefs and their influence on students' mathematics performance. This chapter has four main sections. The first section presents some general literature about mathematics-related affect to capture the role of beliefs in the mathematics-related affect system. The second and the third section addresses important published works in the research literature about students' mathematics-related beliefs. Specifically, the review focuses on different frameworks and crucial constructs of students' mathematics-related beliefs, as well as the effects on mathematics performance. The fourth section presents research on teachers' mathematics-related beliefs and their influence on teachers' practices as well as on students' mathematics performance. Finally, the fifth section summarises relevant research in Vietnam. Within each section, along with reviewing the research literature, the researcher also identifies possible gaps that warrant attention in the context of the present research.

2.1 Research on mathematics-related affect

There is a great deal of research on the nature of belief systems and their relation with academic performance in many subject areas. In the present research the focus is only on their relations in the area of mathematics education. According to many scholars in the field of mathematics education, beliefs (or belief systems) are important components of the mathematics-related affect system (Debellis & Goldin, 2006; Hannula, 2012; Hannula, Op't Eynde, Schlöglmann, & Wedege, 2007). The term 'mathematics-related affect system' is used to refer to the attitudes, motivation, emotions and beliefs that are related to mathematics learning and teaching. In order to elucidate the relationships and the role of mathematics-related beliefs within that system, this section briefly presents an overview of research on mathematics-related affect.

The important role of the affective domains in education in general, and in mathematics education in particular, has been emphasised over the last four decades (DeBellis & Goldin, 1997; Hannula et al., 2019; Leder & Grootenboer, 2005; McLeod & Adams, 1989; Pepin & Roesken-Winter, 2015). Because the main purpose of the present research focuses on mathematics-related beliefs, this section presents a more general literature review of available frameworks, categorisations and models of affect in the field of mathematics education (i.e., mathematics-related affect (Hannula, 2012) and their development over the time. The purpose of this approach is to capture the whole picture of affective domains and the role of mathematics-related beliefs in relation to other facets of affect. The terms 'affect', 'affective factor', 'affective variable', 'affective domain', 'affective component' and 'mathematics-related affect' are used interchangeably in the research literature.

Among various affective variables that have been investigated in the field of mathematics education, many researchers have claimed that anxiety related to mathematics and attitudes towards mathematics are the key constructs of mathematics-related affect (Dowker, Sarkar, & Looi, 2016; Namkung, Peng, & Lin, 2019; Zan et al., 2006). Researchers investigated these constructs by using two widely used measures, namely the *Mathematics Anxiety Rating Scale* (Richardson & Suinn, 1972) and the *Mathematics Attitude Scale* (Fennema & Sherman, 1976). The results of the research on these two constructs confirmed that mathematics anxiety and attitudes towards mathematics are related to mathematics performance (Hunt, Clark-Carter, & Sheffield, 2015; Ma, 1999; Ma & Kishor, 1997).

In the 1980s, there was significant attention from researchers to explore different dimensions of mathematics-related affect and the relationships between mathematics-related affect and mathematical problem-solving (McLeod & Adams, 1989; Schoenfeld, 1985a; Silver, 1985). The publication of *Affect and Mathematical Problem-solving*, edited by McLeod and Adams (1989), covers many particular aspects of affective factors not only in mathematical problem-solving but in also mathematics teaching and learning. When discussing the role of mathematics-related affect, McLeod (1989b) stated that the main purpose of research on mathematics-related affect within the mathematical problem-solving context is to improve mathematics instruction and performance. The author analysed mathematics-related affect in relation to many different aspects of cognition, such as memory, representation processes, consciousness, metacognition, automaticity and mathematical problem-solving strategies. He concluded that the effects on mathematical problem-solving change depending on the phases that the problem solvers are going through, and the types of strategies they choose.

According to McLeod (1989b), affective domains vary in terms of magnitude and direction; in other words, they may change in levels of intensity (e.g., from weak to strong), and they can be positive or negative. For example, much evidence from the research literature showed that students' mathematics self-efficacy has a 'strong' and 'positive' influence on mathematics performance (Liu & Koirala, 2009; Manzano-Sanchez, Outley, Gonzalez, & Matarrita-Cascante, 2018), and students' mathematics anxiety has 'moderate' and 'negative' impact on mathematic performance (Justicia-

Galiano, Martín-Puga, Linares, & Pelegrina, 2017). McLeod claimed that when researchers investigate mathematics-related affect, they should also focus on different aspects of emotions, including magnitude, direction, duration, levels of awareness and level of control. Additionally, when students participate in mathematics classes, some instructional issues should also be considered within the contexts, including different types of cognitive processes, different types of instructional environment, and different types of beliefs. Continuing with the research on mathematics-related beliefs, he summarised most prior research on mathematics-related affect and proposed a theoretical framework for researchers in the field of mathematics education (McLeod, 1992). Three crucial components of this framework were identified, namely, beliefs, attitudes, and emotions.

According to McLeod (1992), beliefs are developed over a long period of time. He proposed different categories of beliefs: beliefs about mathematics, beliefs about the self, beliefs about mathematics teaching, and beliefs about the context. Those beliefs will be discussed in detail in following sections in this chapter. In regards to attitudes toward mathematics, McLeod claimed that they can be developed by automatically repeating emotional reactions to mathematics or by assigning an existing attitude to a new problem. Attitudes towards mathematics interact with mathematics achievement in different ways depending on various types of attitudes. In regard to emotions, McLeod (1992) looked closely at the research literature and claimed that researchers in the area of mathematics-related affect pay insufficient attention to this domain of affect. He argued that one of the important reasons for this is that researchers previously only wanted to investigate affective factors that are more stable and can be measured using questionnaires. McLeod also explained the relationships between these components of affect: "First, students hold certain beliefs about mathematics and about themselves that play an important role in the development of their affective responses to mathematical situations. Second, … students will experience both positive and negative emotions as they learn mathematics …. Third, students will develop positive or negative attitudes toward mathematics as they encounter the same or similar mathematical situations repeatedly" (McLeod, 1992, p. 578). This suggested that there are interactions and interrelations among these components of affect. This framework has been accepted by many researchers, and has been used to investigate the characteristics of each component as well as to examine the relationships between them and mathematics performance.

Fennema (1989) also proposed a generic model for mathematics-related affect research by approaching mathematics-related affect in relation to mathematics education outcomes, as shown in Figure 2.1. This approach was proposed based on a specific model that developed in 1985, namely, *The Autonomous Learning Behavior Model* (ALB) which the author used to explore gender differences in mathematics (Fennema & Peterson, 1985, cited in Fennema, 1989). According to the author, different domains of mathematics-related affect predicted mathematics education outcomes through mediating learning activities. Moreover, the author also recommended that researchers should also focus on external influences, such as characteristics of teachers. Although this model appears quite simple, it provides explanations for many aspects of mathematics teaching and learning. First, there are interrelationships between affect, learning activities, external influences and mathematics education outcomes. Second, in many cases, affect may not have direct links to mathematics education outcomes. Third, affect is also considered as a mediator for other influences.

Removed due to copyright restriction

Figure 2.1: Fennema's model of affective factors (Fennema, 1989, p. 217)

Continuing with McLeod's work, DeBellis and Goldin proposed a tetrahedral model to consider affect (Debellis, 1996; DeBellis & Goldin, 1997, 2006; Goldin, 2002). Figure 2.2 depicts this model, which shows relationships among domains of affect.

Removed due to copyright restriction

Figure 2.2: DeBellis & Goldin's tetrahedral model of affective factors (DeBellis & Goldin, 1997, p. 213)

DeBellis and Goldin adopted three domains, including beliefs, attitudes, and emotions, from McLeod's (1992) work and added values (including morals, and ethical judgments) as another key domain. The authors stated that "*Values*, including *ethics* and *morals*, refer to the deep, 'personal truths' or commitments cherished by individuals. They help motivate long-term choices and shorter-term priorities. They may also be highly structured, forming value systems" (Debellis & Goldin, 2006, p. 135). As can be seen from Figure 2.2, each vertex of tetrahedral represents a domain of affect and where it interacts with other domains (and even interacts with cognition). Furthermore, the authors claimed that each affective domain is influenced by the corresponding environment (including other people's beliefs, attitudes, emotions, and values, as well as social and cultural conditions and external contextual factors) where the person(s) is situated during the process of solving mathematical problems (DeBellis & Goldin, 1997).

Rather than dividing the four domains of affect to show the interactions between them as is the case in Debellis and Goldin's (1997) framework, Grootenboer and Leder (2005) considered the four domains in terms of the overlap between them and the variation regarding levels of cognition/stability and affectivity/intensity. The proposed model of these affective domains is shown in Figure 2.3.

Removed due to copyright restriction

Figure 2.3: A model of conceptions of the affective components (Leder & Grootenboer, 2005, p. 2)

Leder and Grootenboer (2005) argued that beliefs, attitudes and values are inter-related, and that these components are used in interchangeable ways. Emotions (or feelings) are related to attitudes, although they are distinct from beliefs and values. They proposed that emotions (or feelings) are less cognitive than attitudes, attitudes are less cognitive than values, and values are less cognitive than beliefs. Beliefs were considered as the most stable in comparison with other components of affect, which are more intense (Leder & Grootenboer, 2005). Based on this proposed model, Grootenboer and Marshman (2016) conducted a series of studies to investigate different aspects of mathematics-related affect with a sample of middle school students (Year 5 to Year 8) in New Zealand. One of the key findings from this research were that students' beliefs about mathematics, emotional responses to mathematics and mathematics learning, and attitudes toward mathematics are related to their mathematics performance (Grootenboer & Marshman, 2016).

Criticising McLeod's model, Op't Eynde et al. proposed a structure of the affect components and their effects on mathematical activities in classroom as shown in Figure 2.4 (Hannula et al., 2007).

Removed due to copyright restriction

Figure 2.4: Op't Eynde et al.'s structure of the affect components (Hannula et al., 2007, p. 204)

In this model, Op 't Eynde et al. developed a theoretical framework to examine cognition, motivation and affect of both teachers and students in different contexts including classroom and socio-historical contexts.

More recently, based on different theories, Hannula (2012) used the term 'metatheory' and suggested three distinct dimensions to categorise theories related to affect. The first dimension distinguishes among cognitive (e.g., beliefs), motivational (e.g., values), and emotional (e.g., feelings) theories. The next dimension consists of the theories that emphasise on the stability of affect (i.e., traits) and the theories that emphasise on the change of affect (i.e., states). The third dimension is divided into three different traditional types of theories about affect, namely, physiological theories, psychological theories, and social theories. The model is presented in Figure 2.5. This is a complex model and a detailed explanation is beyond the scope of this review. However, it can be seen that Hannula considered beliefs as one of the cognitive factors and they play an important role in relation to other affective domains.

Removed due to copyright restriction

Figure 2.5: Three dimensions for a metatheory of mathematics-related affect (Hannula, 2012, p. 144)

In summary, from the above reviews, it can be seen that there exists a large number of studies and proposed frameworks to investigate mathematics-related affect. Various affective factors influencing mathematics teaching, learning and outcomes have been investigated. Despite the differing approaches, many researchers in the field of mathematics education defined four areas of affect: beliefs about mathematics, attitudes towards mathematics, emotions towards mathematics and values towards mathematics. Recently, many other effective variables have also been investigated such as interest, flow, engagement, and identity (Goldin et al., 2016; Hannula et al., 2019).

Among the different models and affective factors listed above, it is worth noting that beliefs have been considered as a crucial factor (Goldin et al., 2009; Leder, 2019; McLeod, 1992). McLeod (1989b) argued that literature needs more evidence about the relationships among belief systems, affective factors and mathematical problem-solving to provide more understanding about these relationships. According to Leder et al. (2002), although beliefs and belief systems first started to be explored by social psychologists, many researchers in other disciplines have also investigated this construct. There are a few controversial issues within the field of mathematics education related to research on mathematics-related beliefs. First, the question *"Are beliefs a hidden variable in mathematics education?"* (G. C. Leder et al., 2002) has been raised and many studies were implemented to attempt to answer that question. Goldin et al. (2009) reviewed different studies in the field and concluded that "beliefs now constitute 'a no longer hidden variable' in research on the teaching and learning of mathematics" (Goldin et al., 2009, p. 14). However, the criticisms about various issues of measuring beliefs and exploring the relationships among beliefs and other facets of affective and cognitive domains persist. Leder (2019) reviewed the research on mathematics-related beliefs and concluded that most research is about how teachers' and students' mathematics-related beliefs influence mathematics learning and instruction. She also emphasised that researchers tend to use small samples with qualitative approaches and proposed that much more research is needed to generate better understanding about these issues. Second, there have been arguments about the affective and cognitive perspectives of beliefs. While some researchers considered beliefs as an affective factor, many researchers in the field of mathematics education considered them as an element of cognition (Hannula, 2012; McLeod, 1992; Schoenfeld, 1985a).

Within the scope of the present research, the remainder of this chapter covers several issues concerning students and teachers' mathematics-related beliefs as well as their relationships with students' mathematics performance.

2.2 Empirical frameworks of students' mathematics-related beliefs

Over the past four decades, the research on mathematics-related beliefs has received much attention from researchers in the field of educational psychology as well as mathematics education. The main purposes of these studies have been to identify different categories of students' mathematics-related beliefs that influence mathematical learning and performance in order to understand the processes of mathematics teaching and learning through which these beliefs develop and are determined.

There have been quite a few different proposed frameworks to analyse mathematics-related beliefs in the field of mathematics education. This section starts with Schoenfeld's (1983, 1985a, 1989) work as one of the first researchers addressing the important role of mathematics-related beliefs in mathematical problem-solving. His findings have provided the basis for various studies. Following, the empirical frameworks used by Underhill (1988), McLeod (1989a, 1992), Pehkonen (1995) Kloosterman (1996), and Op't Eynde et al. (2002) will be introduced.

2.2.1 Schoenfeld's work on mathematics-related beliefs

Schoenfeld (1983, 1985b) highlighted that mathematical-related beliefs are important because they can be used to explain students' behaviours in mathematical settings (Schoenfeld, 1983). He argued that "the tangible cognitive actions produced by our experimental subjects are often the result of consciously or unconsciously held beliefs about (a) the task at hand, (b) the social environment within which the task takes place, and (c) the individual problem-solver's perception of self and his or her relation to the task and the environment" (p. 330). Specifically, when students solve a mathematical problem, their cognitive actions are influenced by different kinds of beliefs in terms of the problem they are working on (i.e., mathematics knowledge), the social environment where the problem is taking place (i.e., the context of the problem), and their self-perceptions in relation to the problem (i.e., the self as a mathematics learner). This suggested that there are various kinds of beliefs students hold during the process of mathematical problem-solving. He defined that mathematics beliefs (or belief systems) are "one's mathematical world view, the perspective with which one approaches mathematics and mathematical tasks" (Schoenfeld, 1985a, p. 45), or "individuals' sense of mathematics, of themselves, of the context and more, all of which shape what they perceive and what they choose to do" (Schoenfeld, 2011, p. 1). He claimed that "one's beliefs about mathematics can determine how one chooses to approach a problem, which techniques will be used or avoided, how long and how hard one will work on it, and so on. Beliefs establish the context within which resources, heuristics, and control operate" (Schoenfeld, 1985a, p. 45). This suggested that students' beliefs influence different stages of the mathematical problem-solving process, and based on the beliefs that students hold, they would have corresponding actions or behaviour to approach and solve problems. Schoenfeld was also one of the first researchers to claim that students' beliefs are crucial contributors to the success or failure of mathematical problem-solving (Schoenfeld, 1985a).

Schoenfeld (1983) designed a qualitative research program to analyse students' beliefs during the problem-solving process. He proposed a framework that included three different components that are necessary for problem solvers, namely, resources, control, and belief systems. Within Schoenfeld's (1983) framework, resources referred to facts, algorithms, and relevant competencies; the component of control included monitoring, assessment, decision-making, and conscious metacognitive acts; and belief systems included beliefs about the self, beliefs about the environment, beliefs about the topic, and beliefs about mathematics. Schoenfeld designed three problem-solving protocols, selecting two freshman students and one mathematician to participate in the experiment. The two students worked together on two protocols. By observing and interviewing them, he found that students tend to believe that only mathematicians can truly understand mathematics and that mathematical problem-solving should occur quickly or not at all. Within this quantitative approach, Schoenfeld developed a questionnaire to measure students' mathematics-related beliefs (Schoenfeld, 1985b). The questionnaire contained 70 statements and 10 open-ended items related to various student belief constructs: (i) beliefs regarding the attribution of success or failure; (ii) perception of mathematics and school practice; (iii) views of school mathematics, English, and social studies; (iv) beliefs about the nature of geometric proofs, reasoning, and constructions; (v) motivation; and (vi) personal and scholastic performance. He recruited 230 students from 9th grade to 12th grade. Factor analysis and correlations were applied to explore each construct and the relationships among these constructs. The analysis revealed that most students tend to think of mathematics as a domain to be rote memorised rather than an area for understanding and creation. They also thought that mathematics required lots of practice in following procedures and rules. Another result showed that students believed that they only needed about two minutes to solve a typical problem and, if a problem took longer than twelve minutes, they believed that it was impossible to solve that problem.

This questionnaire was also used to assess students' mathematics-related beliefs and to investigate associations with their mathematics performance (Schoenfeld, 1989). He found that students' expected mathematics performance, their perceived mathematics ability and their academic performance were strongly associated with each other. This result suggested that if students believe that they have poor mathematics ability, they would likely expect poor performance in turn. Additionally, the author found that "students who think less of their mathematical ability tend more to attribute their mathematical success to luck … and their failures to lack of ability …, whereas those who think themselves good at mathematics attribute their success to their abilities" (Schoenfeld, 1989, p. 347). Another finding was that there were relationships between beliefs about mathematics as a memorising domain, beliefs about success depending on memorisation, and academic performance. Particularly, the less likely students believed that mathematics was mostly memorising, the less likely students believed success depended on memorisation, and the better students perform at mathematics. Schoenfeld (1992) summarised the important results from his prior research. Figure 2.6. provided seven typical beliefs of students about the nature of mathematics.

Removed due to copyright restriction

Figure 2.6: Students' beliefs about the nature of mathematics (Schoenfeld, 1992, p. 359)

These empirical findings have played an important role in exploring mathematics-related beliefs and the associations with mathematics performance. Using these results, many researchers have developed different frameworks and constructs of mathematics-related beliefs, which are discussed in the next sections.

2.2.2 Underhill's framework

Underhill (1988) discusses beliefs in terms of several dimensions. The author summarised research on learners' beliefs and proposed a framework to examine mathematics-related beliefs based on constructivist principles comprising four different categories. The first category was beliefs about mathematics. He claimed that students of all ages held different beliefs about mathematics as a discipline. The author also argued that "what learners know and what they believe are often indiscernible", and "their beliefs permit them to 'know' mathematics through memorization and algorithms. Their beliefs do not foster integration and relational learning" (Underhill, 1988, p. 58). In other words, if students believe that learning mathematics is about performing procedures and memorising formulas, they tend to think that mathematical knowledge is transmitted to them. Additionally, the author claimed that students' beliefs about the nature of mathematics as a discipline influenced their beliefs about mathematics learning.

The second category was beliefs about mathematics learning. Underhill considered different perspectives of learning, namely, transmission and (re)construction, and argued that "teaching and learning are viewed as complementary parts of a (re) construction process" (Underhill, 1988, p. 59). He emphasised that students, who believe mathematics is an abstract body of language, tend to force themselves to master mathematics knowledge by focusing on memorisation and algorithmic learning. On the other hand, students who believe mathematics is a product of creation and invention tend to discover mathematics knowledge by finding new connections as well as relationships of knowledge during their learning process. The third category was beliefs about mathematics teaching. Even though this is a separate category, Underhill (1988) argued that it is difficult to separate beliefs about the nature of mathematics, beliefs about mathematics learning and beliefs about mathematics teaching. The fourth category was beliefs about the social context. These beliefs consider students' behaviours and the social nature of beliefs (Op't Eynde et al., 2002; Underhill, 1988). He argued that these beliefs influence all three other categories of mathematics-related beliefs, namely, beliefs about the nature of mathematics, beliefs about mathematics learning and beliefs about mathematics teaching.

2.2.3 McLeod's framework

McLeod argued that in the field of mathematics education, researchers need to pay more attention to affective factors to inform improvement in learning and teaching of mathematics (McLeod, 1989b, 1992). He also emphasised the important role of mathematics-related beliefs in relation to affective components and mathematics performance. In the first framework, McLeod (1989a) proposed two different categories, including beliefs about mathematics and beliefs about self. He explained that when students learn mathematics, they are most influenced by their beliefs about the nature of mathematics, and beliefs about themselves as a mathematics learner. According to McLeod (1989a), students develop several types of beliefs about mathematics as a discipline during their learning process. He argued that these beliefs "certainly would tend to generate more intense reactions to mathematical tasks than beliefs that mathematics is unimportant, easy, and based on logical reasoning" (McLeod, 1989a, p. 247).

In 1992, after reviewing the relevant research literature, McLeod (1992) concluded that the research in this field was organised in different ways and there was no consistent framework to investigate mathematics-related beliefs as a system. Moreover, depending on their interests and purposes, researchers tended to emphasise different categories of mathematics-related beliefs (McLeod, 1992). This implied that although various types of beliefs have been explored, researchers only focused on beliefs as a single construct, or a set of constructs in isolation to other constructs rather than examining beliefs as a system. Without considering interrelations among different belief constructs, researchers may miss some important associations because these constructs may relate to each other in different ways. He proposed four categories within his framework for mathematics-related beliefs, namely: (1) beliefs about mathematics; (2) beliefs about self; (3) beliefs about mathematics teaching; and, (4) beliefs about the social context.

In regards to beliefs about mathematics, McLeod argued that, normally, students believe that mathematics is important and difficult, as well as based on procedures, formulas and rules, and beliefs about mathematics as a discipline were mainly cognitive in nature and do not change much in comparison to other types of mathematics-related beliefs. This argument was supported by prior findings from Schoenfeld's (1983, 1989) and Silver's (1985) work. Nevertheless, these beliefs play an important role in developing attitudes and emotions towards mathematics. Additionally, he also highlighted that there are gender differences in these beliefs and they tend to change as students get older. Regarding beliefs about the self, McLeod referred to self-concept, confidence, and causal attributions related to mathematics. He claimed that these beliefs also included self-efficacy and learned helplessness. His third category of mathematics-related beliefs was beliefs about teaching mathematics. He pointed out that students' and teachers' beliefs about mathematics teaching are also important to the research on affect. However, he claimed that little is known from literature about these beliefs and researchers need to pay more attention to this area. The final category in McLeod's framework was beliefs about the social context. He emphasised on the role of social context and argued that social context from the classroom (school) or from home may influence students' beliefs. Students' beliefs about the culture of their mathematics classrooms influenced their problem-solving abilities (McLeod, 1992).

2.2.4 Pehkonen's framework

In Pehkonen' (1995) study, he used the term "pupil's view" instead of "students' beliefs". He proposed four categories of mathematics-related beliefs: (1) beliefs about mathematics; (2) beliefs about oneself within mathematics; (3) beliefs about mathematics teaching; and (4) beliefs about mathematics learning. There was a difference in his approach to categorising beliefs in comparison to other researchers. He attempted to divide each category into different subcategories. For example, within the first category of beliefs about mathematics, he created sub-categories, such as: (1a) beliefs about mathematics as a school subject, (1b) beliefs about the birth of mathematical knowledge, and (1c) beliefs about mathematics as a university discipline. From each sub-category, he suggested that researchers may also split into different areas. For instance, within the sub-category of beliefs about mathematics as a school subject, he suggested different beliefs, such as beliefs about the nature of school mathematics, beliefs about mathematical contents, or beliefs about tests in mathematics. From this point of view, his framework can be viewed as an open framework where researchers can contribute beliefs of interest to one of the four main categories. However, Pehkonen also advised that when researchers focus on beliefs, although they should look at both objective and subjective knowledge, and the author claimed that beliefs are subjective knowledge. Researchers also need to distinguish affective and cognitive beliefs, and accept that beliefs are open to change overtime. Furthermore, research on beliefs also needs to be subject to the context and the research goal within which the beliefs are investigated (Furinghetti & Pehkonen, 2002).

2.2.5 Kloosterman's framework

Based on prior research, Kloosterman (1996) also proposed a framework of mathematics-related beliefs to illustrate the relationships between mathematics-related beliefs, motivation, and achievement (Kloosterman, 1996). Figure 2.7 presents this framework.

Removed due to copyright restriction

Figure 2.7: Kloosterman's model of beliefs/motivation/achievement (Kloosterman, 1996, p. 134)

Unlike other researchers, Kloosterman proposed only two categories of mathematics-related beliefs, namely beliefs about mathematics and beliefs about mathematics learning. Kloosterman (1996) highlighted that the category of beliefs about mathematics in his framework is important because "perceptions of the discipline of mathematics can have substantial influence on what students feel is important to study and how they study it" (p. 133). Evidently, the author shared the idea with McLeod (1992) when he also emphasised the important role of beliefs about mathematics in the process of mathematics teaching and learning. It also implied that these beliefs may influence beliefs about mathematics learning. He argued that if students believe that mathematics includes procedures, they are less concerned with making connections between mathematics knowledge, and they also less concerned with understanding concepts and, as a result, they will be motivated to focus more on memorising these procedures.

The second category in Kloosterman's framework was beliefs about learning mathematics. This category consisted of three subgroups: (1) beliefs about oneself as a learner, (2) beliefs about the role of the teacher, and (3) other beliefs about mathematics learning (Kloosterman, 1996). Looking to McLeod's (1992) framework, three categories of beliefs about self, beliefs about mathematics teaching and beliefs about the social context were included in Kloosterman's category of beliefs about learning mathematics category. The first subcategory is beliefs about self. According to Kloosterman (1996), "beliefs about self are important influences on motivation because, in essence, students who feel they are not capable of learning mathematics see little reason to even try" (p. 133). This suggested that students' beliefs about the self may influence their motivation and, in turn, influence mathematics outcomes. The second subcategory in this framework is the role of the teacher as examined from the students' perspectives. Using non-routine problem-solving to teach fourth grade students, Kloosterman argued that students who believe about transmissive teaching of their teachers might be less motivated to learn than students who believe about constructive teaching of their teachers. The last subcategory is other beliefs about mathematics (Kloosterman, 1996). In order to test the model, Kloosterman (2002) designed a study to measure different aspects related to beliefs about mathematics and beliefs about mathematics learning as proposed in his framework, including: self-confidence, perceptions of ability, feelings about school and about mathematics, natural ability in mathematics, study habits in mathematics, and assessment practices. He conducted interviews and surveys with high school students. The main results of this research showed that students seldomly think about the nature of mathematics, and students tend to focus on memorisation procedures and facts rather than understanding them.

2.2.6 Op't Eynde et al.'s framework

Based on prior research, Op't Eynde, De Corte, and Verschaffel (2002) presented a definition and a theoretical framework of mathematics-related beliefs. They defined "students' mathematics-related beliefs are the implicitly or explicitly held subjective conceptions students hold to be true, that influence their mathematical learning and problem-solving" (Op't Eynde et al., 2002, p. 16). More specifically, this refers to the subjective conceptions students implicitly or explicitly hold to be true about different aspects including mathematics education, themselves as mathematics learners, and the mathematics class context. They suggested that these beliefs are associated with each other and with problem-solving activities. These authors also proposed a triangular model of mathematics-related belief system as shown in Figure 2.8.

Removed due to copyright restriction

Figure 2.8: Op't Eynde et al.'s framework of mathematics-related beliefs (Op't Eynde et al., 2002, p. 27)

Op't Eynde and De Corte (2003) argued that "the analysis of the nature and the structure of beliefs and belief systems points to the social context, the self and the object in the world that the beliefs relate to, as constitutive for the development and functioning of these systems" (p. 5). They argued that a mathematics-related belief system consisted of three categories: (1) beliefs about object (mathematics education); (2) beliefs about the self; and, (3) beliefs about the classroom context (Op't Eynde & De Corte, 2003). Although there is it can be seen that Op't Eynde et al.'s and McLeod's (1992) framework shared some similarities, there is one major difference between them. The category of beliefs about mathematics teaching from McLeod's (1992) framework is not represented in Op't Eynde and De Corte's (2003) model. However, this category could be considered as a sub-component in the category of beliefs about mathematics education because it relates to teaching practices.

Within the first category, beliefs about mathematics education, Op't Eynde et al. (2002) stated that these beliefs refer to students' perceptions of what mathematics is, and the way students approach mathematics and mathematics problems. They proposed three sub-categories, namely: (1) beliefs about mathematics as a subject; (2) beliefs about mathematical learning and problem-solving; and, (3) beliefs about mathematical teaching in general. The second category, beliefs about self, refer to (1) self-efficacy beliefs; (2) control beliefs; (3) task value beliefs; and, (4) goal orientation beliefs (including their intrinsic and extrinsic goal orientation beliefs related to mathematics). These beliefs are also viewed as motivational beliefs. The final category, beliefs about classroom context, consisted of: (1) beliefs about social norms in their own class (including beliefs about the role and function of the teacher and beliefs about the role and function of the students); and, (2) beliefs about the socio-mathematical norms in their own class (Op't Eynde et al., 2002). Although the two frameworks employ different approaches, it can be seen that various aspects of mathematics-related beliefs have been covered. These authors also claimed that this framework has considered mathematics-related beliefs as a system.

Op't Eynde and his colleagues developed the *Mathematics-Related Beliefs Questionnaire* (MRBQ) to validate the structure of mathematics-related belief system and measure students' mathematics-related beliefs (Op't Eynde & De Corte, 2003; Op't Eynde, De Corte, & Verschaffel, 2006a). The MRBQ consisted of items to cover three categories of beliefs, namely, beliefs about mathematics education, beliefs about the self, and beliefs about teachers' role. The questionnaire was developed to measure four different sub-categories rather than the concept as a whole: (1) beliefs about the role and the functioning of their own teacher (16 items); (2) beliefs about the significance of and competence in mathematics (13 items); (3) beliefs about mathematics as a social activity (nine items); and, (4) Mathematics as a domain of excellence (six items). In terms of the results, the authors confirmed the structure of the framework of mathematics-related beliefs in which the category number one belongs to beliefs about the social context, category number two belongs to beliefs about self, and categories number three and four belong to beliefs about mathematics (Andrews, Diego-Mantecón, Vankúš, & Op 't Eynde, 2011; Op't Eynde & De Corte, 2003). The questionnaire was also used in other research to confirm the structure of the framework as well as to measure mathematics-related beliefs of Flemish junior high school students (Op't Eynde et al., 2006a) and Ecuadorian students (De Corte, 2015).

In summary, in this section, the most relevant frameworks of mathematics-related beliefs in the mathematics education literature were introduced. Table 2.1 synthesises different categories of these frameworks. As can be seen in Table 2.1, many researchers have emphasised the important role of mathematics-related beliefs in mathematics teaching and learning processes, and proposed various approaches to define mathematics-related beliefs. All of the above-mentioned frameworks included different kinds of beliefs, as shown in the table. Some researchers claimed that, in general, beliefs about the nature of mathematics affect beliefs about mathematics learning, especially beliefs about the self as a mathematics learner. Based on these frameworks, these authors and other scholars in the field of mathematics education have conducted additional research to investigate how mathematics-related beliefs associate with other domains such as attitudes, motivation, behaviour, and mathematics performance.

Table 2.1. Frameworks of Mathematics-related beliefs

Author	Category
Schoenfeld (1983, 1985a, 1989a)	(1) beliefs about the task in hand (2) beliefs about the social environment within which the task place (3) the individual's perceptions of the self
Underhill (1988)	(1) beliefs about the nature of mathematics (2) beliefs about mathematics learning (3) beliefs about teaching mathematics (4) belief about self within a social context
McLeod (1992)	(1) beliefs about mathematics as a discipline (2) belief about self (3) beliefs about mathematics teaching (4) beliefs about the social context
Pehkonen (1995)	(1) beliefs about mathematics (2) beliefs about oneself within mathematics (3) beliefs about mathematics teaching (4) beliefs about mathematics learning
Kloosterman (1996)	(1) beliefs about mathematics as a subject (2) beliefs about learning mathematics
Op't Eynde et al. (2002)	(1) beliefs about mathematics education (2) beliefs about the self (3) beliefs about the classroom context

However, from the above reviews, it can be seen that these frameworks have undergone considerable changes overtime, especially regarding the way in which different components of mathematics-related beliefs have been categorised. There exists contention when a category in one framework may belong to or overlap with a category of another framework, or some items measuring one construct could be found to measure another construct. Diego-Mantecón et al. (2019) also suggested that researchers in the field of mathematics education need to develop a more consistent framework. Most importantly, only a few scholars have conducted empirical research to validate their frameworks or attempts at validation have not been comprehensive. For example, with Op't Eynde et al.'s (2002) empirical studies to validate their framework of mathematics-related beliefs, only select belief constructs within each belief category were investigated.

The next section reviews the empirical research that has investigated specific mathematics-related beliefs and their associations with mathematics performance.

2.3 Research on specific constructs of mathematics-related beliefs

There have been multiple studies of different constructs of mathematics-related beliefs. As discussed in the previous section, there are some inconsistencies in these frameworks of mathematics-related beliefs. Researchers in the field of mathematics education have paid attention to different separate belief constructs and their influence on mathematics performance. Based on the review of the literature and the focus of the present research on a more complex model investigating relations among mathematics-related beliefs and mathematics performance, this section reviews the research on five categories of mathematics-related beliefs: (1) beliefs about the certainty of mathematics; (2) beliefs about the usefulness of mathematics; (3) beliefs about mathematical problem-solving; (4) beliefs about the self as a mathematics learner (including beliefs about mathematics ability and self-efficacy beliefs); and, (5) students' perceptions of teachers' practices. In the last section, research on the differences in mathematics-related beliefs is also reviewed.

2.3.1 Beliefs about the certainty of mathematics knowledge

Beliefs about the certainty of knowledge form an important component of epistemological beliefs. Before reviewing the research literature on these beliefs and their influence on mathematics performance, a brief review of research on epistemological beliefs will be presented.

2.3.1.1 Epistemological beliefs

Epistemological beliefs are beliefs about the nature of knowledge and the processes of knowing (Bendixen & Feucht, 2010; Hofer, 2002; Hofer & Pintrich, 1997; Khine, 2008; Schommer, 1990). Beliefs about the nature of knowledge refer to what individuals believe knowledge is. Beliefs about the nature of knowing refer to what individuals believe about the process of knowledge acquisition. Within theoretical models, beliefs about the nature of knowledge are further defined as the certainty and simplicity of knowledge to measure whether students believe that knowledge is certain and simple, or something that changes and is complex. Beliefs about the process of knowing are further defined in terms of the source of knowledge, whether it comes from authority or is constructed by the subject, and the justification for knowing, which answers the question how individuals evaluate knowledge claims (Hofer & Pintrich, 1997). Epistemological beliefs are also known as epistemic beliefs or personal epistemology (Bendixen & Feucht, 2010; Buehl & Alexander, 2001; Hofer, 2004). Personal epistemology is defined as "a field that examines what individuals believe about how knowing occurs, what counts as knowledge and where it resides, and how knowledge is constructed and evaluated" (Hofer, 2004, p. 1).

Reviewing relevant research literature showed that research on epistemological beliefs started with Perry's (1968) research on validating a scheme of epistemological development by using in-depth interviews. Inspired by this research, many researchers in different fields of study investigated various aspects of epistemological beliefs (Schoenfeld, 1989). An influential study in this area was Schommer's framework (Schommer-Aikins, 2002, 2004; Schommer, 1990, 1994), which considered epistemological beliefs as a system. Schommer (1990) was interested in investigating how beliefs about the nature of knowledge and the processes of knowing influence approaches to learning. She designed an empirical study on how these beliefs influence comprehension and academic achievement. Unlike other researchers who approached epistemological beliefs as a unidimensional construct, she proposed that these beliefs are multi-dimensional and consist of many independent belief constructs. Specifically, based on Perry's (1968), Dweck's (1986) and Schoenfeld's (1983, 1985a, 1989) work, Schommer (1990) proposed a framework comprising five different constructs of epistemological beliefs: (1) simple knowledge, (2) certain knowledge, (3) omniscient authority, (4) innate ability; and, (5) quick learning. It is important noting that beliefs about the certainty of knowledge examine whether knowledge is considered as something absolute or changing. In order to validate this framework, she developed a questionnaire that consisted of 63 items to cover these five categories known as the Epistemological Beliefs Questionnaire (Schommer, 1990). This questionnaire has been widely used by Schommer and other researchers to measure various aspects of epistemological beliefs as well as their influence on performance.

Although some other research confirmed this framework (Jehng, Johnson, & Anderson, 1993; Schraw, Dunkle, & Bendixen, 1995), there has been criticism of the framework in terms of which beliefs constructs should be represented in an epistemological belief system. Hofer and Pintrich (1997) suggested that the most important limitation of Schommer's work in 1990 is that only two types of beliefs, namely, certain knowledge and simple knowledge, can be considered constructs of epistemological beliefs, whereas other types of beliefs (i.e., innate ability and quick learning) belong to other categories. For example, beliefs about innate ability refer to beliefs about the nature of ability that have been studied by Dweck and colleagues (Dweck, 2000; Dweck & Leggett, 1988) , and beliefs about quick learning may belong to beliefs about learning rather than epistemological beliefs (Hofer & Pintrich, 1997).

Additionally, although different views have been proposed to explain the structure of epistemological beliefs, there is a general agreement that people's epistemological beliefs influence how they learn (Hofer, 2004; Muis, 2004). Hofer (2001) reviewed the literature on epistemological beliefs and there is evidence to support the claim that epistemological beliefs may directly influence beliefs about learning and education, students' motivation, and learning strategy selection. Schraw er al. (1995) examined the relationship between epistemological beliefs and students' responses of well-defined and ill-defined problems. Well-defined problems are defined as problems where they have knowable solutions and students can find correct answers, whereas ill-defined problems are defined as problems where they contain inconsistent assumptions and students may find different solutions (Kitchener, 1983). The results of regression analysis showed that epistemological beliefs are related to ill-defined solutions and concluded that epistemic beliefs play an important role in ill-defined problem-solving. Another review of the literature on epistemological beliefs suggested that epistemological beliefs are: (1) multidimensional and multilayered in nature; (2) significantly related to other learning outcomes; and, (3) can be characterised both as general and domain-specific (Buehl & Alexander, 2001).

2.3.1.2 Domain specificity of epistemological beliefs – The case of mathematics

While much of the prior research has focused on epistemological beliefs in domain general, many researchers have also focused on domain specificity (i.e., academic discipline) of epistemological beliefs such as mathematics (Depaepe et al., 2016; Muis, 2004; Schoenfeld, 1992), science (Elby, Macrander, & Hammer, 2016; Mason, Boscolo, Tornatora, & Ronconi, 2013), history (VanSledright & Maggioni, 2016), and general discussions about domain specificity (Hofer, 2006; Muis, Bendixen, & Haerle, 2006). There is some evidence of relationships of epistemological beliefs between domains. Schommer and Walker (1995) explored the differences of students' epistemological beliefs between two disciplines: mathematics and social science. In this study, students completed a questionnaire about epistemological beliefs twice, each time with each specific domain in mind (mathematics or social science). The authors concluded that epistemological beliefs are likely to vary depending on the academic discipline. For example, they found that students have stronger beliefs about certainty of knowledge in the domain of mathematics than in the domain of social science (Schommer & Walker, 1995).

Other researchers also argued that because of the differences about knowledge structures and models, there are differences about epistemological beliefs among disciplines (Hofer, 2000; Hofer & Pintrich, 1997). For example, Hofer (2000) investigated students' epistemological beliefs and the author examined the differences in these beliefs between psychology and science. The results showed that students tend to view knowledge in psychology as less certain compared with knowledge in science. There is further evidence of this from Baxter Magolda's (2002) research where four disciplines were compared: mathematics, science, humanities, and social science. The results showed that students tended to view mathematics and science knowledge as certain while they viewed humanities and social science knowledge as uncertain. Buehl and Alexander (2005) also found that students tend to believe that knowledge in mathematics is more certain compared with knowledge in history, and that students who believe more in the certainty of knowledge have lower levels of motivation and task performance. Within the scope of the present research, this section will only review research on epistemological beliefs in the domain of mathematics and how they influence learning and mathematics achievement.

Also working in the field of mathematics education, Schommer et al. (1992) examined the effects of students' epistemological beliefs on their comprehension of mathematical text. As mentioned earlier, they designed the first experiment to confirm the structure of an epistemological belief questionnaire. In the second experiment, in order to explore these effects, the authors used two statistical passages, a statistical test, a scale to measure the confidence in their comprehension, a measure of prior knowledge, and a measure of study strategies. Using regression analysis, they found that beliefs about the simplicity of mathematics knowledge have a negative effect on comprehension and meta-comprehension. In other words, the more students believed in simple knowledge the worse they performed on the mathematics text comprehension test. They also found that these effects are mediated by students' study strategies collection. In another study, Schommer (1993) examined the associations between different constructs of epistemological beliefs with academic performance using students' grade point average (GPA) as a measure. The sample included 405 first-year students, 312 second-year students at universities, 274 junior students, and 191 senior students at high schools. The instrument consisted of the *Epistemological Beliefs Questionnaire*. She found that epistemological beliefs significantly change from senior students to first-year students and suggested a longitudinal study should be implemented to confirm this result. In this study, Schommer also found that the construct of beliefs about quick learning predicted academic performance among secondary school students (Schommer, 1993).

Further exploring links between epistemological beliefs and mathematical problem-solving performance, Schommer-Aikins and her colleagues (2005) studied how students' epistemological beliefs, as well as their beliefs about mathematics, influenced their mathematical problem-solving performance with 1,269 7th and 8th grade students. The instruments included the Epistemological Belief Questionnaire (Schommer, 1990) to measure students' epistemological beliefs, and a mathematical problem-solving test including two mathematics problems. Students were asked to solve these problems and explain the way they found the answers. GPA and reading scores were also included in the analysis. The results of a path analysis indicated that beliefs in quick/fixed learning (i.e., learning can occur quickly and learning depends on ability rather than on effort) negatively predicted mathematical problem-solving performance and GPA. Specifically, the more students believed in quick/fixed learning, the lower their grade point average was, and the more likely they were to get low results on mathematical problem-solving. A similar study was designed to explore the associations between epistemological beliefs and mathematics performance among a sample of 701 college students (Schommer-Aikins & Duell, 2013). The instruments included the Epistemological Beliefs Questionnaire (Schommer, 1990), the beliefs about mathematical problem-solving questionnaire and a mathematics performance test. The authors also investigated mathematical cognitive depth (i.e., being able to identify the structural feature of a mathematical problem), and mathematical background (i.e., the number of mathematics courses students completed) in the research. Results showed that beliefs about quick learning indirectly predicted mathematics performance through mathematics background and cognitive depth. It is important to note that the results of this research also showed that there are associations between epistemological beliefs about mathematics and beliefs about the nature of mathematics (i.e., mathematics is useful), and epistemological beliefs about mathematics have indirect effects on mathematics performance through beliefs about the nature of mathematics, students' mathematics background, and cognitive depth (Schommer-Aikins & Duell, 2013).

Apart from Schommer's studies, many other researchers have also explored epistemological beliefs about mathematics and their associations with mathematics behaviour in general, and mathematics achievement in particular. Muis (2004) reviewed the literature on epistemological beliefs about mathematics and introduced the terms 'availing belief' (i.e., one that is positively associated with quality learning and achievement) and 'non-availing belief' (i.e., one that is negatively associated with quality learning and achievement). Based on many studies related to beliefs about mathematics, Muis synthesised some important features of students' epistemological beliefs. First, students usually hold non-availing beliefs (e.g., mathematics knowledge is not changing, belief in innate ability). Second, although studies on epistemological beliefs have been conducted and showed similar results, the problem with the reliability and validity of measuring epistemological beliefs should be given greater focus. Third, the published works on epistemological beliefs showed a lack of theoretical and methodological problems. In terms of effect of epistemological beliefs on behaviour, Muis concluded that there is much evidence about the significant influence of epistemological beliefs on cognitive and motivational factors and mathematics achievement. She proposed that modern statistical techniques should be applied to analyse the complex relationships between these beliefs and students' behaviours, especially mathematics achievement (Muis, 2004). She also explored the relationship between epistemological beliefs and students' self-regulated learning during the mathematical problem-solving process by using a qualitative approach (Muis, 2008). Muis claimed that students who hold sophisticated epistemological beliefs perform better than students with simplistic epistemological beliefs. Recently, Depaepe et al. (2016) also reviewed the research literature on mathematical epistemological beliefs and found that mathematical epistemological beliefs are associated with other types of beliefs about mathematics teaching and learning.

In terms of the using instruments to measure epistemological beliefs about mathematics, some researchers have assumed that there is the same factor structure between general epistemological beliefs and epistemological beliefs about mathematics (Iannone & Simpson, 2019). Schommer-Aikins et al. (2003) and Buehl et al. (2002) adapted the general Epistemological Beliefs Questionnaire (Schommer, 1990) by asking students to think about to domain-specific when they answered the questionnaire. In contrast, Op't Eynde et al. (2006b) proposed that researchers in the field of mathematics education could also approach domain-specific epistemological beliefs from a domain-specific belief system. They stated that "students' beliefs about knowledge and knowing in classroom mathematics seem to be very much the exponents of interactions between the individual and the context, rather than the logical consequence of the epistemological beliefs students hold at a more general level" (Op't Eynde et al., 2006b, p. 68). The authors proposed that when researching students' domain-specific beliefs about knowledge and knowing, researchers should use the term 'epistemic dimensions' rather than epistemological beliefs. Op't Eynde et al. (2006b) demonstrated their idea by analysing their Mathematics-Related Beliefs Questionnaire (Op't Eynde & De Corte, 2003). For the purpose of the analysis, they focused on the category of beliefs about mathematics education and proposed that epistemological beliefs about mathematics are highly domain- and context-specific.

Additionally, some other scholars have developed new instruments for measuring epistemological beliefs about mathematics. For instance, based on Schommer's work about epistemological beliefs, Hoffer (2000) and Buehl et al. (2002) developed their own questionnaires to measure domain-specific epistemological beliefs to compare these beliefs across different domains such as mathematics, social sciences, history, science, and psychology. Based on prior research about epistemological beliefs, and the questionnaire from the Trends in International Mathematics and Science Study (TIMSS), Köller (2001) developed a questionnaire to measure students' epistemological beliefs specifically about mathematics. This instrument has four main types of beliefs: (1) certain knowledge, (2) simple knowledge, (3) constructivism conception, and, (4) relevance of mathematics. This instrument was used to predict mathematics performance using the results from TIMSS. The results showed that all the four types of epistemological beliefs about mathematics are significant predictors of mathematics performance. Specifically, beliefs about the certainty and simplicity of mathematical knowledge had negative effects on mathematics performance, whereas beliefs about constructivism and relativism had positive effects on mathematics performance (Köller, 2001). In order to examine the relationships between mathematics achievement and learner-related variables, Nasser and Birenbaum (2005) investigated the relationships between many different aspects of beliefs, motivational variables, and mathematics achievement. The sample included 478 middle school students in Israel. They designed a five-item scale to measure students' epistemological beliefs. The Israel National Assessment Test in Mathematics was used to measure mathematics performance. They also measured students' self-efficacy, attitude towards mathematics, and mathematics anxiety. The results from structural equation modelling showed that epistemological beliefs had indirect effect on mathematics achievement through students' self-efficacy.

Another instrument to measure epistemological beliefs about mathematics was developed and validated by Wheeler (2007). This instrument consists of seven main types of beliefs: (1) source of knowledge, (2) certainty of knowledge, (3) structure of knowledge, (4) speed of knowledge acquisition, (5) innate ability-general, (6) innate ability- personal, and (7) real-world applicability. Some types of beliefs, such as beliefs about the certainty of knowledge and beliefs about innate ability, are similar to those in Schommer's questionnaire. However, Wheeler added more questions in each of the above types of beliefs and a new type about the real- world applicability of mathematics. This component included 11 questions about the application of mathematics knowledge in real-world life, such as "understanding mathematics is important for mathematicians, economists, and scientists but not for most people", and "mathematics helps us better understand the world we live in" (Wheeler, 2007, p. 96).

2.3.1.3 Beliefs about the certainty of mathematics knowledge and their effect on achievement

To recap, beliefs about the certainty of knowledge examine whether knowledge is considered absolute or changing. In general, the certainty of knowledge is an important construct of epistemological beliefs (Schommer, 1990). Prior research showed that students tended to view mathematics knowledge as more certain than other disciplines (Baxter Magolda, 2002; Buehl & Alexander, 2005; Hofer, 2000) and also indicated that there are relationships between beliefs in the certainty of knowledge and learning (Hofer, 2000; Hofer & Pintrich, 1997; Schommer, 1990; Schraw et al., 1995). For example, the more students believed that knowledge is certain, the more likely they are to misinterpret tentative conclusions (Schommer, 1990). Beliefs in certain knowledge have been found to be negatively related to graded academic performance (Hofer, 2000). Moreover, Schommer (1998) highlighted that while students who believed that knowledge is simple and certain were likely to have single answer to a question, students who believed knowledge is about complex and tentative tend to have complex answers to a question.

In addition to the research on the certainty of knowledge in general, there has been research focusing on specific domains, especially mathematics (Buehl & Alexander, 2005; Schommer & Walker, 1995). Schommer and Walker (1995) investigated the similarities of epistemological beliefs in mathematics and in the social sciences, and examined the effect of these beliefs on comprehension. A questionnaire to measure their epistemological beliefs (including beliefs about the certainty of mathematics knowledge) was completed by 114 undergraduate students. Students also read a passage (mathematics or social sciences) and took a test about their understanding of the passage. Regression analysis showed that the certainty of mathematics and social science knowledge predicted social sciences passage comprehension. In other words, the more students believed in certain knowledge in mathematics or social sciences, the worse they performed on social sciences passage comprehension. In another study, Schommer-Aikins (2008) explored university students' and professors' epistemological beliefs. She applied a qualitative aproach using semi-structured interviews with twenty students and four mathematicians. In regards to beliefs about the certainty of mathematics, she asked participants to write down the percentages responding to statements, such as "Percent of mathematical knowledge that is unchanging", or "Percent of mathematical knowledge that is always changing or evolving" (Schommer-Aikins, 2008, p. 308), following up with in-depth interview questions. She concluded that, generally, students had strong beliefs about the certainty of mathematics knowledge and many students even believed that mathematics knowledge is entirely unchanging.

Buehl (2003) examined the relationships between epistemological beliefs about mathematics and history and other different constructs such as beliefs about ability, expectancies for success, achievement values, and performance. The sample consisted of 482 undergraduate students.

Measures of the research included epistemological beliefs, competency beliefs, achievement values, and a learning performance task. Employing structural equation modelling techniques for the domain of mathematics, Buehl found that although beliefs about the certainty of mathematics do not directly or indirectly predict mathematics performance, these beliefs directly influence beliefs about mathematics ability and mathematics achievement values. Using the data from the above research, Buehl and Alexander (2005) also studied the differences of students' epistemological beliefs in history and mathematics. They claimed that students who believe more in the certainty of mathematics knowledge had lower levels of motivation and task performance.

Cano (2005) also investigated the relationships between epistemological beliefs, approaches to learning, and academic performance of secondary students by adapting the Epistemological Beliefs Questionnaire (Schommer, 1990). The results of path analysis showed that, although beliefs about the certainty of knowledge did not directly predict academic performance, these beliefs had indirect effect on academic performance through the approaches of learning (Cano, 2005). Specifically, the more students believed knowledge is certain, the more students approached learning through surface strategies (instead of deep strategies), and the less students performed well on academic achievement.

Muis and Foy (2010) investigated the associations among teachers' beliefs, students' beliefs, motivation and mathematics performance in elementary schools in USA. For students, beliefs about the certainty and simplicity of knowledge in mathematics were included. Results from structural equation modelling showed that beliefs about the certainty and simplicity of knowledge in mathematics indirectly predicted mathematics performance through achievement goals (both mastery goals and performance goals) and students' self-efficacy. As mentioned elsewhere in this chapter, Schraw et al. (1995) explored the relationship between epistemological beliefs and problem-solving. They also found that beliefs about certain knowledge are related to ill-defined problem-solving.

While the results of the above research showed that beliefs about the certainty of mathematics knowledge have direct or indirect effects on other types of mathematics-related beliefs and mathematics performance, some research also showed that there are non-significant links between these beliefs and achievement. Schommer-Aikins et al. (2005) measured beliefs about the certainty of knowledge and examined the relationships with beliefs about mathematical problem-solving and performance of middle school students. Results of regression analysis showed that there was no statistically significance between these beliefs and achievement. In a similar study in 2013, she and her colleagues also reached the same conclusion a the sample of college students (Schommer-Aikins & Duell, 2013). Lodewyk (2007) investigated the effect of secondary school students' epistemological beliefs on academic achievement and task performance with a sample of 447 tenth grade students. The author also used Schommer's questionnaire which included beliefs about the certainty of knowledge along with students' overall academic achievement (including mathematics) as students' achievement. Results from a regression analysis showed that beliefs about the certainty of knowledge did not predict academic achievement and task performance. Phan (2008) investigated the relationship between epistemological beliefs (including beliefs about certainty of knowledge) and students' self-efficacy. The results of multiple regression analysis showed that beliefs about certainty of knowledge negatively predicted mastery goal orientation but these beliefs did not significantly predict self-efficacy and students' use of self-regulatory strategies.

To summarise, over the last three decades, epistemological beliefs have drawn much attention from researchers in different disciplines, and Schommer's research has been considered influential as she examined epistemological beliefs as a system. Epistemological beliefs have been found to be an important component of students' beliefs in general, and to influence different aspects of learning and educational outcomes. Apart from domain-general epistemological beliefs, many researchers have focused on domain-specific epistemological beliefs, including mathematics. There has been much evidence about how epistemological beliefs about mathematics directly and indirectly influence mathematics performance. Among various beliefs constructs of epistemological beliefs, prior research revealed the important role of beliefs about the certainty of mathematics in the mathematics learning process, especially mathematics achievement. However, from the above review, it can be seen that beliefs about the certainty of mathematics influenced mathematics performance in different ways within different samples. For example, within the sample of undergraduate students, these beliefs directly predicted mathematics performance, whereas for secondary school students, these beliefs indirectly or non-significantly predicted performance.

2.3.2 Beliefs about the usefulness of mathematics

Beliefs about the usefulness of mathematics have been found as an important component of mathematics learning (Arikan, van de Vijver, & Yagmur, 2016; Kloosterman & Stage, 1992; Schommer-Aikins et al., 2005). This section reviewed research on beliefs about the usefulness of mathematics and their influence on mathematics performance.

Although there have been different instruments measuring beliefs about the usefulness of mathematics, a commonly used instrument has been the *Fennema-Sherman Mathematics Attitude Scale* by Fennema and Sherman (1976), and other forms of this instrument which were re-validated and adapted (Broadbooks, Elmore, Pedersen, & Bleyer, 1981; Mulhern & Rae, 1998; Wikoff & Buchalter, 1986). For example, Kloosterman and Stage (1992) designed a questionnaire, the *Indiana Mathematics Beliefs Scale*, to measure beliefs about mathematical problems solving. They also recommended that, when using this scale, researchers should also include beliefs about the usefulness of mathematics from the *Fennema-Sherman Mathematics Attitude Scale*. Using this scale, Schommer-Aikins et al. (2005) investigated the relationships between beliefs about the usefulness of mathematics and their mathematical problem-solving performance. The results of a path analysis indicated that beliefs in the usefulness of mathematics predict mathematical problem-solving performance. In other words, the less students believe in the usefulness of mathematics, the less likely they are to solve mathematical problem successfully. In this research, they also found that beliefs about the usefulness of mathematics act as a mediator between epistemological beliefs and mathematics achievement. Specifically, the more students believe in fixed/quick learning, the less students believe in the usefulness of mathematics, and the less students perform well in solving mathematical problem.

Apart from research using the *Fennema-Sherman Mathematics Attitude Scale*, many other researchers have investigated beliefs about the usefulness of mathematics using other instruments. For instance, Midgley et al. (1989) investigated the relationships between student/teacher relations and attitudes toward mathematics during transition to junior high school, and beliefs about the importance and usefulness of mathematics were included in the category of attitudes toward mathematics. The instrument to measure these beliefs included four items develop by Parsons (1980, cited in Midgley et al., 1989). The sample consisted of 1,301 sixth grade students. They found that, within the group of low-achieving students, their beliefs about the importance and usefulness of mathematics were negatively associated with their mathematics-related achievement. As mentioned earlier in the section about epistemological beliefs, Köller (2001) developed a questionnaire to measure students' epistemological beliefs specifically about mathematics, and the construct of relevance of mathematics had many statements related to the usefulness of mathematics. In this study, Köller found that the construct of relevance of mathematics directly predicted mathematics achievement. Additionally, the results showed that this construct also indirectly predicted mathematics achievement through mathematics interest (Köller, 2001).

Kadijevich (2008) asserted that beliefs about usefulness of mathematics is one of three major dimensions of attitudes toward mathematics. He examined TIMSS data from 33 countries to ascertain associations between beliefs about usefulness of mathematics, mathematics interest, self-confidence and mathematics performance. Kadijevich (2008) used four 4-point Likert scale items to measure beliefs about usefulness of mathematics. The results of regression analysis showed that beliefs about usefulness of mathematics are positively related to mathematics performance for almost all countries. The author also found positive relationships between these beliefs and mathematics achievement considerably varied from country to country (Kadijevich, 2008). Drawing on Kadijevich's (2008) research, Vandecandelaere et al. (2012) examined the relationships between students' perceptions of the learning environment and three aspects of their attitudes toward mathematics, including beliefs about usefulness of mathematics. The authors claimed the higher the mean cognitive abilities of the class group were, the more valuable mathematics was perceived. Espinosa (2018) investigated the relationship between beliefs about the usefulness of mathematics, self-confidence in mathematics ability and performance with a sample of 306 undergraduate students. Results of hierarchical regressions analysis indicated that, with students have low self-confidence, there was a negative relationship between students' beliefs that mathematics is useful and their mathematics performance.

It is worth noting that there is also evidence from prior research showing that beliefs about the usefulness of mathematics do not significantly predict mathematics performance. For example, Pajares & Miller (1994) designed a study using path analysis to examine the relationships between different constructs including beliefs about the usefulness of mathematics and mathematics performance. The researchers recruited 350 undergraduate students to participate in this research and results showed that there was no significant relationship between these two constructs. Using TIMSS data, Arikan et al. (2016) investigated factors influencing Turkish students' achievement in mathematics in comparison to Australian students. In this research, among other factors, they examined the relationship between beliefs about the usefulness of mathematics ('valuing mathematics') and mathematics achievement. The results of structural equation modelling showed that valuing mathematics is not significantly related to achievement in either country.

Looking at the studies mentioned above, it can be seen that beliefs about the usefulness of mathematics are one of the important factors that influence mathematics performance with many different instruments used to measure this construct. However, prior research revealed inconsistencies in the way these beliefs influence mathematics performance. They may directly influence mathematics performance, as well as being a mediator, or can be mediated by other beliefs constructs. For example, beliefs about the usefulness of mathematics have been found to be a mediator between epistemological beliefs and mathematical problem-solving performance in Schommer-Aikins et al.'s (2005) research.

2.3.3 Beliefs about mathematical problem-solving

Mathematical problem-solving has long been seen as one of the most crucial components of mathematics education, including teaching and learning mathematics (Liljedahl, 2019). Within research related to this issue, beliefs about mathematical problem-solving have been considered an important factor. As previously mentioned, Schoenfeld (1985a) emphasised that students' beliefs are crucial contributors to the success or failure of mathematical problem-solving. He found that solving problems is believed to be mostly about the memorisation of rules and procedures (Schoenfeld, 1989). Garofalo (1989) found that many students believed that they could apply procedures and formulas to solve most of mathematics problems, so they tend to try memorising those facts, rules and procedures. However, research literature has shown the inconsistences in categorising beliefs about mathematical problem-solving. For example, while Kloosterman and Stage (1992) proposed that this category of beliefs include five specific constructs (which will be discussed in detail later), Op't Eynde et al. (2002) defined this category as a part of beliefs about learning and problem-solving which fall under beliefs about mathematics education. Although overlap in categorising mathematics-related beliefs has been widely accepted by many researchers because of their dependences in each other (Diego-Mantecón et al., 2019), in order to avoid repetition of different types of beliefs, this section only discusses beliefs about mathematical problem-solving proposed by Kloosterman and Stage (1992) and presents some important results from the research literature using these beliefs.

In order to measure beliefs about mathematical problem-solving, Kloosterman and Stage (1992) designed an instrument known as *The Indiana Mathematics Beliefs Scale*. After pilot testing with 50 items within five different categories of beliefs, the final 30-item scale was administered to 517 college students. The instrument consisted of the five following belief constructs: (1) beliefs regarding whether students can solve time-consuming mathematics problems; (2) beliefs regarding the nature of word problems; (3) beliefs regarding the importance of understanding mathematical concepts; (4) beliefs about word problems; and, (5) beliefs regarding the role of effort in mathematics ability. The issues of word problems are beyond the scope of the present research therefore, this section will discuss only the other four constructs.

2.3.3.1 Beliefs regarding whether students can solve time-consuming mathematics problems

According to Kloosterman and Stage (1992), this category is worth investigating because prior research, such as Schoenfeld's (1988) work, showed that, in general, many mathematics learners believe that every mathematics problems can be solved within five minutes. As a result, they may give up on problems that take more than five minutes to solve. This category appears as "I can solve time-consuming mathematics-problems". There are six items in this scale with three positive wording items (e.g., "Math problems that take a long time don't bother me") and three negative wording items (e.g., "If I can't do a math problem in a few minutes, I probably can't do it at all") (Kloosterman & Stage, 1992, p. 115).

2.3.3.2 Beliefs regarding the nature of word problems

Based on prior research, Kloosterman and Stage (1992) argued that, generally, students believe that there are always rules, formulas, or procedures to follow in mathematics. As a consequence, when they do not find a suitable rule for a problem, they may give up or apply an inappropriate rule. Kloosterman and Stage (1992) developed this category to measure to what extent students believe otherwise, but applied it to word problems rather than mathematics problems in general. This category was called "There are word problems that cannot be solved with simple, step-by-step procedures". There are six items in this scale with three positive wording items (e.g., "Memorising steps is not that useful for learning to solve word problems") and three negative wording items (e.g., "Learning to do word problems is mostly a matter of memorising the right steps to follow") (Kloosterman & Stage, 1992, p. 115).

2.3.3.3 Beliefs regarding the importance of understanding mathematical concepts

Kloosterman and Stage (1992) synthesised different results from prior research and concluded that while some students believe that it is important to understand mathematical procedures behind giving a correct answer, many other students believe that learning mathematics is mostly memorisation. Therefore, they developed the construct, namely, "Understanding concepts is important in mathematics" to measure beliefs about regarding the importance of understanding mathematics. This construct has six items with three positive wording items (e.g., "In addition to getting a right answer in mathematics, it is important to understand why the answer is correct") and three negative wording items (e.g., "It's not important to understand why a mathematical procedure works as long as it gives a correct answer") (Kloosterman & Stage, 1992, p. 115).

2.3.3.4 Beliefs regarding the role of effort in mathematics ability

This scale was first developed by Kloosterman (1988). He investigated effort as a mediator of mathematics ability in relations to other constructs such as self-confidence. He defined effort as a mediator of mathematics ability as “the extent to which students felt that continued effort in mathematics would result in greater ability to do mathematics” (Kloosterman, 1988, p. 348). He developed a Likert-type scale with six items (e.g., “working can improve my mathematics ability”). He claimed that a high score on this scale indicated an incremental view of ability in that a high score indicated a feeling that sustained effort will improve the ability to perform well in mathematics. A low score on this scale indicated a feeling that mathematical ability is fixed and that effort may make no difference. The research explored the relationships between self-confidence and effort as a mediator of mathematics ability (Kloosterman, 1988). Results of regression analysis showed that effort as a mediator of mathematics ability is positively associated with students’ self-confidence. Particularly, students with an incremental view of ability are confident in themselves as learners of mathematics. Kloosterman and Stage (1992) included this construct in the *Indiana Mathematics Beliefs Scale*. This construct has six items with positive wording items (e.g., “By trying hard, one can become smarter in math”) (Kloosterman & Stage, 1992, p. 115).

The *Indiana Mathematics Beliefs Scale* has been used by a number of researchers in different ways to investigate students’ mathematics-related beliefs and the effects of these beliefs on mathematics performance. Some researchers examined the structure this scale (Ayebo & Mrutu, 2019; Berkaliev & Kloosterman, 2009), and confirmed the stability of this structure. Mason (2003) used this scale to examine the effect of Italian students’ beliefs about mathematics on their grades. She recruited 599 high school students in Italy to participate in this research. Twenty-four students who had lowest and highest scores on the questionnaire were also invited to participate in interviews. The students’ mathematics grades were used as measures of mathematics achievement. She found that correlations between scales were low. This suggested that there were only small overlaps between these constructs. Another result showed that the strongest predictor of mathematics achievement is beliefs regarding students’ perceived ability to solve time-consuming problems following by beliefs regarding the nature of word problems and beliefs regarding the importance of understanding mathematical concepts. Mason also found that beliefs about the value of effort to improve mathematical ability did not predict students’ mathematics performance.

In another study, Schommer-Aikins et al. (2005) examined the associations between beliefs about mathematical problem-solving and mathematics performance. The results of a path analysis indicated that these beliefs did not significantly predict mathematics performance (except beliefs about the usefulness of mathematics, as discussed in the previous section). In a similar study, Schommer-Aikins and Duell (2013) also used the *Indiana Mathematics Beliefs Scale* (Kloosterman & Stage, 1992) to measure students' beliefs about mathematical problem-solving. However, due to the very low correlations between these belief constructs with mathematics performance, not all of these beliefs could be used in the final model in this research. Sangcap (2010) studied the relationships between students' beliefs about mathematics and mathematical problem-solving in the Philippines using an adaptation of the *Indiana Mathematics Beliefs Scale* with a sample comprising 336 university students. The results showed that Filipino students believed that effort can increase one's mathematical ability, but they did not believe that word problems are important in mathematics and that they can solve all problems by using simple procedures. Abedalaziz and Akmar (2012) also used this scale to investigate students' beliefs about mathematical problem-solving with 592 Malaysian secondary school students. They reported that Malaysian students had moderate levels of beliefs about mathematical problem-solving.

Prendergast et al. (2018) examined beliefs about mathematical problem-solving of Irish students. They recruited 975 secondary school students across year levels to undertake the *Indiana Mathematics Beliefs Scale* (Kloosterman & Stage, 1992). In terms of beliefs regarding the nature of word problems, results showed that students in higher level of secondary education are more positive in these beliefs in comparison to other groups. Regarding beliefs about the importance of understanding mathematical concepts, results showed that the level of agreement with this construct was higher among the students in the lower level of secondary education.

Going back to Muis and Foy's (2010) work, they found that beliefs about the need for effort to learn mathematics have indirect effect on mathematics achievement through achievement goals (including both mastery goals and performance goals) and students' self-efficacy. Yuanita and Zulnaidi (2018) investigated the role of mathematical representation when they examined the relationships between beliefs about mathematical problem-solving and mathematical problem-solving performance of 426 secondary school students. The authors used the *Indiana Mathematics Beliefs Scale* to measure beliefs about mathematical problem-solving. The results from structural equation modelling showed that these beliefs both directly and indirectly (through mathematical representation) predicted mathematical problem-solving performance.

From the above review, it is worth noting that the *Indiana Mathematics Beliefs Scale* is viewed as an important scale to investigate students' beliefs about mathematical problem-solving. Five belief constructs have been found to directly or indirectly influence mathematical problem-solving and mathematics performance. However, it can be seen that within the *Indiana Mathematics Beliefs Scale*, apart from beliefs about word problems, the other four factors could be divided into two subcategories. The first category included "beliefs regarding the importance of understanding mathematical concepts" and **"**beliefs regarding the nature of word problems**".** In this category, if "word problem" was to be replaced by "mathematics problem", this construct would measure beliefs about mathematical problem-solving in general rather than word problems. By doing so, this construct would reflect the nature of beliefs about mathematical problem-solving, namely that it is about memorisation of rules and procedures, and about finding the correct answers. The second category included the factors "beliefs regarding the role of effort in mathematics ability" and "beliefs regarding whether students can solve time-consuming mathematics problems". If effort is viewed in terms of the amount of time students spend on learning mathematics, the factor of "beliefs regarding whether students can solve time-consuming mathematics problems" in the *Indiana Mathematics Beliefs Scale* may also be considered as a facet of beliefs about the importance of effort.

2.3.4 Beliefs about the self as a mathematics learner

A previous section in this chapter about different frameworks of mathematics-related beliefs showed that all of the examined frameworks included beliefs about the self as a mathematics learner. However, within this category of beliefs, there are different approaches as to what kinds of beliefs they belong to. For example, Op't Eynde et al (2002) proposed that beliefs about the self as a mathematics learner included four different constructs: (1) self-efficacy beliefs; (2) control beliefs; (3) task value beliefs; and (4) goal orientation beliefs. Pehkonen (1995) also proposed beliefs about the self and he recommended researchers could add their desired constructs to this category of beliefs. Roesken et al. (2011) also developed a framework including seven different constructs of students' views about themselves as mathematics learners. Using correlation analysis between these seven constructs, they suggested that three constructs (mathematics ability, difficulty of mathematics, and success) are highly correlated to each other and should be considered core to students' views of themselves as mathematics learners. For the purpose of the present research, this section addresses some research literature about two types of beliefs, namely, beliefs about mathematics ability and self-efficacy beliefs. The justification for choosing these two constructs as a part of beliefs about self as a mathematics learner will be explained in Chapter 3. This section will review the literature about these two constructs.

2.3.4.1 Beliefs about mathematics ability

Before discussing beliefs about mathematics ability, it is necessary to mention the theory of ability. Researchers proposed a distinction between two components of ability theory: the entity theory of ability and the incremental theory of ability (Dweck, 1986, 2000). The entity theory of ability argued that ability is innate, and it do not emphasise the role of effort. Dweck and her colleague (Dweck, 1986; Dweck & Leggett, 1988) suggested that an entity theory promotes students to think about performance because of their innate ability. Dweck and her colleague argued that an entity theory of ability is associated with the traditional beliefs where students pay attention to the correctness. In contrast, according to the incremental theory of ability, ability develops as a consequence of effort and learning. In the incremental theory of ability, students put their efforts to solve problems and they are likely to ignore their intelligence when they explain their difficulty or failure. Research by Dweck and Legget (1988) supported the idea that beliefs about ability also play an important role in the learning process. In general, they found that when students faced a difficult problem, those who believed the ability to learn is innate tend to have helpless behaviour, while those with a strong belief that the ability to learn can improve will persist and try different strategies (Dweck & Leggett, 1988).

As discussed in the section about epistemological beliefs in this chapter, inspired by Dweck's (Dweck, 1986; Dweck & Leggett, 1988) work, beliefs about innate ability is one of four important types of epistemological beliefs (Schommer, 1990). However, researchers recommended that this construct should not belong to the epistemological belief system (Hofer & Pintrich, 1997). Schommer (2004) re-conceptualised epistemological beliefs as a multidimensional construct consisting of two components (beliefs about knowledge and beliefs about learning), and she claimed beliefs about innate ability sit within the latter component. There are a few studies on this construct in the literature. Some researchers adapted the general-domain of this construct from the *Epistemological Beliefs Questionnaire* (Schommer, 1990) and there have been variations in terms of the results. For example, along with other results that were summarised in previous sections from Schommer's (1993) work, she found that beliefs about innate ability negatively predict students' GPA. However, in other research, Schommer concluded that beliefs about innate ability did not predict mathematics performance and mathematical problem-solving ability (Schommer-Aikins & Duell, 2013; Schommer-Aikins et al., 2005). When developing the *Epistemological Beliefs*

Questionnaire for mathematics, Wheeler (2007) also included a scale about innate ability in mathematics. Although Wheeler focused only on validating the construct, it is important to note that this scale measures students' beliefs about mathematics ability.

Many researchers have used the term 'intelligence' instead of 'innate ability', and they have studied beliefs about intelligence as well as the relationships between these beliefs and mathematics performance. Stipek and Gralinski (1996) examined the relationships between beliefs about intelligence and mathematics performance in a longitudinal study. They measured different kind of beliefs, including: (1) ability is stable and unaffected by effort; (2) performance is stable and only modestly affected by effort; (3) intelligence is a specific and global cause of academic performance; (4) effort is a cause of academic performance; and, (5) effort increases intelligence (Stipek & Gralinski, 1996). The results of factor analysis indicated two measures that apply to their research, namely, ability-performance beliefs and effort-related beliefs. Mathematics grades and the mathematics achievement test scores were used as mathematics performance measures. A sample of 319 students from third grade to sixth grade was recruited. The research included two phases. The first phase was conducted within the first four months of the school year, and the second about two months later. The authors found many interesting results. First, children who claim that they cannot do much about intelligence also believe that ability and performance do not change over time. Second, their beliefs about intelligence and performance do not change over their school year. Third, ability-performance beliefs that were measured earlier in the year were strongly and negatively associated with mathematics achievement. Reflecting on Lodewyk's (2007) research, in terms of beliefs about innate ability, Lodewyk found that beliefs about fixed and quick ability to learn predicted overall academic achievement.

Bonne and Johnston (2016) examined the relationships between beliefs about intelligence, mathematics self-efficacy, and mathematics performance through an intervention program. After piloting the instruments with 121 students, the main study was conducted with 91 students aged from seven to nine years old in New Zealand. They used six items from Dweck's (2000) work to measure beliefs about intelligence including incremental and entity beliefs about intelligence. Students were measured through three stages of the research with the intervention. The results showed significant relations between mathematics achievement and incremental or entity beliefs about intelligence. Additionally, at post-intervention, it was found that mathematics self-efficacy beliefs were positively and significantly correlated with incremental beliefs and negatively with entity beliefs. For the intervention group, the results revealed that these correlations were stronger, whereas for the comparison group, only post-intervention self-efficacy and entity beliefs were significantly correlated.

The construct of beliefs about innate ability (intelligence) has drawn attention from many researchers inspired by Dweck's theory of intelligence. These beliefs were also incorporated within the *Epistemological Beliefs Questionnaire* (Schommer, 1990) as one of the constructs of epistemological beliefs. Prior research showed that there are relationships between these beliefs and mathematics performance. However, similar to some other belief constructs, there have been inconsistencies in results that showed direct, indirect, or non-significant associations with mathematics performance. Additionally, some researchers argued that these beliefs should belong to the category of beliefs about learning instead of the epistemological belief system (Hofer & Pintrich, 1997). The researcher of the present research agreed with this approach when considered these beliefs as an important component of beliefs about the self as a learner. This issue will be discussed further in the next chapter.

2.3.4.2 Self-efficacy beliefs about mathematics

There are several definitions of self-efficacy beliefs but Bandura's (1977) definition has been generally accepted among researchers. These beliefs refer to the confidence of individuals about their ability to solve specific problems (Bandura, 1977, 2006). Self-efficacy has been found to have a significant relationship with different constructs, including academic performance (Ferla, Valcke, & Cai, 2009; Pajares & Graham, 1999). Regarding mathematics learning in schools, self-efficacy has been found to be one of the most important factors that influences students' mathematics performance (Pajares, 2006). Indeed, mathematics self-efficacy beliefs have been found to be one of the strongest predictors of success in solving specific mathematical problems over other important constructs such as mathematics anxiety or previous mathematics experience (Pajares & Miller, 1994). There is much evidence about the direct associations between self-efficacy and mathematics performance in the body of scholarship. Liu and Koirala (2009) investigated the relationship between mathematics self-efficacy and mathematics performance of high school students. A sample of 11,726 secondary school students participated in the research. Results of correlation and regression analysis showed that there were relationships between mathematics self-efficacy beliefs and mathematics performance. Specifically, the higher students' self-efficacy beliefs were, the better students performed on mathematics tests. Reflecting to Muis and Foy's (2010) work, they concluded that students' self-efficacy beliefs strongly and directly predict mathematics achievement. Manzano-Sanchez (2018) also reviewed the research literature on the relationships between self-efficacy and academic performance. They found strong and positive relationships between these two constructs at different levels of education.

Along with investigation about self-efficacy beliefs and how they directly influence mathematics performance, some researchers have also examined the relationships among mathematics self-efficacy, other beliefs constructs, and mathematics performance. For example, Kabiri and Kiamanesh (2004) investigated the relationships between mathematics self-efficacy, mathematics attitude, mathematics anxiety, prior mathematics achievement, and present mathematics achievement with a sample of 366 eighth grade students in Tehran. By using a path analysis model, the authors found that there were many direct and indirect associations between those constructs. Results showed that all four constructs of the research directly predicted mathematics achievement. Another result showed that mathematics self-efficacy and mathematics anxiety were mediators between mathematics attitudes as well as prior mathematics achievement and mathematics performance. Hoffman and Schraw (2009) conducted a study to explore the effect of mathematics self-efficacy, working memory capacity on problem-solving efficiency, mathematical problem-solving performance, and the time taken to solve mathematical problems. Along with other results, the authors found that self-efficacy was positively associated with mathematical problem-solving efficiency. Yusuf (2011) investigated the associations between self-efficacy, motivation, leaning strategies and mathematics achievement across a sample comprising 300 undergraduate students. The results of path analysis indicated that self-efficacy beliefs had both direct and indirect effect on mathematics performance. Motivation and learning strategies also directly influenced mathematics performance, as well as acted as mediators between mathematics self-efficacy and mathematics achievement (Yusuf, 2011).

Some researchers have also used modern techniques, such as structural equation modelling, to explore mathematics self-efficacy in relation to other beliefs constructs and mathematics performance. Among other constructs, Nasser and Birenbaum (2005) investigated the associations between epistemological beliefs and self-efficacy beliefs with mathematics achievement within Jewish and Arab groups of students in Israel. The results of structural equation modelling showed that, in both groups, self-efficacy beliefs were the mediator between epistemological beliefs and mathematics performance. Zarch and Kadivar (2006) constructed a structural model to examine the relationship between mathematics ability, mathematics self-efficacy, and mathematics performance. They recruited 848 eighth grade students in Iran to participate in the research. The instrument included the mathematics self-efficacy and mathematics ability scale. The mathematics ability scale had 14 items measuring two different components: conceptual ability and strategic ability. The authors also assessed mathematics performance by using a self-designed test with 15 open-ended items. The results of structural equation modelling indicated that the overall model fit the data reasonably well. Specifically, mathematics ability directly predicted mathematics self-efficacy and mathematics performance, and mathematics self-efficacy also directly predicted mathematics performance. As a result, mathematics ability also indirectly predicted mathematics performance through mathematics self-efficacy beliefs. Hailikari et al. (2008) studied 139 university students regarding the relationships between academic self-beliefs (including self-efficacy), prior knowledge, and students' GPA. Even using a small sample, the proposed model fitted well with the data. They found that prior knowledge and students' GPAs directly predicted mathematics achievement. The results also showed that prior knowledge was a mediator between academic self-efficacy and mathematics achievement. Ünlü and Ertekin (2017) investigated factors (including anxiety, attitude, and self-efficacy) influencing geometry achievement using structural equation modelling. The results showed that these factors strongly and positively predicted geometry achievement. In another study, Xu and Jang (2017) investigated the role of self-efficacy beliefs as a mediator between different extracurricular activities such as video gameplay, internet use ,TV viewing and mathematics performance. The results of structural equation modelling showed that all extracurricular activities included in the research indirectly predicted mathematics achievement through self-efficacy.

Mathematics self-efficacy beliefs are key constructs within the mathematics-related belief system. There have been many studies about these beliefs and how they influence students' learning and outcomes. Most prior research showed strong and positive relationships between these beliefs and mathematics performance. Additionally, mathematics self-efficacy has been found to be also a mediator between several belief constructs of different beliefs categories such as epistemological beliefs, beliefs about the nature of mathematics, and beliefs about the self.

2.3.5 Students' perceptions of teachers' practices

Students' beliefs about mathematics teaching are another important component of mathematics-related beliefs (McLeod, 1992; Op't Eynde et al., 2002; Pehkonen, 1995; Underhill, 1988), and researchers have also focused on these beliefs from students' perspectives. Kloosterman (1996) investigated the relationships between students' beliefs about their teachers' role and students' motivation. Kloosterman (1996) argued that the more students believe in transmissive teaching of their teachers, the less motivated they were to learn mathematics. When proposing the framework of students' mathematics-related belief system, Op't Eynde et al. (2002) stated that beliefs about the mathematics class context forms one of three important categories. Within this component, exists the construct, 'beliefs about the role and the functioning of their teacher'. This construct measures students' perceptions about the friendliness and encouragement of mathematics teachers in teaching and learning. Using this construct to measure students' views of teachers' practices, these authors claimed if students expressed positive beliefs about their teacher, they tended to believe that mathematics is useful, and they felt more confident about doing mathematics (Op't Eynde et al., 2006a).

Some other scholars also used this construct in their research. Tarmizi and Tarmizi (2010) adapted this construct to investigate relationships with other factors. In this research, they also adapted the scale of students' beliefs in mathematical competency from Op't Eynde et al. (2002). Furthermore, the authors also included gender, mathematics grades, and ethnicity in the analysis of data for the research. Results from standard multiple regression analysis showed that students' beliefs about the role and the functioning of their teacher were strongly and positively associated with their beliefs about being mathematically competent. The results also showed significant relationships between students' mathematics grades, gender, and students' beliefs about being mathematically competent. Wang et al. (2019) also used this construct to investigate the associations with Chinese students' perceptions praise from teachers praise and their own perceived mathematics achievement. The results of ANOVA analysis showed that there are relationships between these three constructs. Andrews et al. (2011) also used this construct to conduct a comparative study between three European countries. They found that although no difference in structure of students' beliefs about the role and the functioning of their teachers between those countries could be found, these beliefs still varied across countries. Andrews and Diego-Mantecón also adapted this construct for healthcare research and concluded that, among other belief constructs, the construct is reliable and valid (Andrews & Diego-Mantecón, 2015).

Research literature has also produced evidence about students' perceptions of their teachers' motivational behaviour and their relationships with other facets of learning process. Meiyue et al. (2008) investigated teachers' and students' beliefs about mathematics teaching in China. The results showed that most students believed that the nature of mathematics teaching is the process of their construction and mathematisation, and the purpose of mathematics teaching is to help students understand the concepts, acquire the skills, solve problems, and learn to apply creativity to mathematics. Another finding revealed that students think that the good way to teach mathematics is for a teacher to pay more attention to the textbook, express their own opinion, solve one problem in different ways, lead students to think and explore, and pay attention to students' learning processes. Ampadu (2012) examined students' perceptions of their teachers' teaching methods and how it impacted on their learning experiences with 258 junior high schools students. The results showed that students' perceptions of their teachers' teaching varied because their teachers used both teacher-centred and student-centred teaching approaches. The results also showed that teachers' action and inaction influenced positively or negatively students' learning experience. Murdock and Miller (2003) also studied the relationships between students' perceptions of the teacher and academic outcomes with 206 middle school students participating in the research. In this study, students' perceptions about teacher caring were measured. The results of a hierarchical regression analysis showed that students' perceptions about teacher caring influenced their self-efficacy and their values.

Apart from the research on different aspects of students' beliefs about mathematics teaching, a few researchers have also been interested in their influence on mathematics achievement. Along with reading and english as two domains of the research, You et al. (2016) investigated the effects of students' perceptions of teachers' motivational behaviours on mathematics achievement in a longitudinal research project. The authors recruited 6,227 middle school students in Korea to measure different aspects of students' associated constructs, namely, students' perceptions of teachers' motivational behaviours, mathematics self-efficacy, and intrinsic motivation. The research was implemented over three years, starting when the students were in seventh grade. Results of structural equation modelling showed that although students' perceptions of teachers' motivational behaviours did not directly predict mathematics achievement, they were mediated by students' mathematics self-efficacy and students' intrinsic motivation.

To date, however, little is known about students' perceptions of mathematics teachers' practices and how they influence mathematics achievement.

2.3.6 Differences in mathematics-related beliefs research

This section reviews research on the differences in mathematics-related beliefs between female and male students, as well as between low and high performers.

In terms of gender differences, although De Corte et al. (2002) argued that there are few studies on gender differences about mathematics-related beliefs, recently, along with research on mathematics-related beliefs and their influence on mathematics performance, a large number of studies have investigated gender differences. There has been evidence generated about gender differences in several categories of mathematics-related beliefs. Stage and Kloosterman (1995) argued that both female and male tended to hold non-availing beliefs about the nature of knowledge and the nature of knowing. In many studies, female students tend to hold more sophisticated beliefs in terms of the certainty of knowledge (Bendixen, Schraw, & Dunkle, 1998), and students' mathematics ability (Neber & Schommer-Aikins, 2002; Schommer, 1990). In regard to beliefs about the nature of mathematics, Fennema and Sherman's work (Fennema, 1989; Fennema & Sherman, 1977) investigated middle school students' attitudes about mathematics. Results showed that male students tended to believe mathematics is useful more than female students. Mason's (2003) study with elementary school students showed that male students were less likely to believe that understanding concepts is important than female students. Sangcap (2010) examined students' beliefs about mathematical problem-solving in the Philippines. This author also investigated gender differences about these beliefs and found that girls valued the usefulness of mathematics in their daily lives more so than boys and were also more likely to think that effort is a vital component in increasing mathematical ability. In terms of beliefs about the importance of mathematics, Wilkins and Ma (2003) argued that, while female students in middle schools tended to develop negative beliefs faster than male students, male students at secondary school developed negative beliefs faster than female students. Kiwanuka et al.'s (2017) research also showed that female students had higher scores on the usefulness of mathematics than male students. In terms of beliefs regarding the nature of word problems, Prendergast et al.'s (2018) study revealed that female students have more positive beliefs than male students.

In terms of differences in mathematics-related beliefs between low and high performance, there has been some evidence showing these differences. For example, Hannula and Laakso (2011) argued that in general, high performers are likely to be more positive than low performers in terms of expectations and affect (e.g., beliefs). Op't Eynde and De Corte (2003, cited in Andrews et al., 2011) found that high performers believed that mathematics is useful more than low performers. Grootenboer and Marshman (2016) also investigated the differences in mathematics-related beliefs, and they found that students who have higher mathematics ability tended to believe that mathematics is useful more than students who have lower mathematics ability. However, little is known about these differences from the research literature.

2.3.7 Summary

From the review of research literature in this section, there are many issues that need to be addressed. First, prior research revealed that research on mathematics-related affect has drawn considerable attention and researchers in the field of mathematics education have investigated different facets in relation to these issues. Among several domains of affect, multiple results from published works have shown that mathematics-related beliefs play an important role in the mathematics-related affect system as well as in the improvements of mathematics teaching and learning processes. Several kinds of mathematics-related beliefs have been investigated, such as epistemological beliefs about mathematics (e.g., whether mathematics knowledge is changing or fixed), beliefs about the nature of mathematics (e.g., whether mathematics is useful), and beliefs about the self as a mathematics learner (e.g., beliefs about ability to learn mathematics, self-efficacy beliefs). Additionally, various studies have also investigated the effects of students' mathematics-related beliefs on mathematics performance, and there has been much evidence from the research literature showing that several constructs of mathematics-related beliefs were significantly associated with mathematics performance.

There have been several frameworks to consider mathematics-related beliefs as a system where researchers have proposed different categories, such as beliefs about the nature of mathematics, beliefs about mathematics teaching, beliefs about oneself as a mathematics learner. However, it can be seen from the research literature that: (1) there have been inconsistencies between different proposed frameworks in terms of the number of beliefs categories, the names of each category, and types of belief constructs within each category; (2) although some researchers tried to validate their proposed framework (Op't Eynde & De Corte, 2003), prior research has not shown much empirical research to do so for other frameworks and there is no research to investigate the relationships between different belief constructs within each framework; (3) most research on mathematics-related beliefs has focused on particular belief constructs in relative isolation to other belief constructs. This may be a significant gap in the literature, as many researchers in the field have indicated that there are interactions and interrelationships among different belief constructs and mathematics performance. For example, many researchers claimed that beliefs about the nature of mathematics influence beliefs about mathematics teaching and learning. From the evidence presented in this chapter, there are many non-significant relationships between some belief constructs (e.g., beliefs about the certainty of mathematics, beliefs about effort, and beliefs about the usefulness of mathematics) and mathematics performance.

Second, over the past three decades, epistemological beliefs have been investigated by many researchers, not only generally, but also in domain-specific areas such as mathematics. Prior research has shown that epistemological beliefs should be considered as a system that includes several independent constructs. There has been evidence that these constructs directly influence mathematics performance and mathematical problem-solving. Additionally, researchers also suggested that epistemological beliefs influence beliefs about mathematics and some empirical research has shown that these beliefs are related to other mathematics-related belief constructs such as beliefs about the usefulness of mathematics and mathematics self-efficacy beliefs. Within epistemological beliefs, the construct of beliefs about the certainty of mathematics knowledge is important, and prior research has shown inconsistencies in results in terms of the associations with mathematics performance (i.e., there are some direct or indirect or non-significant links between these beliefs and mathematics performance).

Third, prior research has also shown that students' beliefs about mathematics as a discipline (i.e., the usefulness of mathematics and whether mathematics is interesting) and beliefs about the teaching and learning of mathematics (i.e., the role of teachers and peers, the importance of effort) have received considerable attention over the past thirty years. The research literature also revealed that these beliefs predict students' mathematics performance. From the review in this chapter, it is important to note that there are quite a few studies that show the significant and direct links between these beliefs (e.g., beliefs about the usefulness of mathematics, beliefs about mathematical problem-solving, and beliefs about innate ability or intelligence) and mathematics performance. In addition, prior research also found some non-significant relationships between these beliefs and mathematics performance. Students' self-efficacy beliefs have been found to be one of the most important types of mathematics-related belief systems that not only directly influence mathematics performance but also play an important role as a mediator between other constructs and mathematics performance.

Fourth, students' beliefs about mathematics teaching are an important component of students' mathematics-related beliefs and students' perceptions of teachers' practices were proposed to investigate one facet of these beliefs. Some prior research measured these beliefs and investigated the associations with students' mathematics learning as well as their performance. However, little is known from the research literature.

Fifth, in comparison with other cultural groups, the excellent academic performance in mathematics among Asian students from international assessment programs (e.g., PISA and TIMSS) has drawn great attention from researchers in the field of mathematics education globally. However, the reasons that underlie the achievement for this particular group have rarely been investigated. Among other perspectives, the research on students' and teachers' mathematics-related beliefs on these groups would contribute to greater understanding of these results. Although there is evidence that Asian students have some positive beliefs about mathematics, such as beliefs about the usefulness of mathematics and beliefs about effort, research on students' beliefs remains scarce in Asian countries where some aspects of teachers' and students' mathematics-related beliefs, such as epistemological beliefs about mathematics and the associations of these beliefs with mathematics performance have been rarely studied. Little is known about what kinds of beliefs students and mathematics teachers in Asian countries hold, and how these beliefs promote students' mathematics achievement. Last but not least, prior research also found gender differences in students' mathematics-related beliefs.

2.4 Teachers' mathematics-related beliefs

2.4.1 Teachers' mathematics-related beliefs and their influence on teachers' practices

Mathematics teachers play important roles in supporting students' mathematics learning. Besides knowledge, teacher's mathematics-related beliefs also play an important role in the improvement of mathematics learning processes. (Beswick, 2019). Specifically, Blömeke et al. (2008) recommended that teachers' beliefs should be considered as components of teachers' professional competence. Researchers in the field of mathematics education have paid much attention to these beliefs (Fives & Gill, 2015; Philipp, 2007).

Researchers suggested that teachers' mathematics-related beliefs are often classified into three categories: beliefs about the nature of mathematics, beliefs about mathematics teaching, and beliefs about mathematics learning (Cooney, 2003; Cross, 2009; Ernest, 1989a; Thompson, 1992). Because of the effect on teachers' views about mathematics learning and teaching, teachers' beliefs about the nature of mathematics play a central role in teachers' belief systems (Liljedahl, Rolka, & Rösken, 2007).

Ernest (1989a) proposed that beliefs about the nature of mathematics fall under three views and argued that teachers may hold the combinations of more than one of the three views: "First of all, there is the instrumentalist view that mathematics is an accumulation of facts, rules and skills to be used in the pursuance of some external end. Thus, mathematics is a set of unrelated but utilitarian rules and facts. Secondly, there is the Platonist view of mathematics as a static but unified body of certain knowledge. Mathematics is discovered, not created. Thirdly, there is the problem-solving view of mathematics as a dynamic, continually expanding field of human creation and invention, a cultural product. Mathematics is a process of enquiry and coming to know, not a finished product, for its results remain open to revision" (p. 250). Regarding teachers' beliefs about mathematics teaching, along with three different views about the nature of mathematics, Van Zoest et al. (1994) proposed three corresponding types of beliefs, namely, content-focused with an emphasis on performance (corresponding with instrumentalist view), content-focused with an emphasis on understanding (corresponding with Platonist view), and learner-focused (corresponding with problem-solving view). Beliefs about mathematics learning are clearly interrelated with beliefs about mathematics teaching.

From Ernest's (1989a) work, it can be seen that teachers' beliefs about the nature of mathematics have a hierarchical structure, and for each type of view, teachers' beliefs about the nature of mathematics will have effects on teacher's beliefs about mathematics teaching and learning. Specifically, Ernest (1989a) argued that at the first and lowest level, the instrumentalist view involves the belief that mathematics knowledge includes facts, rules, formulas, and procedures as separate entities. With this kind of view, teachers tend to deliver passive teaching approaches and students may be subject to a passive learning environment. At this level, teachers act like 'instructors' to transfer mathematics knowledge to students. They will strictly follow curricular materials. Furthermore, they believe that learning mathematics involves mastering skills with accuracy. At the next level, the Platonist view involves the belief that mathematics is an objective and consistent structure where mathematics knowledge is a connected system. At this level, teachers act like 'explainers' to help students understand mathematics concepts and the relations between different pieces of mathematics knowledge. Teachers believe they can modify materials as well as enrich the material with more mathematical problems and activities. As a result, teachers' beliefs about learning mathematics is about understanding conceptually, not just procedurally. At the last and highest level, the problem-solving view involves the belief that mathematics is "a dynamically organised structure located in a social and cultural context" (Ernest, 1989a, p. 251). Teachers will act like 'facilitators' to help to be independent learners of mathematics. They believe that they, with the support of schools and colleagues, can construct the mathematics curriculum. At this level, teachers believe that learning mathematics is about student exploring mathematics independently based on their own interests.

There is a large body of literature that investigates the relationships between teachers' beliefs and their practices (Ambrose et al., 2004; Beswick, 2012; Buehl & Beck, 2015; Cross, 2015; Mosvold & Fauskanger, 2014; Pajares, 1992). Anderson et al. (2005) reviewed different schematic models of relationships between teachers' beliefs and their practices. They also proposed a new model to explore these relationships as well as the relationships with other factors that may influence teachers' beliefs and their practices. Barkatsas & Malone (2005) also investigated teachers' beliefs about teaching and learning mathematics and their relations with their practices. They found that the contemporary view (i.e., constructivist orientation) of Greek mathematics teachers rated more highly than the traditional view (i.e., transmission orientation). They also found that school experiences and teaching experiences were the main influence on teachers' beliefs. Philipp (2007) reviewed the research literature on mathematics teachers' beliefs and identified four areas of teachers' mathematics-related beliefs: (1) beliefs about students' mathematical thinking, (2) beliefs about curriculum, (3) beliefs about technology and (4) beliefs about gender. Philipp (2007) emphasised teachers' beliefs related to students mathematical thinking because these beliefs have strong correlations to teachers' instruction and students' learning. Additionally, Francis et al. (2015) explored the structure of teachers' mathematics-related beliefs and how they influence teachers' instructional decisions by reviewing the literature. They found much evidence to support that teachers' beliefs about the nature of mathematics are more strongly related to their instructional practices than beliefs about teaching and learning.

Beswick (2005) conducted a study with secondary mathematics teachers to explore the connections between their beliefs and their classroom practices. Twenty-five teachers and their students participated in the research. The author used the *Constructivist Learning Environment Survey* (Taylor, Fraser, & Fisher, 1993) comprising four scales: autonomy (i.e., whether students can manage their learning and think by themselves); prior knowledge; negotiation; and student-centredness. She found that there were associations between constructivist classroom environments and the teachers who have a problem-solving view of mathematics. In other words, in the classes where teachers believe that mathematics is discovered, students actively construct mathematics knowledge, and they create their own strategies for solving mathematical problems. Since many teachers showed consistency between beliefs about pedagogies and constructivist ideas, the author suggested that teachers may not have a full understanding about constructivism. She also recommended the need for considering the context (e.g., classroom environment) when researchers investigate the relationship between mathematics teachers' beliefs and their practice.

Cross (2009) also explored the relationships between mathematics teachers' beliefs and their practices. The sample included five mathematics teachers from two high schools. Before participating in the semi-structured interviews about teachers' views on the nature of mathematics, mathematics pedagogy, and students' learning, the author conducted two formal observations of each teacher. The author found strong links among teachers' beliefs about the nature of mathematics, beliefs about mathematics pedagogy, and beliefs about students' learning. Similar to Liljedahl et al.'s (2007) conclusion, she also claimed that teachers' mathematics-related beliefs were organised in order, "where their beliefs about mathematics teaching and learning were derived from their mental models of mathematics" (Cross, 2009, p. 340). In other words, teachers' beliefs about the nature of mathematics influenced their views on mathematics teaching and learning. Saadati et al. (2019) studied the relationships among beliefs about traditional teaching, beliefs about reformed teaching, student-centred practices, teacher-centred practices, self-efficacy, and value. The results showed that traditional beliefs directly and significantly predict teacher-centred practices. Additionally, both traditional beliefs and reformed beliefs directly predicted teachers' self-efficacy. Furthermore, reformed beliefs only indirectly predicted student-centred practices through self-efficacy and value.

However, there is evidence that the relationships between teachers' beliefs and their practices are not consistent (Francis et al., 2015; Li & Yu, 2010). There have been two types of explanation for these inconsistencies; either teachers' practices are affected by other factors rather than just teachers' beliefs (Buehl & Beck, 2015) or teachers' beliefs depend on contexts (Schoenfeld, 2015). In recent research, Vosniadou et al. (2020) proposed that the inconsistencies observed between teachers' beliefs and practices may happen because surveys usually focus on finding out if teachers agree with the beliefs that are supposed to be positive predictors of their assumed practices rather than negative predictors and that it is possible that teachers may hold internally inconsistent beliefs about teaching. The authors investigated the relationships between beliefs that are consistent with self-regulated learning (e.g., beliefs in constructive learning, beliefs that the self-regulation of learning improves achievement) and beliefs that are inconsistent with self-regulated learning (e.g., beliefs in transmissive teaching, beliefs that learning cannot change or be taught) in a sample of pre-service teachers. The results of structural equation modelling showed that the participants did held inconsistent beliefs that were negatively related with each other (Vosniadou et al., 2020), a finding confirmed by Darmawan et al. (2020). The pre-service teachers had conflicting beliefs especially between constructive and transmissive teaching.

Researchers have also proposed different categories of mathematics-related epistemological beliefs and beliefs about the nature of mathematics. For example, Blömeke et al. (2008) classified teachers' epistemological beliefs about mathematics into four different categories: (1) a scheme-related perspective (which mathematics consists of a set of rules and formulas; (2) a formalist perspective (which emphasises on logical nature of mathematics); (3) a process-related perspective (which mathematics is about problem-solving); and, (4) an application perspective (which mathematics is strong related to real-life situations). Using this approach, Felbrich et al. (2012) explored teachers' epistemological beliefs about mathematics in 15 different countries.

Some other researchers have used domain-general epistemological beliefs from Schommer's (1990) work to investigate teachers' epistemological beliefs. Using the *Epistemological Beliefs Questionnaire* (Schommer, 1990), Arredondo and Rucinski (1996) measured 126 Chilean teachers' and principals' epistemological beliefs and discovered differences between groups in the sample. Three different groups were identified: teachers who were involved in the school reform effort; teachers who were not involved in the school reform effort; and principals. The authors also compared the differences between Chilean and American teachers and found that there was difference in terms of beliefs about certain knowledge. Significant differences in beliefs about certain knowledge were also found between teachers who were involved in the school reform effort and those who were not involved in the school reform effort. Trakulphadetkrai (2012) explored the relationships between teachers' epistemological beliefs about mathematics and their beliefs about classroom authority. The author found that teachers' beliefs about the source of mathematics knowledge and teachers' beliefs about the certainty of mathematics knowledge are associated with their beliefs about classroom authority. Chrysostomou and Philippou (2010) also used the *Epistemological Beliefs Questionnaire* (Schommer, 1990) to investigate the relationships between epistemological beliefs about mathematics and mathematics teachers' self-efficacy with a sample of 184 pre-service and in-service teachers. The results of regression analysis showed that teachers' epistemological beliefs predicted teachers' self-efficacy.

Researchers also found differences in teachers' beliefs between different cultures. Bryan et al. (2007) compared the findings of research on teachers' mathematics-related beliefs in Mainland China, Hong Kong, Australia, and the US. They found that although the are some similar ways in which mathematics teachers think about effective mathematics teaching and learning, some differences still characterize teachers' views between Eastern and Western cultures. For example, in terms of teachers' beliefs about nature of mathematics, while mathematics teachers in Eastern cultures tended to have Platonic view, where they considered that mathematics as an abstract body of knowledge, mathematics teachers in Western cultures tended to have functional view, where they considered mathematics as a useful tool to solve real-life problems. In terms of teachers' beliefs about mathematics teaching and learning, while mathematics teachers in Mainland China and Hong Kong believed that memorisation can come before and after understanding, mathematics teachers in Australia and the US believed that memorisation can only come after understanding. Andrews (2015) also investigated the differences in teachers' espoused beliefs between English and Hungarian mathematics teachers. Using the results from semi-structured interviews, he found that while English teachers tended to believe that mathematics is a tool for real-world situation application, Hungarian teachers tended to believe mathematics is about problem-solving and logical thinking.

2.4.2 Teachers' mathematics-related beliefs and their influence on students' mathematics performance

Although many studies on teachers' mathematics-related beliefs have focused on the characteristics of these beliefs and the relationships between these beliefs and teachers' practices, the number of studies on how teachers' mathematics-related beliefs may influence students' mathematics performance has dramatically increased during the last decade. Peterson et al. (1989) investigated teachers' constructivist beliefs about learning mathematics. They found that constructivist beliefs were related to student performance. Students who learned in the classes that their teachers had strong constructivist beliefs tended to get higher scores on problem-solving tasks than students who learned in classes that their teachers had less constructivist beliefs. Returning to Muis and Foy's (2010) study, two types of teacher beliefs were used, namely, beliefs about the need for effort in learning mathematics and beliefs about the integration of information and problem-solving in mathematics, as drawn from Bruel et al. (2002). For students, the beliefs studied included the need for effort to learn mathematics; the certainty and simplicity of knowledge in mathematics; their mastery and performance approach goal orientations in mathematics; self-efficacy; and mathematics achievement. Muis and Foy's (2010) used structural equation modeling to test the relationships between related constructs and the results showed that there are positive links between the two types of teachers' beliefs and students' mathematics performance.

Polly et al. (2013) studied the association between teachers' beliefs and students' mathematics learning. They measured teachers' beliefs about mathematics, mathematics teaching, and mathematical learning. The findings of hierarchical linear model indicated a significant association between teachers' beliefs and student learning outcomes. They concluded that there is a significant positive relationship among teachers' beliefs, practices, and students' problem-solving performance. Areepattamannil & Kaur (2013) used data from TIMSS 2011 in Singapore and Australia to explore the effect of teachers' beliefs on students' mathematics performance. The results of structural equation modelling showed that Singaporean and Australian teachers' perceptions of their students were positive linked to students' mathematics performance. Students whose mathematics teachers more positively perceived their students' competence got higher scores than their peers whose mathematics teachers had less positive perceptions. The results also showed that teachers' perceptions were also positively associated with students liking learning mathematics, students' value of learning mathematics, and students' engagement in their mathematics lessons.

Campbell et al. (2014) conducted research on the relationship between teachers' mathematical knowledge, teachers' perceptions, and student performance. Teachers' perceptions included teachers' beliefs and teachers' awareness of interactions between teachers' perceptions and knowledge. The results showed that there were statistically significant associations between middle-grades teachers' knowledge with teachers' beliefs and their students' mathematics performance. Within the *Professional Competence of Teachers, Cognitively Activating Instruction, and the Development of Students' Mathematical Literacy* (COACTIV) project, Voss et al. (2013) explored the structure of mathematics teachers' beliefs and examined the relationships between these beliefs, teachers' instructional practice, and students' mathematics achievement. 155 teachers and 3,483 students participated in the research. Structure of mathematics teachers' beliefs included two different categories: transmissive orientation and constructivist orientation. Results showed that mathematics teachers' beliefs were indirect predictors their students' learning outcomes through their instructional practice. This finding helps to confirm prior research that, while transmissive beliefs were found to be negative predictors on instructional quality and student performance, constructivist beliefs were found to be positive predictors on both outcomes.

Chang (2015) investigated the influence of mathematics teachers' self-efficacy on their students' mathematics self-efficacy and mathematical performance. The findings of regression and ANOVA analyses showed that teachers' self-efficacy predicted fifth-graders' mathematical performance. In another study, Rutherford et al. (2017) investigated teachers' value of professional development, teachers' self-efficacy, and students' mathematics achievement within the context of a mathematics game, namely, *Spatial Temporal Mathematics*. The sample included 395 mathematics teachers and 11,335 elementary school students Results of multilevel structural equation modelling showed that, at a student level, prior achievement directly and indirectly (through *Spatial Temporal Mathematics* progress) predicted mathematics performance. From teacher level, teachers' value for professional development directly and indirectly (through teachers' self-efficacy beliefs about mathematics) predicted students' mathematics performance.

Perera & John (2020) studied the relations of teachers' self-efficacy for teaching mathematics with their job satisfaction and student mathematics achievement. The results showed that teacher self-efficacy for teaching mathematics was also found to positively associate with the class-average of mathematics achievement, indicating that classes taught by teachers with stronger self-efficacy beliefs for teaching mathematics tended to have higher average mathematics achievement.

It can be seen from review in this chapter that there are a large number of studies about this issue, especially with regard to how teachers' mathematics-related beliefs influence their practices. Similar to results pertaining to students, many scholars suggested that teachers' epistemological beliefs influence their beliefs about the nature of mathematics, and beliefs about the nature of mathematics influence beliefs about mathematics teaching and learning. However, empirical research on the relationships between these categories of teachers' mathematics-related beliefs is especially scarce in the research literature. Additionally, although there have been a few studies investigating teachers' mathematics-related beliefs and how they influence students' mathematics performance, these studies have only focused on single beliefs (mainly teachers' self-efficacy about mathematics) rather than examining the relationships among different types of teachers' mathematics-related beliefs and their associations with students' mathematics performance.

As mentioned in previous chapter, research on Vietnamese students' and teachers' mathematics-related beliefs has not been conducted officially in Vietnam. However, the results of relevant studies also revealed some information in terms of factors that influence mathematics learning and teaching in Vietnam. The next section will review these studies.

2.5 Relevant research in Vietnam

In terms of the factors that influence mathematics learning and achievement, there have been a few studies on different aspects of mathematics-related affect regarding Vietnamese students. Palmer (1994) investigated Vietnamese students' attitudes towards mathematics in comparison to non-Vietnamese students. The sample included 30 female Vietnamese students and 111 female non-Vietnamese students studying Year 11 and Year 12 at secondary schools in Perth, Australia. Palmer used the *Fennema-Sherman Mathematics Attitude Scale* (Fennema & Sherman, 1976) to measure six different constructs: self-confidence, the usefulness of mathematics, mathematics as an activity for males, and three constructs measure their perceptions about teachers', mothers', and fathers' attitudes towards them as a mathematics learner. The author used two-tailed *t*-tests to explore the differences between the two groups. The results showed that female non-Vietnamese students' self-confidence in mathematics was significantly lower than that of the female Vietnamese students, and they also tended to believe that mathematics was less useful. However, the author found that female non-Vietnamese students perceived that mathematics was less a stereotypical male activity than did the female Vietnamese students. Regarding students' perceptions about their teachers', mothers', and fathers' attitudes towards them as a mathematics learner, the non-Vietnamese students tended to perceive that their mothers had a less positive attitude. The author also examined the correlations between six constructs within the Vietnamese student sample. The results showed that students' perceptions about teachers' attitudes towards them as a mathematics learner significantly correlated with students' self-confidence and the usefulness of mathematics.

In another study, Hoang (2007) investigated relationships between mathematics learning, instruction, and mathematics achievement. There were five measures used in this research: classroom instructional activities, student and family characteristics, learning resources, out-of-school activities, and mathematics achievement. 565 12-year-old students responded to a questionnaire about the first four constructs. The results of multiple regression analysis showed that students who more regularly try to solve new mathematics problems and discuss real-world problems tended to perform better in mathematics. In terms of classroom instructional activities, the author found that the more frequently teachers presented the strategies to solve mathematics problems during mathematics lessons, the better students performed in mathematics. Hoang (2007) also concluded that students' active learning strategies were significantly associated with mathematics performance.

In 2012, Vietnam participated for the first time in the Programme for International Student Assessment (PISA), organised by OECD. In this assessment (where mathematics was the main domain), Vietnam surprisingly ranked 17th out of 65 countries in mathematics (OECD, 2014). Vietnam also maintained high ranks in the 2015 and 2018 PISA cycles (Gurría, 2016; OECD, 2019).

Researchers around the world have attempted to explain these results. Within the scope of the present research, only relevant results on PISA mathematics performance and research on how relevant factors influenced these results has been reviewed. Regarding the differences in mathematics performance, Darmawan (2016) investigated the equity and quality in academic performance using PISA 2012 data of five Southeast Asian countries (Indonesia, Malaysia, Singapore, Thailand, and Vietnam). He performed multilevel analysis using HLM. The author found that gender differences in mathematics performance are significant within Vietnamese students. Specifically, male Vietnamese students tended to perform better in mathematics than female students. The author also found that economic, social, and cultural status (ESCS) had positive effect on students' mathematics performance.

There is some evidence regarding factors influencing Vietnamese students' mathematical performance in PISA, including evidence from PISA 2012. PISA categorised mathematics problems into four types, namely, formal mathematics, word problems, applied problems in mathematics, and real-world problems. This survey revealed that Vietnam had the lowest index of exposure to word problems and was one of 18 countries showing "no relationship between the frequency of student encounters with applied mathematics problems and the performance of 15-year-olds on PISA" (OECD, 2014, p. 174). In another study, Parandekar and Sedmik (2016) Vietnamese PISA data to explore the profiles of Vietnamese students and teachers. The authors compared Vietnam's results with a group of seven developing countries (Albania, Colombia, Indonesia, Jordan, Peru, Thailand, and Tunisia). The comparison showed that Vietnamese students were more likely to focus on effort. They spent more time studying, especially out of school in extra classes. They tended to have lower levels of mathematics anxiety and higher levels of beliefs about the usefulness of mathematics. An interesting finding in this research was that, in comparison with other developing countries, Vietnamese teachers were more likely to have their performance monitored with higher focus on students' academic achievement with results made public.

More related to mathematics-related belief factors, Sezgin (2017) examined the relationships between ten different factors and mathematics performance using PISA 2012 data. Multiple linear regression analysis was applied for this research. Some important factors studied were mathematics self-efficacy, teacher-student relations, mathematics teachers' classroom management, and mathematics interest. The author found that, within the Vietnamese sample, mathematics self-efficacy beliefs have the most statistically significant influence on mathematics performance and this is a positive relationship. In other words, the more Vietnamese students believe in their ability to overcome difficulties to solve mathematics problems, the better they perform in mathematics. Apart from confirming the positive effect of ESCS on mathematics performance from Darmawan's (2016) work, Sezgin also found significant relationships between mathematics self-concept, mathematics teachers' classroom management, and Vietnamese students' mathematics performance. However, while mathematics self-concept positively predicted mathematics performance, mathematics teacher's classroom management negatively predicted mathematics performance (Sezgin, 2017).

Reviewing literature, only one study was found regarding Vietnamese mathematics teachers' beliefs. Ly and Brew (2010) conducted a comparative study about teachers' beliefs between Vietnamese and Australian pre-service teachers. The sample included 43 Vietnamese students studying at an Education university in Vietnam and 28 Australian students undertaking a Diploma of Education. A mixed method approach was applied for the research. The quantitative instrument used was a questionnaire measuring five different categories of beliefs: (1) mathematics is a collection of rules and procedures; (2) mathematics is a creative endeavour; (3) mathematics is best taught by direct instruction; (4) mathematical problem-solving allows for multiple approaches; and, (5) teachers' self-efficacy (Ly & Brew, 2010). The results showed that Australian pre-service teachers were likely to have more support for constructivist views than their Vietnamese peers, and teachers in both countries had high levels of self-efficacy.

2.6 Summary

It can be seen from the literature that beliefs are one of the important components of mathematics-related affect system. Various categories of students' and teachers' mathematics-related beliefs have been investigated such as epistemological beliefs (e.g., mathematics knowledge is certain or malleable), beliefs about the nature of mathematics (e.g., mathematics is useful), beliefs about mathematics teaching and learning (e.g., mathematical problem-solving is about memorising or understanding procedures, effort is important, transmissive versus constructive teaching), beliefs about the self as a mathematics learner (e.g., mathematics ability can be changed, self-efficacy beliefs). Prior research showed that there are relationships between mathematics-related beliefs as well as their associations with students' mathematics performance. However, the results of the literature reviewed in this chapter also showed some limitations that require greater attention.

While many frameworks have been proposed to consider mathematics-related beliefs as a system, they have not been consistent in categorising different types of beliefs. Furthermore, little is known about the relationships among all beliefs within each framework and their influence on mathematics performance. In other words, prior research only focused on select beliefs in relative isolation to other belief constructs in different proposed frameworks. As a result, some potential relationships between mathematics-related beliefs and mathematics performance were overlooked. Regarding teachers' mathematics-related beliefs, although there was empirical evidence of the relationships between some types of beliefs (e.g., self-efficacy, problem-solving) and students' mathematics performance, little is known about the relationships between many other kinds of beliefs and students' performance.

Another important area where further research is needed has to do with cross-cultural differences in mathematics related beliefs between Asian and Western students and teachers. Most research on the influence of mathematics-related beliefs on mathematics performance has been conducted in Western countries, little research has studied the mathematics-related beliefs of Asian students, and no such research has been carried out in Vietnam. Based on the results of the findings presented in this chapter, the next chapter will address the purposes, the research questions, the context of mathematics education in Vietnam, and the hypotheses of the present research.

CHAPTER 3 THE PRESENT RESEARCH

This chapter describes the purposes and hypotheses of the present research. In the first section, the purposes and research questions of the study are presented. In the second section, the mathematics-related beliefs of students and teachers included in the present research are discussed. In the third section, the context of mathematics education in Vietnam is presented. IN the final section, the hypotheses regarding the relationships among students' mathematics-related beliefs, teachers' mathematics-related beliefs, and students' mathematics performance are presented.

3.1 The Purposes and Research Questions

The main purposes of present research are as follows: (1) to examine, in greater details than previous research, a wide range of possible mathematics-related beliefs of students and understand how these beliefs influence students' mathematics performance; (2) to explore the differences in students' mathematics-related beliefs with regard to gender, and high and low mathematics performers; (3) to investigate teachers' mathematics-related beliefs and understand how these beliefs influence their students' mathematics performance; and (4) to conduct the investigations above with a sample of Vietnamese students and teachers. Three research questions have been identified:

Research Question 1: What are the emergent profiles of Vietnamese students' and teachers' mathematics-related beliefs? And how do these profiles of students differ among female and male students, and among low and high performers of problem-solving?

Research Question 2: How do students' mathematics-related beliefs influence their mathematics performance?

Research Question 3: How do teachers' mathematics-related beliefs influence their students' mathematics performance?

3.2 Students' and Teachers' mathematics-related beliefs investigated in the present research

As indicated in Chapter 2, in the field of mathematics education, students' and teachers' mathematics-related beliefs have received particular attention (Depaepe et al., 2016; Francis et al., 2015; Leder, 2019). Prior research has shown that students tend to believe that mathematics is useful in daily life (Mason, 2003), that mathematics knowledge is certain and unchangeable (Schommer-Aikins, 2008), and that mathematical problem-solving is about memorising procedures and formulas (Schoenfeld, 1992). Some empirical evidence also showed that mathematics teachers tend to believe in constructive teaching (Barkatsas & Malone, 2005). More importantly, there is evidence that many types of students' and teachers' beliefs influence mathematics performance, such as beliefs about the certainty of mathematics knowledge (Cano, 2005; Muis & Foy, 2010), beliefs about the usefulness of mathematics (Kadijevich, 2008; Schommer-Aikins et al., 2005), beliefs about mathematical problem-solving (Mason, 2003; Yuanita & Zulnaidi, 2018), beliefs about effort (Muis & Foy, 2010), beliefs about mathematics ability (Bonne & Johnston, 2016; Lodewyk, 2007), and beliefs about one's self-efficacy in mathematics (Liu & Koirala, 2009; Manzano-Sanchez et al., 2018). Prior research has also shown that there are differences in students' and teachers' mathematics-related beliefs amongst different cultures (Andrews, 2015; Andrews et al., 2011; Bryan et al., 2007). However, researchers have paid little attention to teachers' and students' mathematics-related beliefs in Asian countries, and some aspects of mathematics-related beliefs have, so far, hardly been studied. Specifically, little is known about the effects of teachers' and students' mathematics-related beliefs on students' mathematics performance.

There is evidence from the empirical research literature about the existence of relationships, not only amongst specific beliefs and mathematics performance, but also amongst different belief constructs themselves (Schommer-Aikins et al., 2005; Zarch & Kadivar, 2006). However, there has been no research found in the literature that investigated mathematics-related beliefs in a more systematic way, where researchers explored the relationship amongst various beliefs that represents all categories of existing frameworks including epistemological beliefs, beliefs about nature of mathematics, beliefs about mathematics teaching and learning, and beliefs about the self. The present research focuses on seven belief constructs, based on existing research, to explore the profiles of Vietnamese students and teachers and to understand how they relate to each other and influence students' mathematics performance. Furthermore, the present research investigates whether both students and teachers may hold internally inconsistent mathematics-related beliefs, such as, beliefs that mathematics problem solving is mostly about memorising rules and procedures or about understanding.

The following sections describe the specific beliefs that were chosen for the present research and the hypotheses regarding their relationships and influence on students' mathematics performance.

3.2.1 Students' mathematics-related beliefs investigated in the present research

3.2.1.1 Beliefs about the certainty of mathematics knowledge

Beliefs about the certainty of mathematics knowledge are one kind of epistemological beliefs that have been found to be related to mathematics performance (Schommer, 1990; Wheeler, 2007). Students tend to believe mathematics knowledge is more certain than knowledge in other disciplines, such as history (Buehl & Alexander, 2005), humanities (Baxter Magolda, 2002), and social sciences (Schommer & Walker, 1995). Researchers also found associations among beliefs about the certainty of mathematics knowledge and mathematics learning (Hofer, 2000; Hofer & Pintrich, 1997; Schommer, 1990; Schraw et al., 1995). Research has shown that beliefs that mathematics knowledge is certain and unchanging negatively predicted mathematics performance (Köller, 2001), and that students who believe more in the certainty of mathematics knowledge have lower levels of motivation and task performance (Buehl & Alexander, 2005). Some research has shown significant links between students' beliefs about the certainty of mathematics knowledge and mathematics ability and achievement values with university undergraduates (Buehl, 2003), but not with secondary school students (Lodewyk, 2007; Schommer-Aikins et al., 2005). Beliefs in the certainty of mathematics knowledge have not been found to always predict mathematics performance directly (Lodewyk, 2007; Schommer-Aikins et al., 2005) but have been found to predict mathematics performance through mediators (Cano, 2005; Muis & Foy, 2010). Little is however known about the relationship between beliefs in the certainty of mathematics and other beliefs about the nature of mathematics or mathematics learning.

3.2.1.2 Beliefs about the usefulness of mathematics

Beliefs about the usefulness of mathematics, namely, whether students believe that mathematics is useful in their life, have been found to be an important factor that directly and positively influenced mathematics performance (Espinosa, 2018; Kadijevich, 2008; Schommer-Aikins et al., 2005), although this is not always the case (Arikan et al., 2016; Pajares & Miller, 1995). Prior research has also shown an indirect link between these beliefs and mathematics performance (Köller, 2001). The evidence suggests that beliefs about the usefulness of mathematics may have relationships with other beliefs, and they may indirectly affect mathematics performance through other beliefs as mediators. For these reasons, these beliefs were selected to be included in the research.

3.2.1.3 Beliefs about mathematical problem-solving

There have been many studies investigating beliefs about mathematical problem-solving and their influences on mathematical problem-solving and mathematics performance (Callejo & Vila, 2009; Kloosterman & Stage, 1992; Prendergast et al., 2018; Schoenfeld, 1989; Schommer-Aikins et al., 2005). Many students believe that solving a mathematics problem is a matter of memorisation of rules/procedures, while some other students believed mathematical problem-solving has something to do with understanding and creation. Mason (2003) found that the beliefs that not all mathematics problems can be solved using step-by-step procedures predicted mathematics performance. In view of the current education context in Vietnam, and the move from a content-based curriculum in mathematics to a competence-based curriculum, there has been an emphasis in current policy documents on the importance of mathematical problem-solving. It is, therefore, important to study Vietnamese students' and teachers' beliefs about mathematical problem-solving and to understand how these beliefs may influence students' mathematics performance.

3.2.1.4 Beliefs about the importance of effort

Beliefs about the importance of effort have been found to be related to beliefs about mathematics ability and performance (Kloosterman, 1988; Kloosterman & Stage, 1992; Mason, 2003; Sangcap, 2010). Some studies have shown that beliefs about the importance of effort influence mathematics performance (Muis & Foy, 2010). Prior research has also indicated differences in beliefs about the importance of effort between different cultural groups (e.g., students in Asian countries versus Western countries). For example, Uttal (1997) found that Asian students believe more in the role of effort for student achievement compared to American students who believe that innate ability matters most. There is some evidence that Vietnamese students put more effort into mathematics learning than their peers in other developing countries (Parandekar & Sedmik, 2016). Because of above, it is important to include beliefs about effort in the research.

3.2.1.5 Beliefs about mathematics ability

Beliefs about mathematics ability refer to students' beliefs about whether mathematics ability is innate or whether it can be changed or developed through education. Although some researchers have considered these beliefs as epistemological (Schommer, 1990; Wheeler, 2007), the present research considers them as beliefs about self as a mathematics learner, where students either consider that their mathematics ability is fixed and immutable, or can be changed during the learning process. This decision was supported by Hofer and Pintrich's (1997) recommendation that these beliefs should be separated from epistemological beliefs, and by Bonne and Johnston (2016) and Roesken et al. (2011) who also considered them as beliefs about self as a mathematics learner.

Prior research has shown the relationships between beliefs about mathematics ability and mathematics performance. For example, there is evidence that beliefs that mathematics ability is fixed and immutable negatively predicted GPA (Schommer, 1993). In the case of mathematics, Bonne & Johnston (2016) also found negative correlations between entity view of ability (i.e., ability is fixed) and mathematics achievement. However, other research revealed that beliefs about innate ability did not predict mathematics performance and mathematical problem-solving (Schommer-Aikins & Duell, 2013; Schommer-Aikins et al., 2005). These inconsistent results may lead to an assumption that there are other ways beliefs about mathematics ability may influence mathematics performance. In the present research, beliefs that mathematics ability can be changed via education were chosen, and it was hypothesised that these beliefs positively and indirectly predict mathematics performance.

3.2.1.6 Self-efficacy beliefs

Prior research has shown that students' mathematics self-efficacy beliefs are one of the most critical predictors of mathematics performance (Liu & Koirala, 2009; Manzano-Sanchez et al., 2018; Muis & Foy, 2010). Moreover, self-efficacy beliefs have been found to be an important mediator between several other beliefs and mathematics performance (Kabiri & Kiamanesh, 2004; Zarch & Kadivar, 2006). For these reasons, it was decided to include an investigation of these beliefs in the present research.

3.2.1.7 Students' perceptions of teachers' practices

Students' beliefs about mathematics teaching are an important belief category in students' mathematics-related belief system. There has been some research on different aspects of students' beliefs about teaching, such as students' perceptions of their teachers' teaching styles

(Kurniatia & Suryab, 2017), and their teachers' classroom effectiveness (Ibrahim, 2014). As indicated in Chapter 2, when proposing a framework to study students' mathematics-related belief system, Op't Eynde et al. (2002) argued that beliefs about the mathematics class context were one of three important categories. Within this component, there were beliefs about the role and the functioning of the teacher designed to measure students' perception of the friendliness and encouragement offered by mathematics teachers. Using these beliefs to measure students' views of teachers' practices, it was found that students who held positive beliefs about their teacher considered mathematics more valuable and felt more confident about it (Op't Eynde et al., 2006a). However, little is known from the literature about the relationships between students' perceptions of teachers' practices and their own mathematics performance. With the intention to contribute further evidence to the research literature in this area of study, this construct was adopted from the *Mathematics-related Beliefs Questionnaires* (Op't Eynde et al., 2006a) to investigate the associations with Vietnamese students' mathematics performance.

3.2.2 Teachers' mathematics-related beliefs selected to be investigated

Some of the mathematics-related beliefs selected to be investigated with students are also relevant to teachers. It is hypothesised that teachers' beliefs about the certainty of mathematics knowledge, beliefs about the usefulness of mathematics, and beliefs about mathematical problem-solving may be interrelated with other beliefs, and may have indirect or direct effects on students' mathematics performance. Some additional mathematics-related beliefs were also included in the present research, such as their beliefs about mathematics teaching, self-efficacy beliefs, beliefs about students' effort, and beliefs about students' mathematics ability. These beliefs are discussed below.

3.2.2.1 Teachers' beliefs about mathematics teaching

A distinction is often made between constructive teaching and transmissive teaching. While a constructive teaching approach focuses more on developing students' strategies to explore knowledge by themselves, a transmissive teaching approach focuses more on providing knowledge to students. Different approaches to teaching are likely to have different effects on students' mathematics performance. For example, Peterson et al. (1989) found that students who learn in classes that their teachers express strong constructivist beliefs get higher scores on problem-solving tasks than students who learn in classes that their teachers express weak constructivist beliefs. In the case of Vietnamese mathematics teachers, a more transmissive teaching approach is common, although new approaches to teaching are being introduced. In the current context it would be of interest to investigate how teachers' beliefs about mathematics teaching may influence students' mathematics performance.

3.2.2.2 Teachers' self-efficacy beliefs

There have been many studies about mathematics teachers' self-efficacy (Chang, 2015; Woolfolk-Hoy, Woolfolk-Hoy, & Davis, 2009). Some prior research showed that mathematics teachers' self-efficacy beliefs are positive predictors of students' mathematics performance (Chang, 2015; Perera & John, 2020). The present research chose these beliefs to examine the direct association with students' mathematics performance as well as to explore these beliefs as potential mediators between other teachers' beliefs and students' mathematics performance.

3.2.2.3 Beliefs about students' effort and beliefs about students' mathematics ability

There has been evidence that teachers' beliefs about students' learning influence students' mathematics performance (Areepattamannil & Kaur, 2013; Muis & Foy, 2010). In order to explore what Vietnamese teachers believe about students as mathematics learners, two constructs used to study students' mathematics-related beliefs about themselves as mathematics learners were adapted. These were beliefs about students' effort, namely, what teachers believe with respect to the role of effort in students' mathematics performance, and beliefs about students' mathematics ability, namely, whether teachers believe that students' mathematics performance is related more to their innate ability or to their effort.

It is important to emphasise that prior research has investigated mathematics-related beliefs by considering each type of beliefs as an entire construct without internal inconsistency, and explored the positive or negative relationships of that type of beliefs and mathematics performance. However, there is still a case where students (or teachers) hold inconsistent beliefs. For example, teachers may agree with statements about transmissive teaching and they also agree with statements about constructive teaching. As mentioned in the previous chapter, Vosniadou and her colleagues recently investigated the internal inconsistency of pre-service teachers' beliefs (Darmawan et al., 2020; Vosniadou et al., 2020). The present research also emphasised to explore the internal inconsistency of Vietnamese teachers' mathematics-related beliefs.

3.3 Mathematics Performance

With the intention to examine the relations between Vietnamese teachers' and students' mathematics-related beliefs, not only in terms of curriculum-based assessment but also novel mathematical problem-solving performance, an assessment was designed for this study comprising two parts. The first part included items that students encounter in mathematics curriculum at schools. The second part included mathematical problems similar to those used in the PISA tests (OECD, 2006; Stacey & Turner, 2015) which were novel to the students and required transfer of knowledge acquired in the school setting to new situations. Mathematics scores from district mathematics exam were also included in the research as one type of mathematics performance.

3.4 The mathematics education context in Vietnam

In Vietnam, across different sectors, education is considered a top priority in the national policy agenda. This sector has seen significant development since the *Đổi Mới* economic reform in 1986. In 1993, the Government restructured the education system to include many levels of education. This structure of the national education system is shown in Figure 3.1. Within the context of the present research, the researcher focused only on general education. Students attend in primary school from the age of six to eleven years old. The ages for lower secondary school are from 11 and 15 years old, from sixth grade to ninth grade. Tenth to twelfth grade occurs in upper secondary schools.

In the national general curriculum, mathematics is one of the core subjects. As mentioned in the Introduction, when the present research was implemented, the mathematics curriculum in Vietnam had been developed using a content-based approach. As a result, teachers had focused on teaching mathematics concepts and theories (including rules and procedures) using mathematics textbooks rather than focusing on mathematical problem-solving in real-world situations (T. T. Nguyen et al., 2019). Nguyen and his colleagues (2019) reviewed the practical situations of mathematics teaching and learning in Vietnam and argued that, because of the lack of working with real-world problems, students usually find it difficult to practically apply mathematics knowledge. They also concluded that Vietnamese mathematics teachers mainly provide students with content knowledge of mathematics and students usually learn mathematics through examples. Most teachers think that the purpose of teaching mathematics is to help students get high scores on exams. These mathematics teachers usually help their students practise forms of problems which usually appear in the exams (T. T. Nguyen et al., 2020).

Removed due to copyright restriction

Figure 3.1: Structure of the Vietnam national education system (C. L. V. Le, 2009, p. 221)

Over the past ten years, Vietnamese researchers in the field of mathematics education have paid more attention to different aspects of realistic mathematics education and mathematical problem-solving. In regards to the development of mathematical problem-solving, C. Nguyen (2012) reviewed the research literature and proposed five different aspects related to problem-solving in mathematics: (1) problem-solving processes; (2) problem posing; (3) looking back in problem-solving; (4) evaluation of problem-solving; and, (5) problem solvers. He recommended that mathematics teachers should pay more attention to developing students' mathematical and problem-solving skills. In 2015, Le proposed some techniques for teachers to teach mathematics in order to develop competence for students by giving specific examples (N. S. Le, 2015), although all the problems in that paper focused on pure mathematics rather than applied mathematics. In other words, there were no real-world problems where students need to apply mathematics knowledge to solve practical problems. In another study, P. Nguyen (2014) reviewed the process of assessment standards development and proposed a framework and assessment standards of problem-solving skills in Vietnam. She also presented an assessment framework of learners' competence and defined a process for assessing problem-solving skills (T. L. P. Nguyen, 2015). She and her colleague also proposed methods to develop mathematical problem-solving instruments (T. L. P. Nguyen & Dang, 2015). These research results were published in national journals.

Recently, some scholars in the field of mathematics education have also been working on realistic mathematics education and proposed some strategies in terms of implementing realistic mathematics education in Vietnam. Pham and Pham (2018) applied Freudenthal's theory of realistic mathematics education (Streefland, 1991) to assess the mathematics teaching situation in primary schools in Vietnam. They proposed some suggestions to improve the use of authentic problems in teaching mathematics. T. T. Nguyen et al. (2019) studied the theories of realistic mathematics education and didactical situations in mathematics to propose real teaching situations that teachers can apply when teaching students mathematics. Based on their analysis, the authors concluded that students were not ready to participate in practical learning activities and solving authentic problems in mathematics, and when students face an authentic problem that they are not familiar with, they find it difficult to recall relevant knowledge to solve that problem. They also found that students' reading comprehension ability is limited and they tended to ignore the contexts of the authentic problems. In another study, T. T. Nguyen et al. (2020) reviewed the policies and practices of implementing realistic mathematics education in Vietnam. They developed an analytical framework consisting of four components: (1) national vision and strategy; (2) curriculum and educational materials; (3) teacher training program; and, (4) tests and examination. They found that, although realistic mathematics education had been mentioned in some national policies, no detailed official plan had been implemented. Recently, many kinds of materials such as books and websites have focused on realistic mathematics education. They also stated that formal mathematics assessment moving towards a realistic mathematics education approach will be developed. However, Vietnam still needs different kinds of resources to implement realistic mathematics education for all four components, especially in training mathematics teachers as well as to incorporate realistic mathematics education into new mathematics curriculum.

As mentioned in Chapter 1, in 2018 Vietnamese Government issued the new general curriculum which was developed using competence-based approach (Minister of Education and Training, 2018b). Within mathematics curriculum, the new objectives for learning and teaching mathematics emphasised forming and developing key mathematical competencies such as mathematical problem-solving (Minister of Education and Training, 2018a).

In summary, within the Vietnamese education context, a content-based curriculum is currently used to teach mathematics. Consequently, students are being taught mathematics by being told the correct solutions to mathematical problems rather than by being provided with situations that will help them develop the problem-solving skills required to find the solutions themselves (T. T. Nguyen et al., 2020; T. T. Nguyen et al., 2019). As a result, many Vietnamese students find it difficult to deal with unfamiliar problems (e.g., real-world problems) and they tend to give up or apply an inappropriate strategy to solve these problems (T. T. Nguyen et al., 2020). However, as in many other countries, Vietnam is going through a change from a content-based approach to a competence-based approach in mathematics education. Consequently, mathematics curricula and instruction have been undergoing a process of changing from an emphasis on the provision of content knowledge to a focus on developing problem-solving skills and their application in everyday life situations. As a result, researchers and stakeholders are interested not only in whether students can reproduce the well-defined mathematical procedures they learn at school, but also on whether they can apply their mathematical knowledge in less well-defined contexts that are more similar to real-life situations. During this process, the role of students' and teachers' mathematics-related beliefs in mathematics teaching and learning and their influence on students' mathematics performance should be paid more attention. Understanding the Vietnamese students' and teachers' mathematics-related beliefs and the way they influence mathematics performance can contribute to the improvement of mathematics instruction. This transition requires fundamental changes, not only in the way mathematics is taught but also in teachers' and students' beliefs about mathematics teaching and learning. If teachers are not convinced about the need for change, they are not going to change their teaching. It is important to study teachers' and students' beliefs to understand such possible roadblocks to change.

The present research is the first study to investigate students' and teachers' mathematics-related beliefs in Vietnam. This research is important, especially in the Vietnamese education context, because the Vietnamese government has implemented curriculum reform, and prior research has shown the important relationships between mathematics-related beliefs and curriculum reform (Handal & Herrington, 2003). It is also worth noting that ninth grade students were chosen to participate in the present research. At this level of education in Vietnam, students have completed their lower secondary education and are soon to move on to the upper secondary level. Ninth grade also represents the end of compulsory education in Vietnam. The investigation on mathematics-related beliefs of ninth grade students may contribute to greater understanding of the high PISA assessment results of the Vietnamese students, as the PISA assessment was conducted in Vietnam with ninth grade students. Additionally, the results of the present research may provide useful information in terms of students' and teachers' beliefs and their influence on mathematics performance that may help stakeholders in Vietnam gain more understanding about mathematics teaching and learning in Vietnam.

3.5 Hypotheses of the Present Research

Prior research has indicted that mathematics-related beliefs are interrelated and influenced by each other. For example, it has been claimed that epistemological beliefs about mathematics may influence beliefs about the nature of mathematics which, in turn, may influence beliefs about mathematics learning (in the case of students) and beliefs about mathematics teaching (in the case of teachers) (Depaepe et al., 2016; McLeod, 1989a; Schommer-Aikins, 2004; Schommer-Aikins et al., 2010; Schommer-Aikins & Duell, 2013; Thompson, 1992). A general model of the hypothesised relations between students' and teachers' mathematics-related beliefs and student performance is presented above in Figure 3.2.

Regarding students' mathematics-related beliefs, seven belief constructs were divided into two different categories. The first category (beliefs about mathematics) consisted of five belief constructs (beliefs about the certainty of mathematics knowledge, beliefs about the usefulness of mathematics, beliefs about mathematical problem-solving, beliefs about the importance of effort). The second category (beliefs about self as a mathematics learner) consisted of two belief constructs (beliefs about mathematics ability, self-efficacy beliefs). It was hypothesised that students' beliefs about mathematics might influence students' beliefs about self as a mathematics learner. It was also hypothesised that students' beliefs about the self as a mathematics learner may influence students' mathematics performance. Additionally, prior research has also shown the differences in students' mathematics-related beliefs among different groups, such as between male and female students, (Prendergast et al., 2018; Sangcap, 2010), between low and high performers (De Corte & Op't Eynde, 2003; cited in Andrews & Diego-Mantecón, 2015). The present research sought to investigate further possible differences in students' mathematics-related beliefs between female and male students as well as between low and high performers of problem-solving.

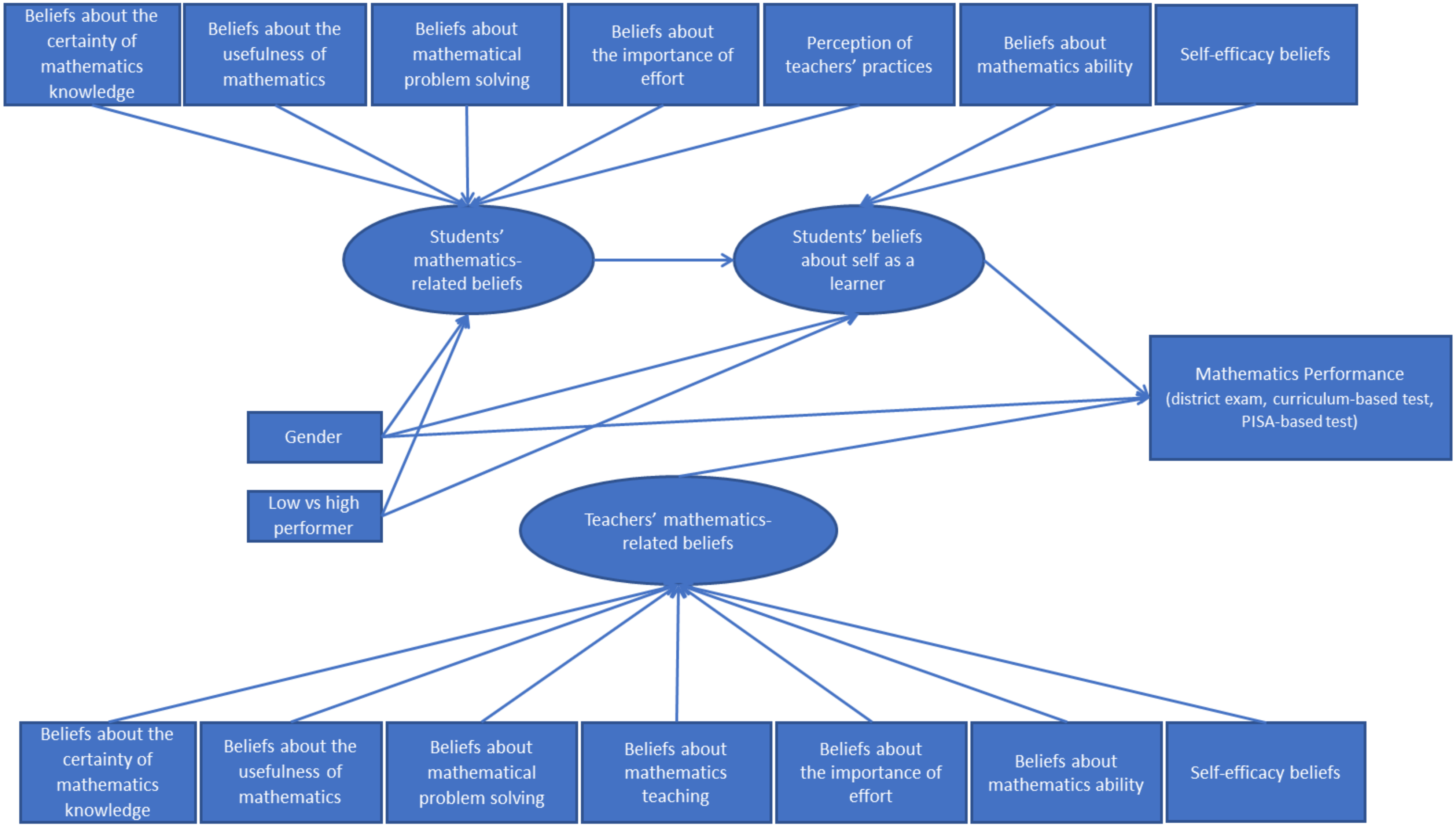


Figure 3.2: Proposed model of the present research

In terms of teachers' mathematics-related beliefs, prior research has suggested that teachers' epistemological beliefs and beliefs about the nature of mathematics may influence their beliefs about mathematics teaching and learning (Cross, 2009; Ernest, 1989a; Liljedahl et al., 2007; Schommer-Aikins, 2004), and that teachers' beliefs about mathematics teaching and learning may influence students' problem-solving performance (Muis & Foy, 2010; Polly et al., 2013). Some of these possible influences will also be explored in the present research.

3.6 Summary

This chapter presents the purposes, the research questions, the context of mathematics education in Vietnam, and the hypotheses of the present research. Seven constructs representing mathematically related beliefs of students, and a further seven constructs representing mathematically related beliefs of teachers were selected to be investigated based on prior research indicating possible relations amongst them and their influence on student performance. A mathematics performance test including curriculum-based part and PISA-based part was developed. The methodology used to conduct this investigation is presented in the next chapter.

CHAPTER 4 METHODOLOGY

In this chapter, the methodology applied to the present research is described and justified. This chapter is structured in four major sections. The first section addresses the participants of students and mathematics teachers included in the present research; the second section presents the materials development process; the third section describes the data collection procedures of the main research; and, the fourth section outlines statistical procedures that applied for the present research.

4.1 Participants

Participants in this study included 620 students and 46 mathematics teachers. This section describes the characteristics of these participants.

4.1.1 Students

The target participant demographic for this research was Vietnamese school students in ninth grade from Nghe An Province in Vietnam. Nghe An is one of 63 Vietnamese provinces, and is located in central Vietnam. This province was selected for the data collection phase because it is the largest province in Vietnam and includes rural, urban, and remote areas. As a result, the sample was more likely to be representative of Vietnamese students in other areas of the country. Two districts were selected from this province, namely, Cua Lo and Thanh Chuong. Cua Lo is a district located nearby Vinh city, the centre of Nghe An Province, and is an urban area, and Thanh Chuong is 100 kilometres from Vinh city in a rural area.

Table 4.1: Sample of research participants–ninth grade students

District	School	Gender		Total
		Female	Male	
Thanh Chuong	1	23	41	64
	2	18	17	35
	3	22	32	54
	4	42	63	105
	Sub-total	***105***	***153***	***258***
Cua Lo	5	48	60	108
	6	57	46	103
	7	77	74	151
	Sub-total	***182***	***180***	***362***
Total		**287**	**333**	**620**

According to school data, there was a total of 647 ninth grade students in the seven selected schools at the time of data collection. During the days that the data collection was conducted, 620 ninth grade students willingly participated in the research (27 students were absent from school on these days). Table 4.1 provides further information about the student sample. Among 620 students, 287 (46.3%) identified as female and 333 (53.7%) identified as male. Most participants were born in 2004. On average, they were 14 years and 6 months old, and the age range of participants was 13 years and 6 months to 15 years 11 months at the time of the data collection.

4.1.2 Mathematics teachers

In order to investigate the relationships between teachers' mathematics-related beliefs and students' mathematics performance, all teachers from the selected schools were invited to participate in the research. Since the targeted participants included ninth grade students, the researcher attempted to specifically contact and invite all mathematics teachers who were teaching ninth grade classes to capture their perspectives. According to information from the schools, there were 53 mathematics teachers within the seven selected schools. A total of 46 mathematics teachers (39 females and 7 males) took part in the research (see Table 4.2). Among these teachers, there were 28 teachers who were teaching mathematics from sixth grade to eighth grade, and 16 mathematics teachers teaching 20 ninth grade classes within the seven selected schools. It is worth noting that all teachers in the sample had teaching experience with ninth grade students, with the 28 teachers not teaching ninth grade classes at the time of the research had previously taught ninth grade classes. Most teachers in the sample had more than ten years' experience in teaching mathematics. There were 45 teachers with more than ten years' experience in teaching mathematics and 14 teachers with more than 20 years of experience.

Table 4.2: Sample of research participants–mathematics teachers

District	School	Gender		Total
		Female	Male	
Thanh Chuong	1	4	0	4
	2	5	0	5
	3	1	4	5
	4	5	2	7
Cua Lo	5	7	0	7
	6	6	1	7
	7	11	0	11
Total		**39**	**7**	**46**

4.1.3 Recruitment process

The sample size for the present research was determined in order to meet the needs of the structural equation modelling used to analyse the data. Further discussion about this issue is provided later in the Statistical Procedures section. With the objective of recruiting a total of approximately 600 ninth grade students for the sample, the researcher worked with the Nghe An Department of Education and Training and the principals of schools to recruit participants. First, based on information pertinent to the research in terms of geography, the researcher and the representative of the Nghe An Department of Education and Training agreed on Cua Lo and Thanh Chuong districts as locations to implement the research. Second, the Department of Education and Training worked with the principals of secondary schools within these two districts to identify schools that were willing to participate in the research. As a result, three lower secondary schools in Cua Lo and four in Thanh Chuong were selected. Third, the researcher worked directly with selected schools to recruit participants. The researcher approached principals individually to make detailed plans about a week before the data collection was conducted. The principals were asked for advice in relation to which teachers within their schools might be interested in participating in the research project. The researcher organised to meet face-to-face with interested teachers to provide an accurate and clear description of the research. Teachers were allowed some time to consider their participation on an entirely voluntary basis. In terms of students, the purpose of the research was explained to them in their classrooms to ensure that they fully understood the process of data collection and their participation was appreciated. They were made aware that they could opt to withdraw their consent at any time with no adverse consequences.

This research was granted final ethics approval by the Flinders University Social and Behavioural Research Ethics Committee before data collection was conducted. All participants were asked to sign a printed consent form indicating that they reserved the right to withdraw from the research at any time. In addition, all participants were provided an introduction letter and an information sheet explaining the research. All documents were provided to the participants in Vietnamese version. A copy of the ethics approval, letters of introduction, and information letters are included in Appendix A to Appendix E.

4.2 Procedures

4.2.1 Procedures for students

About a week before the data collection commenced, the researcher worked with the selected schools to distribute the introduction letter and information sheets that included information regarding the purpose of the research, procedures of data collection, benefits and risks, and the confidentiality of participants to all students and mathematics teachers in the sample. The principals then informed all mathematics teachers and ninth grade students within their schools, and emphasised that participation was voluntary and participants were free to withdraw at any time. Participants also received a consent form to express their willingness to participate.

To manage the process of data collection, the researcher designed a protocol to assign a unique identification code to each student and teacher who participated in the research. These codes helped the researcher to keep track of information about the locations (Cua Lo or Thanh Chuong) and schools of participants. These codes were created by schools based on the researcher's guidelines. Within each school, the students were divided into different classrooms based on their identification codes. On average, each ninth grade class in the sample school normally had around 35 students. However, in order to avoid students from copying or discussing the results among each other, 20-24 students were placed in each classroom. In each room, one designated teacher was appointed to invigilate the test and questionnaire.

During the data collection days, the researcher had about 20 minutes to discuss the procedure they should follow in each class with each appointed teacher, such as reminding students to fill in their identification code (both questionnaire and test) at the beginning and at the end of each section; informing students of the allowed time for the test (every 15 minutes); and preventing students from copying and discussing the instruments with each other. The researcher answered their questions about the research and procedures, and also emphasised to teachers that they should encourage students to try their best on the test and to answer the questionnaire honestly.

Before students took the test, teachers explained to them how to complete the test. Students took the test within 45 minutes then they had a 15-minute break before they returned to the classroom to answer the questionnaire that took about 25 minutes to complete.

4.2.2 Procedures for teachers

The researcher distributed the teacher information sheet and consent form, and spent about 20 minutes with all mathematics teachers within each school to talk about the purpose and importance of the research, as well as what was expected by the researcher. Teachers were informed that participation was voluntary. The researcher also explained to teachers how to complete the questionnaire. Again, the researcher encouraged them to answer honestly by carefully reading each statement in the questionnaire. Although it took only about 20 minutes to complete the questionnaire, due to their teaching commitments, teachers were allowed to complete the questionnaire during the day and the completed questionnaires were returned to the researcher at the end of the school day.

4.3 Materials

Multiple instruments were used in the collection of data. The data collection tools consisted of a questionnaire designed to investigate students' mathematics-related beliefs, namely, the *Mathematics-Related Beliefs Questionnaire for Students* (MBQ-S); a questionnaire designed to investigate teachers' mathematics-related beliefs, namely, the *Mathematics-Related Beliefs Questionnaire for Teachers* (MBQ-T); two assessments of students' mathematics performance (Test 1 and Test 2); and students' scores from the district mathematics exam. This section presents these materials and discusses the process of their development.

4.3.1 Considerations related to the development of the MBQ-S and MBQ-T

Along with the development process two questionnaires, some issues related to the number of items within each construct, the length of the questionnaires, and the solutions to obtain high rates of responses were considered to make sure the success of the research and the effectiveness of the instruments.

4.3.1.1 Number of items with each construct and the length of the questionnaires

The number of items within each construct was duly considered as some scholars recommended that there should be more than three items to measure a construct (Hair, Black, Babin, & Anderson, 2019). Within each belief construct chosen for the present research, based on different existing scales have been developed by other researchers in the field, it was decided to start with at least six items for each construct so as to have enough items at the end of the validation process to investigate each belief. Consequently, each belief construct in the MBQ-S and MBQ-T had between six and seventeen items.

The length of the questionnaires also concerned the researcher. As mentioned earlier, the present research investigated the relationships between students' and teachers' mathematics-related beliefs and students mathematics performance using systematic approaches by choosing many different belief constructs to demonstrate these relationships. As a consequence, the questionnaires may be long due to attempting to cover so many items. This may lead to some unexpected results, such as students either providing incomplete questionnaires or losing focus towards the end of the questionnaire. Furthermore, this was the first time Vietnamese students and teachers had been exposed to mathematics-related belief scales. If the questionnaires were too long, the researcher feared that students would likely ignore many items, or give up, or provide unusual patterns of responses. In other words, the more items the questionnaire had, the more difficult it may prove to manage students' responses, the more likely students would give up, or have bias responses. Therefore, in order to avoid student fatigue, it was decided to include only the most important belief constructs in the questionnaire.

4.3.1.2 Other issues

(1) The items in the questionnaires were measured using 4-point Likert scale (strongly disagree, disagree, agree, strongly agree), so participants could focus on reading items and selecting their options. Without the "neutral" option, the researcher assumed that participants may be encouraged to read statements more carefully to avoid careless choices. The use of a 4-point scale, rather than a 6-point or 8-point scale, helped participants save time in completing the long questionnaire while still providing quality data.

(2) The MRBQ-S and MRBQ-T used simple and accessible language. The wording of each item, the quality of the translation, and recommendations from experts in the field of mathematics education, as well as of the secondary school teachers and students, were taken into account to ensure that each item was clearly expressed.

(3) The items in the final MRBQ-S and MRBQ-T were presented in a random order.

The next sections explain the development process of the MBQ-S and MBQ-T.

4.3.2 Mathematics-Related Beliefs Questionnaire for Students (MBQ-S)

The first draft of MBQ-S consisted of seventy-eight 4-point Likert scale items (1-strongly disagree; 2-disagree; 3-agree; 4-strongly agree) belonging to seven belief constructs: (1) beliefs about the certainty of mathematics knowledge (CMK); (2) beliefs about the usefulness of mathematics (UoM); (3) beliefs about mathematical problem-solving (MPS); (4) beliefs about the importance of effort (BaE); (5) beliefs about mathematics ability (MA); (6) students' self-efficacy beliefs (SelfE); and, (7) students' perception of teachers' practices (Per). Within each construct, some items were created by the researcher and others were taken from a number of existing questionnaires and scales including the *Epistemological Beliefs Questionnaire* (Schommer, 1990), the *Epistemological Beliefs for Mathematics Questionnaire* (Wheeler, 2007), the *Indiana Mathematics Beliefs Scale* (Kloosterman & Stage, 1992) and the *Mathematics-related Beliefs Questionnaire* (Op't Eynde & De Corte, 2003; Op't Eynde et al., 2006a). Within each belief construct, there were some items representing beliefs that were hypothesised to be negative predictors of mathematics performance and others that were assumed to be negative predictors of mathematics performance.

4.3.2.1 Validation of the MBQ-S

Validation is critical to improve and ensure the quality of the instruments in terms of structure, format, and quality of the items within each construct before undertaking data collection (Creswell, 2014). The validation process helps detect any potential problems in the instruments such as time required to answer the questions and ambiguity of wording, and rectify them before commencing data collection. In the case of the present research, the student questionnaire included items from various existing scales. Although these scales had been widely used to investigate students' mathematics-related beliefs, they were mainly applied to measure students' beliefs in Western countries rather than students in Asian countries. Furthermore, within each construct of the MBQ-S, the researcher also developed some new items that were deemed suitable for each particular construct. The main purposes of the validation process in the present research was to identify problems regarding the structure of the MBQ-S, the meaning and expression of the items, and the processes students employed in answering the items in the questionnaire to make sure that participants fully understood each of the statements in the questionnaire. The MBQ-S was validated by experts and by conducting cognitive interviews with students. Experts were recruited (nationally and internationally) to assess the quality of the MBQ-S in terms of quality of items within each belief construct, how well the items represented the meaning of the belief constructs, and the quality of the translated version. The cognitive interviews were also conducted with a small sample of students to get feedback about their understanding of each item in the questionnaires. The main purpose was to ensure that participants fully understood the items in the way intended by the researcher. Based on the information obtained from these processes, the researcher modified the MBQ-S before it was used to collect the main data.

In the following section, general information about the validation process will first be presented. The detailed results of the validation stages, and how these results were used to revise the questionnaire, will be explained in a later section which discusses each construct of the MBQ-S.

4.3.2.1.1 Testing the construct validity of the MBQ-S using experts

Seven experts participated in the process to revise the MBQ-S, following a number of different steps. First, the researcher worked with another expert, a Professor in Cognitive Psychology conducting research on students' and teachers' beliefs about learning and teaching, to develop the first draft of the MBQ-S. The first draft of the instrument consisted of 78 items belonging to seven types of beliefs: beliefs about the certainty of mathematics knowledge (CMK-8 items); beliefs about the usefulness of mathematics (UoM-11 items); beliefs about mathematical problem-solving (MPS-12 items); beliefs about effort (BaE-12 items); beliefs about mathematics ability (MA-9 items); mathematics self-efficacy beliefs (SelfE-9 items); and students' perceptions of teachers' practices (Per-17 items).

In the next step, two professors from the College of Education, Psychology and Social Work at Flinders University, who were working in the field of mathematics education and in educational psychology, were invited to review the structure and the expression of each item. Comments from these scholars were invaluable to help the researcher improve the MBQ-S. After revising the questionnaire based on their advice, the researcher translated the MBQ-S into Vietnamese. At this point, the researcher worked on the wording of each item so that the intended meaning was communicated in the Vietnamese language as faithfully as possible.

In the third step, both the English and Vietnamese versions of the MBQ-S were sent to a Vietnamese expert in English teaching at the College of Humanities, Arts and Social Sciences of Flinders University, who had a PhD in Education Management, to check the translation and obtain additional comments on the questionnaire. Based on her comments, the MBQ-S was further revised. In the fourth step, the revised version was sent out to two experts in the field of mathematics education from the Vietnam National Institute of Educational Sciences. The structure of the questionnaire and the number of items for each belief construct were not changed in this stage and only the wording of some items in Vietnamese was modified. Further information about these changes will be provided later when the researcher addresses each construct in detail. Having undertaken each of the above steps and modified the questionnaire accordingly, the revised version of the questionnaire was then used for the cognitive interviews.

4.3.2.1.2 Testing the construct validity of the MBQ-S using cognitive interviews

Cognitive interviews are typically used in the drafting of questionnaires to be piloted with a small sample of people representing the intended demographic of the participant sample. The purpose of the cognitive interview is to determine whether participants interpret the items as intended by the researchers (Presser et al., 2004). Cognitive interviews help to identify and prevent potential misinterpretations of the meaning of the items, and to improve the ways they are expressed.

One of the most important reasons for using cognitive interviews in the present research was that this was the first time Vietnamese students would respond to a questionnaire about mathematics-related beliefs. Beliefs about learning and teaching are rarely mentioned in school contexts in Vietnam. As a result, it was presumed that students might find it difficult to understand some of the items in the MBQ-S. Two lower secondary schools in Nghe An Province, which had agreed to participate in the research, were chosen to implement the cognitive interviews. One school was located in Vinh City, located centrally in Nghe An Province, and the other was in Cua Lo District, one of the districts of Nghe An Province nearby Vinh City. Within each school, ten students were invited to participate in the cognitive interviews.

The students were informed that the questionnaire was designed to measure their beliefs about mathematics and about mathematics learning and teaching. They were also informed that the purpose of the research was to investigate the influence of these beliefs on students' mathematics performance. They were thanked for their willingness to participate voluntarily, then they were given the MBQ-S to read and answer. The students were asked to mark the items that they did not understand or thought could be expressed with greater clarity. When the students finished their answers (in about 25 minutes), they were interviewed. During the interviews, the researcher asked the participants to how well they understood each item and also asked the students to explain their understanding of some selected items to ensure that they fully understood their intended meaning. The participants' comments about each item were noted.

Overall, the students appeared to have good understanding of most items in the MBQ-S. Their detailed comments for each construct will be presented later, though it is important to note that they did not seem to fully understand the intended meaning of the words ‘strategy’ and ‘mathematics ability’. In Vietnamese, the term ‘strategy’ is used in business settings rather than in education, so it was difficult for students to understand this term in the educational context. Additionally, in Vietnam, the term ‘ability’ is interchangeably used with the terms ‘capability’ and ‘competence’. In order to clarify these, the researcher added the explanations shown below to the beginning of the questionnaire. When the questionnaire was delivered to participants for the main data collection, the researcher explained these terms before the participants answered the questionnaires. The explanations in the questionnaire were as follows:

- The term ‘ability’ refers to innate ability. People have different beliefs regarding mathematics ability and the extent to which it can change via education. We would like you to tell us what are your beliefs.
- The term ‘strategies’ refers to the processes and actions you can use during learning. General strategies that help you learn (learning strategies), include things such as ‘repeat thing to remember’, ‘break the problem into smaller problems’, ‘make a table’, and ‘work backwards’. Sometimes, strategies do not help you to find a correct answer, but they help you to find the solutions or the appropriate way to solve the problems. In other words, strategies are not procedures that always lead you to the correct answers.

At the beginning of the questionnaire, there were four items designed to capture students’ demographic information. Students were asked to provide information about their age, gender, ethnicity, and the particular grade they were in at the time of the research. Students were also provided with unique identification codes (written at the top of the questionnaire) to connect the questionnaire to the mathematics performance test. Each unique identification code also allowed the researcher to know which district and school students came from. Along with the specific grade they indicated in a demographic item, this information also allowed the researcher to connect student data with those of their teachers.

4.3.2.2 Seven constructs of the MBQ-S

Based on comments from experts and results of the cognitive interviews, the researcher finalised the questionnaire. In addition to three demographic items, the final MBQ-S version consisted of 76 items measuring seven constructs about mathematics-related beliefs. A cover letter and a consent form were placed at the front of the questionnaire.

All items in the final questionnaire are showed in Table 4.3. Within each construct, some items represented beliefs that were assumed to be positive predictors of students' mathematics performance (+), and some items represented beliefs that were assumed to be negative predictors of students' mathematics performance (-).

Table 4.3: The items in the Mathematics-Related Beliefs Questionnaire for Students (MBQ-S)

Total item No.	Construct item No.	Statement
I. Beliefs about the Certainty of Mathematics Knowledge (CMK)		
1	CMK1	Most of what is true in math is already known (-)
2	CMK2	Math is really just knowing the right formula for the problem (-)
3	CMK3	I prefer a math teacher who shows students lots of different ways to look at the same problem (+)
4	CMK4	Mathematical theories are the product of creativity (+)
5	CMK5	In math, the answers are always either right or wrong (-)
6	CMK6	There is no place for students to be creativity in math class (-)
7	CMK7	Math problem has always only one true answer (-)
8	CMK8	Answers to questions in math change as mathematicians gather more information (+)
II. Beliefs About the Usefulness of Mathematics (UoM)		
9	UoM1	I study math because I know how useful it is (+)
10	UoM2	Knowing math will help me earn a living (+)
11	UoM3	Math is a worthwhile and necessary subject (+)
12	UoM4	Math will not be important to me in my life's work (-)
13	UoM5	Math is of no relevance to my life (-)
14	UoM6	Studying math is a waste of time (-)
15	UoM7	Understanding math is important for mathematicians, economists, and scientists but not for most people (-)
16	UoM8	The only reason I would take a math class is because it is a requirement (-)
17	UoM9	It is important to see the connections between the math I learn in class and real-world applications (+)
18	UoM10	Math provides the foundation for most of the principles used in science and business (+)
19	UoM11	Math helps us better understand the world we live in (+)
III. Beliefs About Mathematical Problem-Solving (MPS)		
20	MPS1	There are math problems that just can't be solved by following a predetermined sequence of steps (+)
21	MPS2	Math problems can be solved without remembering formulas (+)
22	MPS3	Memorising steps is not that useful for learning to solve math problem (+)

23	MPS4	Learning to do math problems is mostly a matter of memorising the right steps to follow (-)
24	MPS5	Time used to investigate why a solution to a math problem works is time well spent (+)
25	MPS6	A student who doesn't understand why an answer to a math problem is correct hasn't really solved the problem (+)
26	MPS7	In addition to getting a right answer in math, it is important to understand why the answer is correct (+)
27	MPS8	It's not important to understand why a mathematical procedure works as long as it gives a correct answer (-)
28	MPS9	Getting a right answer in math is more important than understanding why the answer works (-)
29	MPS10	It doesn't really matter that you understand a math problem as long as you can get the right answer (-)
IV. Beliefs About the Importance of Effort (BaE)		
30	BaE1	Math problems that take a long time don't bother me (+)
31	BaE2	I feel I can do math problems that take a long time to complete (+)
32	BaE3	I find I can do hard math problems if I just keep trying (+)
33	BaE4	If I can't do a math problem in a few minutes, I probably can't do it at all (-)
34	BaE5	If I can't solve a math problem quickly, I quit trying (-)
35	BaE6	I'm not very good at solving math problems that take a long time to figure out (-)
36	BaE7	By trying hard, one can become smarter in math (+)
37	BaE8	Studying hard can improve one's ability in math (+)
38	BaE9	I can get smarter in math by trying hard (+)
39	BaE10	Ability in math increases when one knows the right strategies (+)
40	BaE11	Appropriate study skills can increase one's ability to do math (+)
41	BaE12	I can get smarter in math if I know how to solve math problems (+)
V. Beliefs About Mathematics Ability (MA)		
42	MA1	When I'm having trouble in math class, better study habits can make a big difference (+)
43	MA2	I'm confident I could learn math if I had better study strategies (+)
44	MA3	When I don't understand something, I keep asking questions (+)
45	MA4	Learning good study skills can improve my math ability (+)
46	MA5	Math is like a foreign language to me and even if I work hard, I'll never really understand it (-)
47	MA6	I knew at an early age what my math ability was (-)
48	MA7	It is frustrating when I have to work hard to understand a problem (-)
49	MA8	I can learn new things in math, but I can't really change the math ability I was born with (-)
50	MA9	I'm just not a math student (-)
VI. Students' Self-Efficacy Beliefs (SelfE)		
51	SelfE1	I feel confident enough to ask questions in my mathematics class (+)
52	SelfE2	I am certain that I can do well in my math tests (+)
53	SelfE3	I can complete all of the assignments in my mathematics course (+)
54	SelfE4	I will be able to use mathematics in my future career when needed (+)
55	SelfE5	I feel that I will be able to do well in future mathematics courses (+)
56	SelfE6	I feel confident when using mathematics outside of school (+)
57	SelfE7	I am afraid to give an incorrect answer during my mathematics class (-)

58	SelfE8	I worry that I will not be able to get a good grade in my mathematics course (-)
59	SelfE9	I feel stressed when listening to mathematics teachers in class (-)
VII. Students' Perceptions of Teachers' Practices (Per)		
60	Per1	My math teacher is friendly to us (+)
61	Per2	My math teacher listens carefully when we ask questions or say something (+)
62	Per3	My math teacher understands the problems and difficulties we experience (+)
63	Per4	My math teacher tries to make math lessons interesting (+)
64	Per5	My math teacher does not really care how we feel in class (-)
65	Per6	My math teacher appreciates it when we have tried hard, even if our results are not so good (+)
66	Per7	My math teacher really wants us to enjoy learning new things (+)
67	Per8	My math teacher thinks mistakes are bad (-)
68	Per9	My math teacher thinks mistakes are okay as long as we are learning something from them (+)
69	Per10	My math teacher first shows step by step how to solve a specific mathematical problem, before giving us similar exercises (+)
70	Per11	My math teacher explains why math is important (+)
71	Per12	My math teacher gives us time to really explore new problems and try out possible solution procedure (+)
72	Per13	My math teacher wants us to understand the content of this math course, not just memorise it (+)
73	Per14	My math teacher lets us do a lot of group work in this math class (+)
74	Per15	My math teacher shows us different strategies to solve problems (+)
75	Per16	My math teacher teaches us strategies to remember important math procedures (+)
76	Per17	My math teacher teaches us strategies to evaluate our problem solutions (+)

The next section describes each of the beliefs within the MBQ-S. Within each construct, information is provided about the meaning and the purpose of each item, where the items were taken from, how they changed after the validation process, and how they appeared in the final version.

4.3.2.2.1 Beliefs about the certainty of mathematics knowledge (CMK)

The construct of beliefs about the certainty of mathematics knowledge measures whether students believe that mathematics consists of a body of knowledge that is certain and unchangeable or something malleable that can be changed. Items from this construct were adopted from the *Epistemological Beliefs Questionnaire* (Schommer, 1990) and the *Epistemological Beliefs for Mathematics Questionnaire* (Wheeler, 2007). In the final version, this construct consisted of eight items, as shown in Table 4.3. The items representing the beliefs that mathematics is certain and unchangeable, such as “In math, the answers are always either right or wrong”, were assumed to be negative predictors of mathematics performance on the basis of the results of prior research (Köller, 2001; Schommer, 1990). The opposite was assumed for the items representing beliefs that mathematics knowledge is malleable, such as “Mathematical theories are the product of creativity” and “Answers to questions in math change as mathematicians gather more information”.

As indicated in previous chapters, prior research showed that students usually hold non-availing beliefs, i.e., beliefs that are negatively associated with quality learning and achievement (Muis, 2004). Students tend to believe that mathematics knowledge is more certain than knowledge of other disciplines (Baxter Magolda, 2002; Buehl & Alexander, 2005), and there is evidence that students’ beliefs that mathematics knowledge is certain and unchanging negatively predicted mathematics performance (Köller, 2001). Additionally, due to the fact that the content-based curriculum was in use in Vietnam at the time this research was conducted, the researcher divided this construct into two different sub-constructs; beliefs that mathematics knowledge is certain and unchanging, and beliefs that mathematics knowledge is malleable. The experts also agreed with this decision.

The first sub-construct consisted of three items (CMK3, CMK4, CMK8), which the researcher defined as beliefs that mathematics knowledge is malleable (CMK_P). The second sub-construct consisted of five items (CMK1, CMK2, CMK5, CMK6, CMK7), with this sub-construct referred to as beliefs that mathematics knowledge is certain (CMK_N).

4.3.2.2.2 Beliefs about the usefulness of mathematics (UoM)

This construct consisted of eleven observed variables, with six items taken from the *Indiana Mathematics Beliefs Scale* (Kloosterman & Stage, 1992) and five items from the *Epistemological Beliefs for Mathematics Questionnaire* (Wheeler, 2007). The items representing the beliefs that mathematics is useful, such as “I study math because I know how useful it is” and “It is important to see the connections between the math I learn in class and real-world applications”, were assumed to be positive predictors of mathematics performance. There were also five items representing the beliefs that mathematics is not useful, such as “Studying math is a waste of time” and “The only reason I would take a math class is because it is a requirement”. These items were assumed to be negative predictors of mathematics performance. This scale received positive feedback from both experts and students during the validation process in that they were all stated clearly and easy to understand. Therefore, in the final questionnaire, all eleven items (see Table 4.3) were used to measure this construct.

4.3.2.2.3 Beliefs about mathematical problem-solving (MPS)

As mentioned in Chapter 2, the *Indiana Mathematics Beliefs Scale* includes five different sub-scales to measure different aspects of beliefs about mathematical problem-solving (Kloosterman & Stage, 1992). In the present research, in order to avoid overlap with other constructs, beliefs about mathematical problem-solving was used to refer specifically to whether mathematical problem-solving are about memorising steps and procedures and finding correct answers, or whether it is about understanding why certain mathematical procedures will provide the correct answers.

This construct included twelve items that measured students' beliefs about mathematical problem-solving. It was adopted from two sub-scales of the *Indiana Mathematics Beliefs Scale*, namely, "There are word problems that cannot be solved with simple, step-by-step procedures" with six observed variables and "Understanding concepts is important" with six observed variables (Kloosterman & Stage, 1992). In the *Indiana Mathematics Beliefs Scale*, the first sub-scale was used to measure the beliefs about word problems. In the present research, the researcher changed the term 'word problem' to 'math problem' in all of six items in this scale. The second sub-scale was not altered from the original, however, when this construct was discussed with the experts, two items were recommended to be removed due to repetition. They were "Any word problem can be solved if you know the right steps to follow" and "Most word problems can be solved be using the correct step-by-step procedures". In the final version, this construct consisted of ten items.

Within this construct, six items representing the beliefs that understanding rather than memorisation is important, such as, "Memorising steps is not that useful for learning to solve math problems", were assumed to be positive predictors of mathematics performance. Four items representing the beliefs that what is important is memorising formulas rather than understanding why formulas work, such as, "Getting a right answer in math is more important than understanding why the answer works", were assumed to be negative predictors of mathematics performance. All items for this construct are listed in Table 4.3.

Prior research showed students tend to believe mathematical problem-solving is about memorising procedures (Schoenfeld, 1992). Additionally, Vietnamese mathematics teachers generally apply transmissive teaching approaches (T. T. Nguyen et al., 2019). As a result, Vietnamese students also tend to memorise procedures and formulas, and they may also believe that mathematical problem-solving is about memorising procedures. Therefore, similar to beliefs about the certainty of mathematics, in order to investigate Vietnamese students' beliefs about mathematical problem-solving, the researcher divided this construct into two sub-constructs, namely, beliefs that mathematical problem-solving is about memorising procedures and formulas, and beliefs that mathematical problem-solving is about understanding procedures. During the validation process, the experts also agreed with this decision.

The first sub-construct consisted of four items (MPS1, MPS 5, MPS6, MPS7) with beliefs that solving problems is not about memorising the rules. The researcher named this sub-construct, positive beliefs about mathematical problem-solving (MPS_P). The second sub-construct consisted of four items (MPS4, MPS8, MPS9, MPS10) with beliefs that solving problems is about memorising the rules. The researcher named this sub-construct, negative beliefs about mathematical problem-solving (MPS_N).

4.3.2.2.4 Beliefs about the importance of effort (BaE)

The construct of beliefs about the importance of effort in the present research consisted of two sub-scales with twelve items from the *Indiana Mathematics Beliefs Scale* (Kloosterman & Stage, 1992). One sub-scale was "I can solve time-consuming mathematics problems" with six items, and the other, "Effort can increase mathematical ability", also with six items. Nine items representing the beliefs that effort is importance to improve mathematics outcomes were assumed to be positive predictors of mathematics performance, such as "I find I can do hard math problems if I just keep trying". Three items representing the beliefs that effort is not importance to improve mathematics outcome were assumed to be negative predictors of mathematics performance, such as, "If I can't solve a math problem quickly, I quit trying". As shown in Table 4.3, this construct retained all twelve items in the final version.

4.3.2.2.5 Beliefs about mathematics ability (MA)

As mentioned previously, beliefs about mathematics ability measured the students' beliefs as to whether mathematics ability is fixed and immutable (-) or can be changed during the learning process (+). This construct consisted of nine items that were adopted from the *Epistemological Beliefs Questionnaire* (Schommer, 1990) and the *Epistemological Beliefs for Mathematics Questionnaire* (Wheeler, 2007). There was no change within this construct in the pilot study phase. Four items representing the beliefs that mathematics ability can be changed during learning process were assumed to be positive predictors of mathematics performance, for example, "Learning good study skills can improve my math ability". Five items representing the beliefs that mathematics ability is fixed and immutable, such as, "Math is like a foreign language to me and even if I work hard, I'll never really understand", were assumed to be positive predictors of mathematics performance. After the validation process, all nine items (see Table 4.3) were retained to measure this construct.

4.3.2.2.6 Students' self-efficacy beliefs (SelfE)

In order to measure students' self-efficacy beliefs, nine items of this construct were developed by the researcher based on different scales about mathematics self-efficacy beliefs from the literature. There were six items that expressed positive beliefs, such as, "I feel confident enough to ask questions in my mathematics class". Three items expressed negative beliefs, including, "I am afraid to give an incorrect answer during my mathematics class". The researcher made some minor modifications to some items based on the results from the validation process. The full nine items of this construct were provided in Table 4.3.

4.3.2.2.7 Students' perception of teachers' practices (Per)

This construct consists of seventeen observed variables measuring students' perception regarding teachers' friendliness and the encouragement given by mathematics teachers in teaching and learning. There were thirteen items which were adopted from the *Mathematics-related Beliefs Questionnaire* (Op't Eynde et al., 2006a). Because this scale from the *Mathematics-related Beliefs Questionnaire* has been previously used to measure secondary school students' beliefs, the researcher retained the same items in the scale. In addition to these items, four new items were developed. These four items asked students whether they believed that their teachers implement constructive teaching in mathematics classes. The experts did not express any negative comments regarding this construct. Through the cognitive interviews, students stated that these items were easy to understand. There were 15 items expressing positive beliefs about teachers, such as, "My math teacher is friendly to us" and "My math teacher listens carefully when we ask questions or say something". There were two items expressing negative beliefs, such as, "My math teacher does not really care how we feel in class". All seventeen items within this construct were provided in Table 4.3.

To summarise, the MBQ-S was developed based on a number of existing scales designed to measure mathematics-related beliefs from the research literature. The validation process was applied by seeking critique from experts and conducting cognitive interviews with Vietnamese students and mathematics teachers. The final MBQ-S consisted of the following nine constructs of mathematics-related beliefs: (1) beliefs that mathematics knowledge is malleable (CMK_P) with three (03) items; (2) beliefs that mathematics knowledge is certain (CMK_N) with five (05) items; (3) beliefs that mathematics is useful (UoM) with eleven (11) items; (4) beliefs that mathematical problem-solving is about understanding procedures (MPS_P) with six (06) items; (5) beliefs that mathematical problem-solving is about memorising procedures and formulas (MPS_N) with four (04) items; (6) beliefs about the importance of effort (BaE) with twelve (12) items; (7) beliefs that mathematics ability can be change during learning process (MA) with nine (09) items; (8) students' self-efficacy beliefs (SelfE) nine (09) items; and, (9) students' perceptions of teachers' practices (Per) with seventeen (17) items.

4.3.3 Mathematics-Related Beliefs Questionnaire for Teachers (MBQ-T)

With the exception of three background items about their gender, number of years of mathematics teaching experience, and the class they were teaching, the original MBQ-T consisted of 69 4-point scale items (1-strongly disagree; 2-disagree; 3-agree; 4-strongly agree). These items were first grouped into seven constructs, namely, (1) beliefs about the certainty of mathematics knowledge (CMKt); (2) beliefs about the usefulness of mathematics (UoMt); (3) beliefs about mathematical problem-solving (MPSt); (4) beliefs about mathematics teaching (Teach); (5) beliefs about the role of student effort in their learning (BaEt); (6) beliefs about students' mathematics ability (MAt); and, (7) teachers' self-efficacy beliefs (SelfEt). Similar to the MBQ-S, within each belief construct in the MBQ-T, there were some items representing beliefs that were assumed to be negative predictors of students' mathematics performance and some items representing beliefs that were assumed to be negative predictors of students' mathematics performance.

The MBQ-T was subjected to the same validation process used for the students' questionnaire. First, seven experts (as indicated in the relevant section about the MBQ-S) were invited to assess the structure and the content of the questionnaire in both English and Vietnamese versions using the same procedure as was used for the MBQ-S. Cognitive interviews with mathematics teachers were then conducted at two selected schools. Five mathematics teachers from each school participated in this process. The mathematics teachers were informed of the purposes of the research and of the voluntary nature of their participation, and were then given the questionnaire to read and answer by themselves. The teacher interviews were conducted afterward. In general, the teachers said that they understood the items in the questionnaire. However, similar to students, there were two terms, 'strategies' and 'mathematics ability', that teachers expressed some confusion around. Therefore, in order to clarify these terms to avoid misunderstanding, the researcher added explanations of these terms at the beginning of the teachers' questionnaire, as was done for the students' questionnaire.

The final MBQ-T had 61 items measuring seven different constructs about mathematics-related beliefs and their teaching practices. Table 4.4 shows all items in the final version of MBQ-T.

As mentioned in the previous chapter, in order to investigate in greater detail of the possible inconsistency in each type of beliefs as well as their influence on students' mathematics performance, the researcher decided to divide each construct into two sub-constructs. The first sub-construct consisted of items representing beliefs that were assumed to be positive predictors of students' mathematics performance (+), and the second sub-construct consisted of items representing beliefs that were assumed to be negative predictors of students' mathematics performance (-). Similar to student participants, teachers were assigned a unique identification code to identify the district and schools to which they belonged. Along with the information about the specific classes they were teaching, this information helped the researcher to connect the teachers with their students.

Table 4.4: The items in the Mathematics-Related Beliefs Questionnaire for Teachers (MBQ-T)

Total Item No.	Construct item No.	Statement
I. Beliefs about the Certainty of Mathematics Knowledge (CMKt)		
1	CMKt1	Most of what is true in math is already known (-)
2	CMKt2	Math is really just knowing the right formula for the problem (-)
3	CMKt3	It is better when a math teacher shows students lots of different ways to look at the same problem (+)
4	CMKt4	Mathematical theories are the product of creativity (+)
5	CMKt5	In math the answers are always either right or wrong (-)
6	CMKt6	There is no place for students to be creative in math class (-)
7	CMKt7	Math problem has always only one true answer (-)
8	CMKt8	Answers to questions in math change as mathematicians gather more information (+)
II. Beliefs about the Usefulness of Mathematics (UoMt)		
9	UoMt1	I teach math because I know math is useful (+)
10	UoMt2	Knowing math will help people earn a living (+)
11	UoMt3	Math is a worthwhile and necessary subject (+)
12	UoMt4	Math will not be important to students in their life's work (-)
13	UoMt5	Understanding math is important for mathematicians, economists, and scientists but not for most people (-)

14	UoMt6	The only reason students would take a math class is because it is a requirement (-)
15	UoMt7	It is important to see the connections between the math students learn in class and real-world applications (+)
16	UoMt8	Math provides the foundation for most of the principles used in science and business (+)
17	UoMt9	Math helps us better understand the world we live in (+)
III. Beliefs about Mathematical Problem-solving (MPSt)		
18	MPSt1	There are math problems that just can't be solved by following a predetermined sequence of steps (+)
19	MPSt2	Math problems can be solved without remembering formulas (+)
20	MPSt3	Memorising steps is not that useful for learning to solve math problems (+)
21	MPSt4	Learning to do math problems is mostly a matter of memorising the right steps to follow (-)
22	MPSt5	Time used to investigate why a solution to a math problem works, is time well spent (+)
23	MPSt6	A student who doesn't understand why an answer to a math problem is correct hasn't really solved the problem (+)
24	MPSt7	In addition to getting a right answer in math, it is important to understand why the answer is correct (+)
25	MPSt8	It's not important to understand why a mathematical procedure works as long as it gives a correct answer (-)
26	MPSt9	Getting a right answer in math is more important than understanding why the answer works (-)
27	MPSt10	It doesn't really matter whether or not students understand a math problem as long as they can get the right answer (-)
IV. Beliefs about math teaching (Teach)		
28	Teach1	It is important for teachers to teach students strategies to solve math problems (+)
29	Teach2	Students will learn math better if teachers let them have opportunities to discuss math (+)
30	Teach3	Teachers should teach students ways to integrate new information with their existing knowledge of math (+)
31	Teach4	Teaching math involves mostly the transmission of math knowledge from teachers to students (-)
32	Teach5	Telling students the correct answers in math is the most important task for teachers (-)
33	Teach6	The main goal of teaching math is to increase the amount of knowledge in the students' memory (-)
34	Teach7	Teachers can help students learn math when they teach them problem-solving strategies (+)
35	Teach8	Repeating math knowledge in class is necessary for students learning (-)
36	Teach9	Math learning depends mostly on how good the teacher is to tell students what they need to know about math (-)
37	Teach10	Students learn best when they develop their mathematical problem-solving skills (+)
38	Teach11	The main goal of teaching math is to help students develop learning strategies in math (+)

V. Teachers' self-efficacy beliefs (SelfEt)		
39	SelfEt1	I am certain I can teach math concepts effectively (+)
40	SelfEt2	I have no difficulties answering students' math questions (+)
41	SelfEt3	I am certain that I am a good teacher of math (+)
42	SelfEt4	I often wonder whether I am effective in monitoring math activities (-)
43	SelfEt5	Sometimes I doubt whether I understand math concepts well enough to be an effective math teacher (-)
44	SelfEt6	I wonder if I have the necessary skills to teach math (-)
45	SelfEt7	Given a choice, I will not invite the principal to evaluate my math teaching (-)
46	SelfEt8	Often I do not know what to do to turn students on to math (-)
VI. Beliefs about the role of students' Effort in their learning (BaEt)		
47	BaEt1	By trying hard, one can become smarter in math (+)
48	BaEt2	Studying hard can improve one's ability in math (+)
49	BaEt3	Students can get smarter in math by trying hard (+)
50	BaEt4	Ability in math increases when one knows the right strategies (+)
51	BaEt5	Appropriate study skills can increase one's ability to do math (+)
52	BaEt6	Students can get smarter in math if they know how to learn math problems (+)
VII. Beliefs about students' Mathematics Ability (MAt)		
53	MAt1	Better study habits are the key to success for students who struggle in math (+)
54	MAt2	Student who doesn't have high natural ability in math is still capable of learning difficult math material (+)
55	MAt3	Learning good study skills can improve a students' math ability (+)
56	MAt4	Some people are born with great math ability and some aren't (-)
57	MAt5	Math ability is really just something students are born with (-)
58	MAt6	The smartest math students don't have to do many problems because they just get them quickly (-)
59	MAt7	It is frustrating for students to have to work hard to understand a problem (-)
60	MAt8	Students can learn new things in math, but they can't really change the math ability they were born with (-)
61	MAt9	Most people know at an early age whether they are good at math or not (-)

4.3.3.1 Beliefs about the certainty of mathematics knowledge (CMKt)

This construct consisted of eight items, seven of which were also used in the students' questionnaire. Based on the experts' recommendations, the item "I prefer a math teacher who shows students lots of different ways to look at the same problem" from the students' questionnaire was changed to "It is better when a math teacher shows students lots of different ways to look at the same problem" to be suitable to capture the teachers' perspective. There were five items representing the beliefs that mathematics is certain and unchangeable (CMKt_N), and three items representing the beliefs that mathematics knowledge is malleable (CMKt_P). All items are listed in Table 4.4.

4.3.3.2 Beliefs about the usefulness of mathematics (UoMt)

This construct was developed by selecting items from the student questionnaires that were applicable to teachers. As a result, this construct is similar to the one used in the MBQ-S, with the exception of two items that were not considered relevant to teachers based on recommendations from experts. Specifically, those items were, "Math is of no relevance to my life" and "Studying math is a waste of time". In the final questionnaire, nine items were used in this construct, as shown in Table 4.4. Six items represented the beliefs that mathematics is useful (UoMt_P) and three items represented the beliefs that mathematics is not useful (UoMt_N).

4.3.3.3 Beliefs about mathematical problem-solving (MPSt)

This construct consisted of the same ten items as the MBQ-S, appropriately modified to capture a teacher's perspective. As shown in Table 4.4, six items represented beliefs that what is important is understanding solutions and their mathematical procedures as opposed to memorisation of formulas and steps (MPSt_P), such as "a student who doesn't understand why an answer to a math problem is correct hasn't really solved the problem", were assumed to be positive predictors of students' mathematics performance. Four items represented the beliefs that what is important is memorising formulas as opposed to understanding solutions and their mathematical procedures, such as "getting a right answer in math is more important than understanding why the answer works". These items were hypothesised to be negative predictors of students' mathematics performance (MPSt_N).

4.3.3.4 Beliefs about mathematics teaching (Teach)

The purpose of this construct was to investigate teachers' beliefs regarding the nature of teaching, focusing on the distinction between constructive and transmissive teaching. It consisted of 11 items that were partly based on, or influenced by, existing research in this area. The experts mostly agreed on the eleven items in this construct. The mathematics teachers in the cognitive interviews expressed the opinion that all of these items were easy to understand. As can be seen from Table 4.4, six items represented beliefs consistent with constructive approaches to teaching, such as, "Students will learn math better if teachers let them have opportunities to discuss math" (Teach_P). Five items represented beliefs more consistent with transmissive views about teaching, such as, "The main goal of teaching math is to increase the amount of knowledge in the students' memory" (Teach_N).

4.3.3.5 Teachers' self-efficacy beliefs (SelfEt)

In order to measure teachers' self-efficacy beliefs, eight observed variables of this construct were developed based on existing scales on teachers' self-efficacy beliefs from the research literature. Some minor modifications were made to some items based on results from the validation process. As shown in Table 4.4, four items expressed beliefs in high self-efficacy, such as "I have no difficulties answering students' math questions" (SelfE_P), and four items expressed beliefs in low self-efficacy, such as, "I wonder if I have the necessary skills to teach math" (SelfE_N).

4.3.3.6 Beliefs about the role of students' effort in their learning (BaEt)

This construct was developed by selecting items from the student questionnaires that were suitable for teachers. In the original questionnaire, there were 12 items adapted from the student questionnaire. However, some experts recommended removing items related to spending time to solve problem (six items). In the final version, six items in this construct represented beliefs that students' effort is important to improve their mathematics outcomes, such as, "studying hard can improve one's ability in math", that were assumed to be positive predictors of students' mathematics performance.

4.3.3.7 Beliefs about students' mathematics ability (MAt)

All items from this construct were adopted from the *Epistemological Beliefs Questionnaire* (Schommer, 1990) and the *Epistemological Beliefs for Mathematics Questionnaire* (Wheeler, 2007). Experts and mathematics teachers had no comments on these items. Three items represented beliefs that mathematics ability can be changed during the learning process (MAt_P), such as, "a student who doesn't have high natural ability in math is still capable of learning difficult math material", were assumed to be positive predictors of students' mathematics performance. Six items represented beliefs that students' mathematics ability is fixed and immutable (MAt_N), such as, "students can learn new things in math, but they can't really change the math ability they were born with", which were assumed to be negative predictors of students' mathematics performance.

To summarise, the MBQ-T was developed based on many existing scales of teachers' mathematics-related beliefs from the research literature. Similar to the MBQ-S, a validation process was applied by seeking advice from experts and conducting cognitive interviews with Vietnamese students and mathematics teachers. The final MBQ-T consisted of the 13 following sub-constructs of teachers' mathematics-related beliefs: (1) beliefs that mathematics knowledge is malleable (CMKt_P) with three (03) items; (2) beliefs that mathematics knowledge is certain (CMKt_N) with five (05) items; (3)

beliefs that mathematics is useful (UoMt_P) with six (6) items; (4) beliefs that mathematics is not useful (UoMt_N) with three (03) items; (5) beliefs that mathematical problem-solving is about understanding procedures (MPSt_P) with six (06) items; (6) beliefs that mathematical problem-solving is about memorising procedures and formulas (MPS_N) with four (04) items; (7) beliefs in constructive teaching of mathematics (Teach_P) with six (06) items; (8) beliefs in transmissive teaching of mathematics (Teach_N) with five (05) items; (9) high self-efficacy beliefs (SelfEt_P) with three (03) items; (10) low self-efficacy beliefs (SelfEt_N) with five (05) items; (11) beliefs about the importance of students' effort (BaEt) with six (06) items; (12) beliefs that students' mathematics ability can be change during learning process (MA) with three (03) items; and, (13) beliefs that students' mathematics ability is innate (MAt_N) with six (06) items;

4.3.4 Assessments of students' mathematics performance

The main purpose of the present research is to investigate the influence of students' and teachers' mathematics-related beliefs on students' mathematics performance. In order to do so, the researcher used various indicators of mathematics performance to explore these relationships. First, the student scores in the district's end-of-term exam for mathematics, which were collected from schools, were used. Second, the researcher designed an assessment of students' mathematics performance that assessed the ability to solve Vietnamese mathematics curriculum-based problems and PISA-based problems. This section presents relevant issues regarding these performance measures.

4.3.4.1 District mathematics exam

Students' scores in the district exam were chosen as one of the measures of students' mathematics performance. This exam is designed by the Bureau of Education and Training from each district in Vietnam, and all ninth grade students participate in the assessment. By choosing a common mathematics assessment, mathematics scores are comparable among students within each district. This district end-of-term exam is carefully designed to cover all important aspects of mathematics in the current mathematics curriculum covered during the first semester.

4.3.4.2 The assessment of mathematics performance designed for the purpose of the research

This assessment consisted of two parts. The first part included items that students commonly encounter in the context of their mathematics curriculum at school. The second part included mathematical problems used in recent PISA tests (OECD, 2006; Stacey & Turner, 2015). This section describes these two parts of assessment.

4.3.4.2.1 Part 1: Mathematics assessment based on the current mathematics curriculum in Vietnam

This part of the assessment was designed to test how much the students had benefited from mathematics instruction at school. When the research was implemented, the ninth grade students had just finished their Semester 1 in Vietnam. In the mathematics curriculum, there were two important areas covered in ninth grade mathematics at this stage: algebra and geometry. Therefore, an assessment instrument was designed to include two multiple-choice questions in algebra and two multiple-choice questions in geometry.

When ninth grade students finish the first semester, they are expected to have algebraic knowledge about square root and linear functions, and this is emphasised in the mathematics curriculum. The following item was considered an easy item where students need only find the numbers that have the square root. If students have learned about square root, they would know that only 0 or positive numbers can have a square root. The answer for this item is B (4 numbers).

Item 1. Let's take the following numbers $-\frac{1}{4}$; 4; -1; 0; $\frac{1}{4}$; 1; -4. How many numbers have the square root?

A. 3 numbers B. 4 numbers
C. 5 numbers D. 7 numbers

The mathematics curriculum emphasises that students should have introductory knowledge about linear functions, and they should be able to indicate or check whether the point presented in the form of (x, y) satisfies the designated equation in the graph. The following item in the test measured the understanding of the relationships between linear functions and their graphs. This item was a medium-level item, where students should know to identify which the coordinates of *x* and *y* in the given options satisfied the function. In this case, the answer is D.

Item 2. Which of the following is on the graph of function $y = -4x + 1$?

A. (1; 5) B. (–1; 3)
C. (1; –5) D. (1; -3)

Within the area of geometry, the mathematics curriculum emphasises that students know the right-angled triangle altitude theorem and can use this theorem to calculate relevant elements of the right-angled triangle. In the test, there were two items in this area. The following item measured geometry knowledge about the relationships of ratios in similar right-angled triangles by asking students to calculate the length of *x* and *y*. This was a medium-level item where students had to find the ratios between sides and hypotenuse of the similar right-angled triangles. In this case, the answer is B.

Item 3. The results for x and y in Figure 1 are:

A. $x = 4$ and $y = 16$

B. $x = 4$ and $y = 2\sqrt{5}$

C. $x = 2$ and $y = 8$

D. $x = 2$ and $y = 2\sqrt{2}$

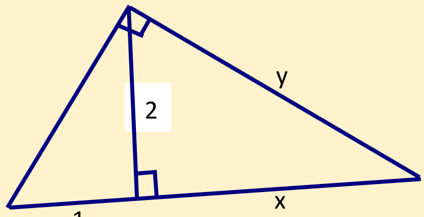


Figure 1

The second item also measured geometry knowledge about the relationships of ratios. However, this is a difficult item where students have to indicate right answer for ratios using sides, hypotenuse, and altitude of the right-angled triangle. In this case, the answer is D.

Item 4. In Figure 2, which equation is correct?

A. $a^2 = cb'$

B. $b^2 = ca'$

C. $c^2 = a'b'$

D. $h^2 = a'b'$

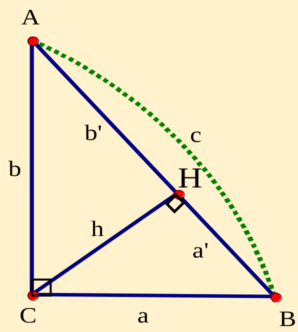


Figure2

4.3.4.2.2 Part 2: Mathematics performance assessment based on PISA problems

One of the purposes of the present research is to investigate the influence of mathematics-related beliefs on mathematical problem-solving performance in general, beyond the strict limits of the taught curriculum. In order to achieve that purpose, two problems from the *Programme for International Student Assessment* (PISA) were chosen. The Organization for Economic Co-operation and Development (OECD) has implemented PISA since 2000, and the assessment is conducted every three years. In the PISA, fifteen-year-old students are assessed in the domains of mathematics, science, and reading. After each cycle, OECD released some problems as samples for stakeholders who are interested in investigating different aspects of the PISA assessment. In the present research, the sample was also ninth grade students, so the researcher selected two real-world problems with four questions representing ninth grade algebra and geometry, where students could use mathematics knowledge that they have learned to solve these problems.

Being aware that Vietnamese students rarely work on real-world problems at school, and based on PISA analysis of the item difficulty and the level of mathematics knowledge and skills ninth grade students had when they finished Semester 1, two problems were selected. Within each problem, there was one easy item and one item of medium-level difficulty. These problems assess whether students can transfer current knowledge and skills to solve problems that relate to everyday situations. The problems were adopted from OECD sources (OECD, 2006; Stacey & Turner, 2015). The following section presents these two problems.

Problem 1: APPLES. A farmer plants apple trees in a square pattern. In order to protect the apple trees against the wind he plants conifer trees all around the orchard. Here you see a diagram of this situation where you can see the pattern of apple trees and conifer trees for any number (n) of rows of apple trees:

Removed due to copyright restriction

There were two items for this problem. The first item was quite easy with students asked to complete a table indicating the number of apple trees and number of conifer trees based on the number of apple tree rows.

***Item 5: APPLES*.** Complete the table:

N	Number of apple trees	Number of conifer trees
1	1	8
2	4	
3		
4		
5		

The second item related to solving a simple quadratic equation. From experience of practical situations with Vietnamese students, although the quadratic equation is simple, students were expected to find it difficult to extract information from the item to obtain the equation. This item is shown as follows:

Item 6: APPLES. There are two formulae you can use to calculate the number of apple trees and the number of conifer trees for the pattern described above: Number of apple trees are n^2; Number of conifer trees are $8n$ where n is the number of rows of apple trees. There is a value of n for which the number of apple trees equals the number of conifer trees. Find the value of n and show your method of calculating this.

The second problem also had two items. This problem related to geometry knowledge. The reason these particular geometry problems were selected was because the students had just finished their first semester of study, and the theorem of Pythagoras was the focus of the curriculum. The two PISA problems were selected from the same topic field in order to find out how students could use their knowledge and skills that they had just been instructed at school in order to solve a new, unfamiliar real-world problem.

Problem: SAILING SHIPS. Ninety-five percent of world trade is moved by sea, by roughly 50 000 tankers, bulk carriers and container ships. Most of these ships use diesel fuel. Engineers are planning to develop wind power support for ships. Their proposal is to attach kite sails to ships and use the wind's power to help reduce diesel consumption and the fuel's impact on the environment.

Removed due to copyright restriction

In the first item of this problem, students needed to realise that the objective was to apply trigonometric ratios in right-angled triangle to calculate hypotenuse. In fact, this question appeared to be easier for students when the image with the right-angled triangle was shown.

Item 7: SAILING SHIPS. Approximately what is the length of the rope for the kite sail, in order to pull the ship at an angle of 45° and be at a vertical height of 150 m, as shown in the following diagram?

A. 173 m

B. 212 m

C. 285 m

D. 300 m

Removed due to copyright restriction

The second item of this problem was considered the most difficult item in this part. Although the nature of the mathematics behind this item is quite easy, the students might struggle with the information provided in this item.

Item 8: SAILING SHIPS. Due to high diesel fuel costs of 0.42 zeds per litre, the owners of the ship NewWave are thinking about equipping their ship with a kite sail. It is estimated that a kite sail like this has the potential to reduce the diesel consumption by about 20% overall.

Removed due to copyright restriction

The cost of equipping the NewWave with a kite sail is 2 500 000 zeds. After about how many years would the diesel fuel savings cover the cost of the kite sail? Give calculations to support your answer.

A rubric was developed to score each item of the performance test. Each multiple-choice item had three codes: 0 for incorrect answer, 1 for correct answer, and 9 for missing answer. Two open-ended questions (Item 6 and Item 8) were scored from 0 to 2 as follows:

Item 6 scoring

Code 2: n=8, algebraic method explicitly shown

$n^2 = 8n$, $n^2 - 8n = 0$, $n(n - 8)=0$, $n = 0$ & $n = 8$, so $n = 8$

Code 1: n=8, no clear algebra presented, or no work shown

Code 0: Other responses

Code 9: Missing.

Item 8 scoring

Code 2: A solution from 8 to 9 years is provided with adequate (mathematical) calculations.

- Diesel consumption per year without a sail: 3.5 million litres, price 0.42 zed/litre, costs for diesel without a sail 1 470 000 zeds.
- If 20% is saved with the sail this results in a saving of 1 470 000 x 0.2 = 294 000 zeds per year.
- Thus: 2 500 000 / 294 000 = 8.5, i.e., after about 8 to 9 years, the sail becomes (financially) worthwhile.

Code 1: Solve up to 2 steps above or equivalent.
Code 0: Other responses.
Code 9: Missing.

To summarise, the mathematics performance test had two different parts with four items in each part. Students were allocated 45 minutes to complete the test. During the development of the performance test, the test was first translated from English to Vietnamese by the researcher. Two researchers in the field of mathematics education from the Vietnam Institute of Educational Sciences were invited to review the test. These two experts generally agreed with the content of the test and stated that this test was suitable to assess Vietnamese ninth grade students' mathematics performance.

4.4 Statistical Procedures

There were three stages of data analysis for the current investigation. First, the collected data was entered and analysed using SPSS 25. The purpose of this stage was to provide descriptive statistics about the participants and preliminary data analysis. In the second stage, Confirmatory Factor Analysis (CFA) was employed to develop a measurement model and to test if the data was sufficient to implement Structural Equation Modelling (SEM). Stage three tested how well the part of the proposed structural model for students fitted the data and investigated the relationships between proposed constructs of mathematics-related beliefs and mathematics performance using SEM. The present research also employed multilevel SEM analysis to explore the effects of teachers' mathematics related beliefs on students' mathematics performance. Results of the data analysis are discussed in Chapter 5. This section provides basic background information about the statistical procedures employed.

4.4.1 Structural Equation Modelling

4.4.1.1 Why SEM is selected for the present research

Recently, Structural Equation Modelling (SEM) has been widely used among researchers in behavioral and social sciences (Khine, 2013). SEM is considered as the second generation of multivariate analysis to explore relationships among independent and dependent variables (Kline, 2011). Similarly, Hair et al. (2019) defined SEM as a combination of multiple regression and factor analysis to test the hypotheses on relationships simultaneously. Some researchers also provided specific recommendations to utilise SEM for educational research (Khine, 2013; Schreiber, Nora, Stage, Barlow, & King, 2006). In terms of the investigation of mathematics-related beliefs and their associations with mathematics' performance, some scholars also recommended that modern techniques of data analysis (e.g., SEM) should be employed to explore such complex relationships (Muis, 2004).

The selection of SEM as the main analysis technique was based on a number of reasons. First, as discussed above, compared to other traditional techniques (such as regression or path analysis), SEM is more appropriate when one independent variable within a relationship may become a dependent variable in another relationship. Second, the purpose of the present research is to explore a complex structural model between various constructs of students' and teachers' mathematics-related beliefs and students' mathematics performance. These constructs were considered as latent variables which were measured by different observed variables. In this context, SEM offered more advantages than other analysis techniques. Finally, the present research also explored the complex relationships between different belief constructs of teachers' mathematics related beliefs with three different types of mathematics performance. Currently, only multilevel SEM analysis allows this complexity of analysis. The present research employed Mplus version 8.4 (Muthén & Muthén, 1998-2017) as the main software for data analysis.

According to Hair et al. (2019), six stages in SEM analysis are following: "Stage 1: Defining individual constructs; Stage 2: Developing the overall measurement model; Stage 3: Designing a study to produce empirical results; Stage 4: Assessing the measurement model validity; Stage 5: Specifying the structural model; Stage 6: Assessing structural model validity" (Hair et al., 2019, p. 625). The first four stages are measurement model examination where CFA techniques are applied, and the last two stages are structural model examination where SEM techniques are applied. They also recommended that researchers should evaluate the structural model using this two-step approach. In other words, researchers should first evaluate the measurement model and then the structural model.

4.4.1.2 Sample size

Sample size within the targeted population is a critical issue for quantitative researchers. Although the use of a large sample cannot guarantee precision, small samples may impact the accuracy of the results (Hair et al., 2019). With the application of SEM, sample size is always carefully considered because of the demand of the analysis. There are different rules of thumb about sample size proposed by researchers. For example, Kline (2011) proposed that, in order to analyse a complicated path model, the sample size should be 200 or more. Moreover, Schreiber et al. (2006) recommended that with the model, there should be at least 10 respondents for each estimated parameter. Hair et al. (2019) suggested that the sample size estimation should be based on the number of participants per estimated parameter. They also suggested that if the model has many constructs and/or fewer than three observed variables within some constructs, the sample size should be at least 500. In line with the above recommendations and based on the complexity of the proposed model of the present research, it was decided that the sample size for this research should be at least 600.

4.4.1.3 Coding and Missing data

In the case of the present research, the questionnaire and the test papers completed by participants were screened before the process of data entry. One of the main purposes of this process was to check for missing background information, missing answers, and choice of more than one option for some statements. This process also provided a general view about how students and teachers responded to the questionnaires and the performance test. Although this step was simple, it still provided important information before moving on to the data entry process. In regard to the coding of the questionnaires, apart from the demographic items, both students' and teachers' questionnaires included 4-point-scale items. Accordingly, it was coded 1 as 'strongly disagree', 2 as 'disagree', 3 as 'agree', and 4 as 'strongly agree'. The code of 8 indicated missing answers. Regarding statements where students chose two options, if the two options were both of agreement (i.e., 'strongly agree' and 'agree'), it was coded as 'agree'. If two options were both of disagreement (i.e., 'strongly disagree' and 'disagree'), then it was coded as 'disagree'. If the two options crossed agreement and disagreement scores (i.e., 'agree' and 'disagree'), it was coded as a missing value.

In terms of the mathematics performance test, the researcher marked the scores of open-ended items (in the part of PISA problems) directly on the students' test papers based on the developed rubric. When this process was complete, IBM SPSS Statistics 25.0 (IBM Corp, Released 2017) was employed to enter all student and teacher data. The data were then cleaned using the descriptive statistics for each item to ensure all data fully reflected the participants' responses and ensure data accuracy prior to the main analysis.

Dealing with missing data is one of the considerations in data analysis because it may influence the results of the research (Hair et al., 2019), and it is even more critical with structural equation modelling (Schumacker & Lomax, 2004). Some researchers suggested that it is considered acceptable if the percentage of missing data is less than 5%. After the above process, the rate of missing data was also calculated. There was a total of 331 (0.651%) missing values in the student data. The teacher data had only 37 (1.319%) missing values. Because the percentage of the missing data was low, the researcher did not apply the imputation procedure for the present research data.

4.4.2 Confirmatory Factor Analysis

When researchers developed the constructs and the observed variables of each construct based on theories and/or empirical research, Confirmatory Factor Analysis (CFA) was normally used in preference to Exploratory Factor Analysis (EFA) to examine the extent to which items from a construct may reflect that construct based on collected data. In other words, "CFA reveals the degree of confirmation for our preconceived measurement theory" (Hair et al., 2019, p. 661). As mentioned elsewhere in this chapter, the proposed structural model of the present research was developed based on multiple theories and results from prior research. Additionally, each construct was designed based on valid and reliable scales from the research literature. These constructs were assessed carefully through the pilot study. Consequently, the present research employed CFA techniques rather than EFA in order to revalidate and examine the characteristics of all beliefs constructs from the proposed mathematics-related belief system. Many issues should be considered when researchers conduct CFA. This section covers issues relevant to the present research.

4.4.2.1 Measurement Model Identification

Identification refers to the information used to estimate a set of unknown parameters based on known information (Brown, 2006). When a model is identified, it means that a unique set of estimates can be calculated for the unknown values of parameters in the model. Researchers proposed three levels of identification based on the degree of freedom (*df*): under-identified, just-identified, and over-identified. A model is 'under-identified' (i.e., unidentified) if there are more parameters to be estimated than the known information (i.e., $df < 0$). A model is 'just-identified' if there is a unique solution to estimate the unknown parameter (i.e., $df = 0$). A model is 'over-identified' if there is more than one estimate for unknown parameters in the model (i.e., $df > 0$) (Brown, 2006; Hair et al., 2019; Harrington, 2008; J. Wang & Wang, 2012). An example for just-identified model is the model with a latent variable measured by three observed variables since this model has six known parameters (three variances and three covariances) and six unknown parameters (three factor loadings and three error variances). In practice, from the research design, all of the proposed models are over-identified. At this point, a solution of measurement models should be provided with a positive value of the degree of freedom and a corresponding value of chi-square goodness-of-fit. This issue will be discussed in the next section. In terms of the number of items per construct to ensure model identification, many researchers recommended that there should be at least three items for each construct (Hair et al., 2019).

4.4.2.2 Construct validity

The term 'validity' refers to many different aspects. Firstly, face validity and content validity need to be checked before conducting any statistical procedures. The present research carefully assessed the face validity and content validity during the validation process, as previously discussed, using methods of expert assessment and cognitive interviews. Secondly, 'construct reliability' (CR) is normally used with SEM analysis to measure the reliability of the constructs (Hair et al., 2019), and this was also applied for the present research. A value of 0.70 or higher for CR is considered a good reliability, however, values between 0.60 and 0.70 are acceptable (Hair et al., 2019). Details of the construct reliability of each construct in the present research will be provided in Chapter 5.

4.4.2.3 Fit indices

All observed variables in the questionnaire were categorical variables, so Weight Least Square Mean and Variance (WLSMV) estimation was used for all SEM analyses and subsequent path models to estimate parameters, standard errors, χ^2, and other fit indices. Assessing fit is an important step when CFA is applied to examine whether the proposal measurement model is valid. There are many different types of fit indices. In educational research, common use of fit indices includes Chi-square goodness-of-fit (χ^2), the ratios of Chi-square goodness-of-fit and degree of freedom, Comparative Fit Index (CFI), Tucker-Lewis Index (TLI), Root Mean Square Error of Approximation (RMSEA), and Standardized Root Mean Square Residual (SRMR) (Khine, 2013; Schreiber et al., 2006). The present research also used these fit indices to assess how well the collected data fitted the proposed structural model of the study.

Chi-square goodness-of-fit (χ^2) indicates the fit between a proposed model and the corresponding model obtained from the sample (Hu, 2018, p. 66), and a high p-value suggests a closer fit between these two models. In other words, when considering a good fit for a model, the chi-square (χ^2) index will first be used to examine the overall fit if it is not significant. However, this index is dependent on sample size of the research, and if the sample size is large enough, χ^2

statistic might show the significance. Kline (2011) suggested that this influence can be reduced by dividing the χ^2 index by the degree of freedom. Therefore, in research where sample size is large, as in the present research, the ratio of chi-square and degree of freedom (χ^2/df) index, which is less dependent on sample size, is used with values ranging from 2 to 3 and as high as 5 to indicate a good fit. In the present study, we used the χ^2/df index with values less than 3. Because of this reason, some other fit indices were also used to evaluate the overall fit.

Comparative Fit Index (CFI) assesses model fit of independence model (Khine, 2013). This index was first introduced by Bentler (1990). The values of this index vary from 0.0 to 1.0, with values closer to 1.0 indicating good fit (with higher values representing better fit). Another fit index, the *Tucker-Lewis Index* (TLI) compares a proposed model against a null model (Kelloway, 2015). Similar to CFI, the values of this index vary from 0.0 to 1.0 with values closer to 1.0 indicating good fit (higher values represent better fit). Researchers proposed different rules of thumb about the cut-off value for these two fit indices. While some researchers proposed that the cut-off values should be greater than 0.95 (Schreiber et al., 2006), others argued the cut-off values of these fit indices depend on the sample size of the research (Hair et al., 2019). With a sample of 620 participants, the present research used the cut-off values of 0.90 for these two fit indices (Hair et al., 2019).

Root Mean Square Error of Approximation (RMSEA) measures the error of approximation per model degree of freedom. Unlike the χ^2, RMSEA is not influenced by the large sample size and it has become one of the most informative fit indices. The RMSEA index tells us how well the model would fit the population's covariance matrix. There are some recommendations for RMSEA cut-off points. However, more recently, a cut-off of .06 is considered as a reasonable value (Hair et al., 2019). The final fit index used in this research, *Standardized Root Mean Square Residual* (SRMR), is defined as the standardised difference between an observed correlation and a predicted correlation. It may be biased when a sample has a small size and a low degree of freedom. A value of 0 in the SRMR indicates a perfect fit. In practice, values of 0.8 or lower are considered a good fit.

4.4.2.4 Factor loadings

Factor loadings are the coefficients that link the observed variables and their underlying constructs (J. Wang & Wang, 2012). A factor loading indicates the correlation between an observed variable and a factor, with higher loadings making the variable representative of the factor (Hair et al., 2019). Normally, factor loadings with completed standardised solutions will be reported. The value of the factor loadings is one crucial consideration (Hair et al., 2019). In other words, the size of factor loadings is an important consideration in the process of assessing measurement model. At minimum, all factor loadings should be statistically significant. In terms of the rules of thumb, Brown (2006) recommended that the cut-off point should be 0.30. Hair et al. (2019) suggested the cut-off values of standardised loadings vary based on the sample size and the number of constructs and items within each construct. Although standardised loadings should be 0.50 or higher, the cut-off value of 0.4 was acceptable. The present research applied the cut-of value at 0.4 for factor loadings of all items in the questionnaires.

4.4.3 Multilevel structural equation modelling

Over the past two decades, multilevel structural equation modelling has become a mainstream data analysis tool to explore relationships between different factors in different levels of sample structures (Heck & Thomas, 2015). Multilevel structural equation modelling enables us to propose questions of the data that cannot be adequately investigated using other single-level techniques such as multiple regression, path analysis, or structural equation modelling. The procedures of multilevel SEM analysis are the same as SEM analysis regarding analysis steps, requirements of fit indices, and factor loadings (Finch & Bolin, 2017). In the present research, multilevel SEM through Mplus software was used to examine the relationships between different constructs of teachers' mathematics-related beliefs and students' mathematics performance.

4.5 Summary

This chapter has presented the methodology of the present research and outlined the sample of 620 ninth grade students and 46 mathematics teachers at seven secondary schools who participated. The instruments, including the questionnaires for students and teachers, and the assessment of mathematics performance, were described. The questionnaires used in the present research were developed across a number of different stages to validate them before the final versions were used to collect the main data. This chapter also addressed issues of statistical procedures applied to the present research for data analysis. The next chapter will present the results of the main data analysis.

CHAPTER 5 RESULTS

This chapter presents the analysis of the data and results of the present research and is divided into two main sections. The first section reports the analysis of the student data. It presents the validation of the MBQ-S; the characteristics of Vietnamese students' mathematics-related beliefs; the investigation of the relationship between students' mathematics-related beliefs and their mathematics performance; the investigation of gender differences; and differences between low and high mathematics performers. The second section reports on the results of the analysis of the teacher data. It presents the validation of the MBQ-T, the characteristics of the Vietnamese mathematics teachers' beliefs and the relationships between teachers' mathematics-related beliefs and students' mathematics performance.

5.1 Analysis of Student Data

5.1.1 Preliminary data analysis

The data, once entered into SPSS, was analysed first to assess the outliers and to test the normality of the data.

5.1.1.1 Outliers

Outliers are defined as "an observation that is substantially different from the other observations on one or more characteristics" (Hair et al., 2019, p. 48). Outliers may have substantial effects on the results of multivariate analysis (Fidell & Tabachnick, 2014; Hair et al., 2019). There are two types of outliers, namely, univariate outliers and multivariate outliers. Univariate outliers refer to cases with such an extreme value on one variable, and multivariate outliers refer to unusual combinations of scores on two or more variables (Fidell & Tabachnick, 2014).

IBM SPSS Statistics 25.0 was used to identify the univariate and multivariate outliers. In the first step, the boxplots were used to identify univariate outliers. The results showed that 57 out of 76 items in the student questionnaire had univariate outliers (a total of 133 students). All values of outliers were valid values (from 1 to 4). There were 66 students who had outliers in two variables and 34 students who had outliers in more than three variables. The Mahalanobis distance (D^2) was used to identify multivariate outliers. According to Fidell and Tabachnick (2014), the Mahalanobis distance "is the distance of a case from the centroid of the remaining cases where the centroid is the point created at the intersection of the means of all the variables" (p. 108). According to Hair et al. (2019), the conservative level of significance of 0.001 can be used as the threshold value. In other words, all the cases having Mahalanobis D^2 measures with p-value < 0.001 would be considered as influential multivariate outliers. In the present research, Mahalanobis D^2 measures and the p-values of these measures were calculated by using IBM SPSS Statistics 25.0. The results showed that there were 36 students who had multivariate outliers. Eleven of these students had both multivariate and univariate outliers in over two variables, so it was decided to remove these 11 students. The sample size at this stage became 609 students. Data from these 609 students were used for the main data analysis.

5.1.1.2 Normality testing

Testing the presence of normality from a selected sample is an essential consideration in multivariate analysis, because if the data is not normally distributed, then it may affect the results of the analysis, although lack of normality may only have serious influence on results in small samples (Hair et al., 2019). Hae-Young (2013) recommended that with a sample size of over 300, the absolute value of skewness should be less than 2 and the absolute value of kurtosis should be less than 3. Because the sample size of the present research was over 300, following this recommendation, all items in the dataset for both samples were found to be normally distributed, except item UoM6 under the construct of beliefs about the usefulness of mathematics (skewness value is 2.881 and kurtosis value is 9.426). Exploring the item UoM6, "Studying math is a waste of time", nearly the entire sample (97.2%) disagreed with this statement. This suggested that nearly all Vietnamese students in the sample are valuing mathematics learning, and that they believe in the usefulness of mathematics. Because there was no reason to ignore these facts, this item was retained for the main analysis. The detailed skewness and kurtosis values of each item of the questionnaire are provided in Appendix F.

The next section presents the assessment of measurement models using the data from MBQ-S. It is important to emphasise that, in order to investigate the influence of students' mathematics-related beliefs on students' mathematics performance, the researcher first used only data from student questionnaire and their mathematics performance. The main reason for this was to examine these influences in separation to the teachers' data to avoid unnecessary causes that may affect the nature of these influences.

5.1.2 Measurement models

Mplus version 8.4 (Muthén & Muthén, 1998-2017) was employed for the main data analysis. Due to categorical variables of MBQ-S, the Weighted Least Squares Mean and Variance (WLSMV) estimator was applied in Mplus.

As indicated in Chapter 4, CFA, rather than EFA, was used to explore the characteristics of each construct. In the first stage, the CFA was conducted by placing items from all of the constructs together into one measurement model. Table 5.1 provides the standardised factor loadings of the items in different CFA models. The factor loading value of 0.40 was used as the minimum value of acceptable loading coefficient. In other words, only items with factor loadings greater or equal to 0.4 were maintained, and they were considered as good observed variables of their corresponding constructs.

In regard to goodness of fit indices, there are several types of fit indices that should be considered in order to assess a model's goodness-of-fit. First, because the χ^2 was found to be too sensitive for the sample size of 600 in the present research, the ratio of the χ^2 statistic to its degree of freedom (χ^2/df) was used, and the threshold level for χ^2/df was 2.5 (Hair et al., 2019). Based on Schreiber et al.'s (2006) recommendations, the researcher used four different types of fit indices: Root Mean Square Error of Approximation (RMSEA), Comparative Fit Index (CFI), Tucker-Lewis Index (TLI), and Standardized Root Mean Square Residual (SRMR). Because the sample size of the present research was over 250 and the number of observed variables were over 30, Hair et al.'s (2019) suggestions that CFI and TLI should be over 0.90, RMSEA should be less than 0.7, and SRMR should be less than 0.8 were observed.

5.1.2.1 Assessing the measurement model

There were nine intended constructs of students' mathematics-related beliefs as follows: (1) beliefs that mathematics knowledge is malleable (CMK_P); (2) beliefs that mathematics knowledge is certain (CMK_N); (3) beliefs that mathematics is useful (UoM); (4) beliefs that mathematical problem-solving is about understanding procedures (MPS_P); (5) beliefs that mathematical problem-solving is about memorising procedures and formulas (MPS_N); (6) beliefs about the importance of effort (BaE); (7) beliefs that mathematics ability can be changed (MA); (8) students' self-efficacy beliefs (SelfE); and, (9) students' perceptions of teachers' practices (Per).

According to Hair et al. (2019), it is necessary to place all the constructs within a single measurement model rather than conducting separate analyses for each construct. He argued that it is easier to meet the requirement of fit indices if researchers assess separate constructs rather than the entire model. Additionally, the assessment of the entire model helps to identify potential problems across the model, such as cross-loadings, because the items between different constructs may have high correlations to each other which can be avoided by putting all constructs together in a single, whole measurement model (Hair et al., 2019). Therefore, the present research conducted a CFA analysis for all nine belief constructs of the MBQ-S to assess the measurement model.

The constructs beliefs about the certainty of mathematics knowledge and beliefs about mathematical problem-solving were each divided into two sub-constructs (positive and negative predictors), and the original responses of participants were kept without reversing. All other constructs were considered as single constructs and, therefore, the responses to the items representing beliefs that were assumed to be negative predictors of mathematics performance were reversed before conducting the analysis.

Several measurement models were run using the Mplus version 8.4 software. Table 5.1 provides the standardised factor loadings of the items in the main three CFA models. The fit indices of the first model (see table 5.1 Model 1), were not very good [χ^2 = 4597.976; df = 2591; χ^2/df = 1.775; RMSEA = 0.036; CFI = 0.817: TLI = 0.809; SRMR = 0.068]. There were 23 out of 74 items with factor loadings under 0.4, and six of these factor loadings were not significant. All items with non-significant factor loadings were removed one by one from the model to form the next models. In the subsequent models, due to deletion of item CMK8, the construct of beliefs that mathematics knowledge is malleable (CMK_P) was reduced to only two items. According to Hair (2019), this is acceptable with the model consisting of many constructs and a sample size of over 500. Within the model, after removing these six items, the fit indices were improved even though they still did not meet the requirement for good fit [χ^2 = 4054.138; df = 2174; χ^2/df = 1.865; RMSEA = 0.038; CFI = 0.831: TLI = 0.822; SRMR = 0.068]. At this stage, all items in the measurement model had significant factor loadings.

Table 5.1. Results of the Confirmatory Factor Analysis of the MBQ-S (N = 609)

Construct	Total Item No.	Item Name	Item	Mplus 8.4 Factor loadings		
				Model 1*	Model 2*	Model 3*
I. Beliefs About the Certainty of Mathematics Knowledge (CMK)						
CMK_N	1	CMK1	Most of what is true in math is already known	0.533	0.541	0.532
	2	CMK2	Math is really just knowing the right formula for the problem	0.444	0.463	0.466
	3	CMK5	In math, the answers are always either right or wrong	-0.074		
	4	CMK6	There is no place for students to be creativity in math class	0.759	0.746	0.753
	5	CMK7	Math problem has always only one true answer	0.076		
CMK_P	6	CMK3	I prefer a math teacher who shows students lots of different ways to look at the same problem	0.421		
	7	CMK4	Mathematical theories are the product of creativity	0.284		
	8	CMK8	Answers to questions in math change as mathematicians gather more information	0.049		
II. Beliefs About Usefulness of Mathematics (UoM)						
	9	UoM1	I study math because I know how useful it is	0.501	0.495	0.514
	10	UoM2	Knowing math will help me earn a living	0.24		
	11	UoM3	Math is a worthwhile and necessary subject	0.647	0.639	0.65
	12	UoM4	Math will not be important to me in my life's work	0.556	0.557	0.543
	13	UoM5	Math is of no relevance to my life	0.558	0.555	0.547
	14	UoM6	Studying math is a waste of time	0.625	0.645	0.652
	15	UoM7	Understanding math is important for mathematicians, economists, and scientists but not for most people	0.486	0.501	0.482
	16	UoM8	The only reason I would take a math class is because it is a requirement	0.581	0.592	0.581
	17	UoM9	It is important to see the connections between the math I learn in class and real-world applications	0.624	0.607	0.621

	18	UoM10	Math provides the foundation for most of the principles used in science and business	0.295		
	19	UoM11	Math helps us better understand the world we live in	0.335		
III. Beliefs About Mathematical Problem-solving (MPS)						
MPS_P	20	MPS1	There are math problems that just can't be solved by following a predetermined sequence of steps	0.025		
	21	MPS2	Math problems can be solved without remembering formulas			
	22	MPS3	Memorising steps is not that useful for learning to solve math problems			
	23	MPS5	Time used to investigate why a solution to a math problem works is time well spent	0.534	0.47	0.472
	24	MPS6	A student who doesn't understand why an answer to a math problem is correct hasn't really solved the problem	0.177		
	25	MPS7	In addition to getting a right answer in math, it is important to understand why the answer is correct	0.676	0.621	0.618
MPS_N	26	MPS4	Learning to do math problems is mostly a matter of memorising the right steps to follow	0.408	0.416	0.412
	27	MPS8	It's not important to understand why a mathematical procedure works as long as it gives a correct answer	0.65	0.643	0.637
	28	MPS9	Getting a right answer in math is more important than understanding why the answer works	0.47	0.471	0.479
	29	MPS10	It doesn't really matter that you understand a math problem as long as you can get the right answer	0.702	0.703	0.706
IV. Beliefs About the Importance of Effort (BaE)						
	30	BaE1	Math problems that take a long time don't bother me	0.386		
	31	BaE2	I feel I can do math problems that take a long time to complete	0.269		
	32	BaE3	I find I can do hard math problems if I just keep trying	0.45	0.431	0.439
	33	BaE4	If I can't do a math problem in a few minutes, I probably can't do it at all	0.534	0.528	
	34	BaE5	If I can't solve a math problem quickly, I quit trying	0.43	0.427	

	35	BaE6	I'm not very good at solving math problems that take a long time to figure out	-0.046		
	36	BaE7	By trying hard, one can become smarter in math	0.5	0.5	0.546
	37	BaE8	Studying hard can improve one's ability in math	0.45	0.448	0.514
	38	BaE9	I can get smarter in math by trying hard	0.544	0.552	0.62
	39	BaE10	Ability in math increases when one knows the right strategies	0.533	0.531	0.606
	40	BaE11	Appropriate study skills can increase one's ability to do math	0.564	0.562	0.618
	41	BaE12	I can get smarter in math if I know how to solve math problems	0.454	0.454	0.532
V. Beliefs About Math Ability (MA)						
	42	MA1	When I'm having trouble in math class, better study habits can make a big difference	0.421	0.423	0.456
	43	MA2	I'm confident I could learn math if I had better study strategies	0.579	0.583	0.63
	44	MA3	When I don't understand something, I keep asking questions	0.496	0.482	0.506
	45	MA4	Learning good study skills can improve my math ability	0.491	0.493	0.534
	46	MA5	Math is like a foreign language to me and even if I work hard I'll never really understand it	0.584	0.581	
	47	MA6	I knew at an early age what my math ability was	0.141		
	48	MA7	It is frustrating when I have to work hard to understand a problem	0.547	0.53	
	49	MA8	I can learn new things in math, but I can't really change the math ability I was born with	0.198		
	50	MA9	I'm just not a math student	0.511	0.502	0.493
VI. Students' Self-Efficacy Beliefs (SelfE)						
	51	SelfE1	I feel confident enough to ask questions in my mathematics class	0.502	0.578	0.567
	52	SelfE2	I am certain that I can do well in my math tests	0.369	0.458	0.467
	53	SelfE3	I can complete all of the assignments in my mathematics course	0.271		
	54	SelfE4	I will be able to use mathematics in my future career when needed	0.562		
	55	SelfE5	I feel that I will be able to do well in future mathematics courses	0.542	0.68	0.677
	56	SelfE6	I feel confident when using mathematics outside of school	0.426	0.509	0.518
	57	SelfE7	I am afraid to give an incorrect answer during my mathematics class	0.346		

	58	SelfE8	I worry that I will not be able to get a good grade in my mathematics course	0.055		
	59	SelfE9	I feel stressed when listening to mathematics teachers in class	0.557		
VII. Students' Perception of Teachers' Practices (Per)						
	60	Per1	My math teacher is friendly to us	0.554	0.572	0.567
	61	Per2	My math teacher listens carefully when we ask questions or say something	0.654	0.664	0.661
	62	Per3	My math teacher understands the problems and difficulties we experience	0.556	0.555	0.548
	63	Per4	My math teacher tries to make math lessons interesting	0.623	0.622	0.619
	64	Per5	My math teacher does not really care how we feel in class	0.546	0.559	
	65	Per6	My math teacher appreciates it when we have tried hard, even if our results are not so good	0.247		
	66	Per7	My math teacher really wants us to enjoy learning new things	0.597	0.603	0.614
	67	Per8	My math teacher thinks mistakes are bad	0.148		
	68	Per9	My math teacher thinks mistakes are okay as long as we are learning something from them	0.313		
	69	Per10	My math teacher first shows step by step how to solve a specific mathematical problem, before giving us similar exercises	0.456	0.468	0.484
	70	Per11	My math teacher explains why math is important	0.539	0.547	0.544
	71	Per12	My math teacher gives us time to really explore new problems and try out possible solution procedure	0.582	0.589	0.596
	72	Per13	My math teacher wants us to understand the content of this math course, not just memorise it	0.328		
	73	Per14	My math teacher lets us do a lot of group work in this math class	0.394		
	74	Per15	My math teacher shows us different strategies to solve problems	0.507	0.509	0.512
	75	Per16	My math teacher teaches us strategies to remember important math procedures	0.587	0.592	0.596
	76	Per17	My math teacher teaches us strategies to evaluate our problem solutions	0.456	0.45	0.474

Considering the potential interrelations among observed variables within and between constructs, serious consideration was given to the items that should be removed from the original model. The decision of removing an item was based on many considerations, such as the size of factor loadings, the meaning of each item, the number of items within each construct, and the modification indices. This step started with looking at the items that had the smallest factor loading. Within each model, only one item was removed, and the whole process was repeated for the next model. The main purpose of this process was to obtain the best measurement model for the present research. As a result, several models were assessed until all items in the model had factor loadings equal to, or higher than, 0.4.

The first result of the whole measurement model satisfying these criteria consisted of 49 items. All information about the factor loadings of this model are provided in the sixth column (Model 2) in Table 5.1. At this point, the construct of beliefs that mathematics knowledge is malleable (CMK_P) was removed from the model because two items in this construct had factor loadings lower than 0.4. The fit indices for this measurement model were [χ^2 = 2266.541; df = 1099; χ^2/df = 2.062; RMSEA = 0.042; CFI = 0.877: TLI = 0.868; SRMR = 0.064]. According to the requirements of cut-off score for the fit indices, the results showed that three fit indices (χ^2/df= 2.062, RMSR=.064; RMSEA=.042) were acceptable, but two other fit indices (CFI =.877; TLI=.868) were not. The low CFI and TLI suggested that some items within some constructs were not appropriate, and that the model needed additional re-specification.

5.1.2.2 Measurement model re-specification

Different methods have been proposed to increase fit indices when assessing measurement model. However, Hair et al. (2019) identified three common types of poor practices that researchers have used to increase fit indices. The first type is to reduce the number of items within each construct. Their wariness about this method was that many researchers have reduced the number of items to two or even one item within constructs, as it “likely diminishes the construct’s theoretical domain and ultimately its validity” (Hair et al., 2019, p. 641). The second type, as mentioned earlier, is to only assess the fit of the measurement model through single constructs separately rather than doing so with the entire measurement model. The third type is to reduce the sample size which is considered unacceptable practice (Hair et al., 2019).

Because of these problems, the researcher chose to reduce the number of items based on the use the modification indices (MI) with careful consideration of the number of items within each construct. This is one of the effective solutions to improve model fit (Hair et al., 2019). One of the main functions of MI is to identify parameter constraints that are poorly selected. Therefore, under the process for MI, all fixed parameters are assessed to identify if there are parameters, if freely estimated, would lead to significant decreases in the χ^2 statistic. Some researchers suggested that items that reveal a very high covariance with other items, and/or demonstrate high regression weights with other constructs, should be removed from the model (Byrne, 2012; Hair et al., 2019). The results from Mplus software provided information on MI and the researcher used this information to improve the model fits for the measurement model. Within this stage, the MIs were assessed carefully by removing items one by one. The MI information for the items removed is shown in Table 5.2.

Table 5.2. High modification indices between variables in the measurement model

No	Removed Item	Path	Modification Indices
1	BaE4	CMK_N BY BaE4	79.680
2	BaE5	CMK_N BY BaE5	84.591
3	MA7	UoM BY MA7	45.441
4	MA5	CMK_N BY MA5	45.084
5	Per5	Per5 WITH UoM	29.048

As shown in Table 5.2, in using MI to assess the initial measurement model, the MI showed that item BaE4 had very high covariance with different items and high regression weights with some other constructs. The highest value of MI was found on ON/BY Statements in Mplus between item BaE4 (belonged to beliefs about the importance of effort) and the construct CMK_N with the value of 79.680. This suggested that if BaE4 is considered as an item within the construct CMK_N (i.e., cross-loading or secondary factor loading), χ^2value would decrease by approximately 79.680. Since item BaE4 was designed to measure beliefs about the importance of effort and it could not be considered as an item within the construct CMK_N, the researcher decided to remove this item from the model. In regard to the subsequent model after removing this item, the fit indices were improved even though they still did not meet the requirement for good fits [χ^2 = 2095.771; df = 1052; χ^2/df = 1.992; RMSEA = 0.040; CFI = 0.887: TLI = 0.878; SRMR = 0.062]. This process of using MI was repeated and four other items (as shown in Table 5.2) were also removed from the model. During this process, before removing any item, the researcher again based this decision not only on the results of Mis, but on also the meaning of the items and the number of existing items within each construct. For example, the results of MIs showed quite high covariance between the item CMK6 of beliefs that mathematics knowledge is certain (CMK_N) with some other items and high regression weight with some other constructs. However, the researcher decided to keep this item because there were only three items within this construct. Instead of removing item CMK6, the researcher chose to remove item Per5 in the construct of students' perceptions of teachers' practices for three reasons: (1) there were other items within this construct that might convey the meaning of this item (i.e., "My math teacher does not really care how we feel in class"); (2) MIs showed this construct had high covariance with many other items; and, (3) there were many items (12 of 17 items) represented for this construct. At the end of this process, the best measurement model that the researcher could obtain had 44 items (with factor loadings above 0.4 and being significant) belonging to eight constructs, and all type of fit indices met the requirements to run the SEM model [χ^2 = 1614.670; df = 874; χ^2/df = 1.847; RMSEA = 0.037; CFI = 0.909: TLI = 0.902; SRMR = 0.058]. The seventh column (Model 3) in Table 5.1 showed the final estimates for the factor loadings of items within each construct in the final measurement model.

5.1.2.3 Reliability

Construct reliability (CR) is normally used with SEM analysis and it is also used to assess construct validity (Hair et al., 2019). The equation to calculate CR is as below, where λ_i is factor loading and e_i is error variances.

$$CR = \frac{\left(\sum_{i=1}^{n} \boldsymbol{\lambda_i}\right)^2}{\left(\sum_{i=1}^{n} \boldsymbol{\lambda_i}\right)^2 + \left(\sum_{i=1}^{n} \boldsymbol{e_i}\right)}$$

CR was calculated based on the results of factor loadings within each construct and using the above equation. Table 5.3 shows the CR of each construct.

Table 5.3. Construct reliability of each students' belief construct

No	Construct	CR
1	Beliefs that mathematics knowledge is certain and unchanging (CMK_N)	0.62
2	Beliefs that mathematics is useful (UoM)	0.8
3	Beliefs that mathematical problem-solving is about understanding procedures (MPS_P)	0.48
4	Beliefs that mathematical problem-solving is about memorising procedures (MPS_N)	0.65
5	Beliefs that effort is important (BaE)	0.76
6	Beliefs that mathematics ability can be changed (MA)	0.63
7	Students' self-efficacy beliefs (SelfE)	0.64
8	Students' perception of teachers' practices (Per)	0.85

Hair et al. (2019) suggested that CR should be greater than 0.60 to show good reliability. The table above shows that seven out of eight constructs had CR over 0.60. This suggested that these seven constructs were reliable. The construct of beliefs that mathematical problem-solving is about understanding procedures (MPS_P) had CR of 0.48. The reason for this low value was because there were only two observed variables for this construct in the final measurement model. Therefore, this value for the construct MPS_P might be acceptable. Additionally, constructs with fewer than three observed variables are acceptable if it is included in a model with many constructs and the sample size is over 500 (Hair et al., 2019). Therefore, it was decided to keep this construct in the final SEM model.

The final measurement model consisted of eight belief constructs: (1) beliefs that mathematics knowledge is certain (CMK_N); (2) beliefs that mathematics is useful (UoM); (3) beliefs that mathematical problem-solving is about understanding procedures (MPS_P); (4) beliefs that mathematical problem-solving is about memorising procedures and formulas (MPS_N); (5) beliefs about the importance of effort (BaE); (6) beliefs that mathematics ability can be changed (MA); (7) students' self-efficacy beliefs (SelfE); and, (8) students' perceptions of teachers' practices (Per). In the final model, 44 out of 76 items belonging to eight constructs were retained and all fit indices met the requirements [χ^2 = 1614.670; df = 874; χ^2/df = 1.847; RMSEA = 0.037; CFI = 0.909: TLI = 0.902; SRMR = 0.058]. Before reporting the results of the structural model, the next section presents the characteristics of the Vietnamese students' mathematics-related beliefs.

5.1.3 The profiles of the Vietnamese students' mathematics-related beliefs

As discussed in a previous chapter, within each belief construct, there were some items representing beliefs that were assumed to be negative predictors of mathematics performance and others representing beliefs that were assumed to be negative predictors. Based on the descriptive statistics of students' data for these items, this section presents the percent and mean agreement for each item. It also presents differences in data between female and male students.

Table 5.4 provides the percentage of agreement, mean of agreement and the standard deviation (SD) for each item associated with each construct of the primary research in the final measurement model. For example, the first item under beliefs that mathematics knowledge is certain, "CMK1. Most of what is true in math is already known", had a mean of 2.22 (out of 4). 31.20% of students agreed with this statement indicating that, on average, the students showed quite low levels of agreement with beliefs that mathematics knowledge is certain on this item.

Additionally, in Table 5.4, within each belief construct, the mean for each group of items that represented beliefs that were assumed to be positive predictors of mathematics performance, as well as the mean for each group of items that represented to beliefs that were assumed to be negative predictors, were calculated to provide more information about what Vietnamese students believed with respect to each belief construct. In this section, based on the above descriptive statistics, the characteristics of students' beliefs within each construct will be presented.

5.1.3.1 Beliefs about the certainty of mathematics knowledge (CMK)

As mentioned earlier, the final measurement model had only the sub-construct of beliefs that mathematics knowledge is certain in the model. Agreement with this sub-construct was low, however, there were between twenty and thirty per cent of the Vietnamese students who expressed agreement with beliefs that mathematics knowledge is certain and unchangeable. Mean agreement with this sub-construct was 1.95. It was expected that this sub-construct would be a negative predictor of mathematics performance.

5.1.3.2 Beliefs about the usefulness of math (UoM)

There was overwhelming agreement amongst the Vietnamese students that mathematics is useful. This construct had eight items representing students' beliefs that mathematics is useful and mean agreement with this construct was 3.43. Specifically, 92.3% of students agreed that they learn mathematics because mathematics is useful (item UoM1), 95.4% of students agreed that mathematics is a worthwhile and necessary subject (item UoM3), and 89.1% of students agreed that it is important to see the relationship between school mathematics and real-world applications (item UoM9). It was expected that this belief construct would be a positive predictor of mathematics performance.

Table 5.4. Percent and mean agreement of students with items representing belief constructs

	Total item No.	Construct item No.	Statement	Percentage of agreement	Mean	SD
1. Beliefs that mathematics is certain and unchangeable					**1.95**	
	1	CMK1	Most of what is true in math is already known (-)	31.20%	2.22	0.788
	2	CMK2	Math is really just knowing the right formula for the problem (-)	23.30%	2.01	0.74
	3	CMK6	There is no place for students to be creativity in math class (-)	11.50%	1.62	0.775
2.1. Beliefs that mathematics is useful					**3.43**	
	4	UoM1	I study math because I know how useful it is (+)	92.30%	3.41	0.656
	5	UoM3	Math is a worthwhile and necessary subject (+)	95.40%	3.58	0.623
	6	UoM9	It is important to see the connections between math I learn and real-world applications (+)	89.10%	3.31	0.725
2.2. Beliefs that mathematics is not useful					**1.64**	
	7	UoM4	Math will not be important to me in my life's work (-)	17.70%	1.77	0.89
	8	UoM5	Math is of no relevance to my life (-)	13.90%	1.57	0.856
	9	UoM6	Studying math is a waste of time (-)	2.80%	1.20	0.50
	10	UoM7	Understanding math is important for mathematicians, economists, and scientists but not for most people (-)	16.40%	1.75	0.835
	11	UoM8	The only reason I would take a math class is because it is a requirement (-)	23.40%	1.9	0.878
3.1. Beliefs that mathematical problem-solving is about understanding procedures					**3.43**	
	12	MPS5	Time used to investigate why a solution to a math problem works is time well spent (+)	90.60%	3.22	0.679
	13	MPS7	In addition to getting a right answer in math, it is important to understand why the answer is correct (+)	94.90%	3.63	0.608
3.2. Beliefs that mathematical problem-solving is about memorising procedures					**1.99**	
MPS_N	14	MPS4	Learning to do math problems is mostly a matter of memorising the right steps to follow (-)	45%	2.36	0.885
	15	MPS8	It's not important to understand why a mathematical procedure works as long as it gives a correct answer (-)	22%	2.01	0.765
	16	MPS9	Getting right answers in math is more important than understanding why answers works (-)	31.70%	2.1	0.835
	17	MPS10	It doesn't really matter that you understand a math problem as long as you can get the right answer (-)	8.60%	1.5	0.689
4. Beliefs that effort is important					**3.31**	

	18	BaE3	I find I can do hard math problems if I just keep trying (+)	89.30%	3.22	0.646
	19	BaE7	By trying hard, one can become smarter in math (+)	87.60%	3.28	0.723
	20	BaE8	Studying hard can improve one's ability in math (+)	88.20%	3.22	0.678
	21	BaE9	I can get smarter in math by trying hard (+)	94.60%	3.53	0.631
	22	BaE10	Ability in math increases when one knows the right strategies (+)	92.80%	3.35	0.638
	23	BaE11	Appropriate study skills can increase one's ability to do math (+)	91%	3.31	0.651
	24	BaE12	I can get smarter in math if I know how to solve math problems (+)	89.50%	3.26	0.689
5.1. Beliefs that mathematics ability can be changed					**3.34**	
	25	MA1	When I'm having trouble in math class, better study habits can make a big difference (+)	90.30%	3.25	0.656
	26	MA2	I'm confident I could learn math if I had better study strategies (+)	95.70%	3.48	0.601
	27	MA3	When I don't understand something, I keep asking questions (+)	90.30%	3.3	0.657
	28	MA4	Learning good study skills can improve my math ability (+)	91.80%	3.33	0.666
5.2. Beliefs that mathematics ability is innate					**1.60**	
	29	MA9	I'm just not a math student (-)	9.30%	1.6	0.706
6. High self-efficacy beliefs					**2.82**	
	30	SelfE1	I feel confident enough to ask questions in my mathematics class (+)	75.30%	2.94	0.765
	31	SelfE2	I am certain that I can do well in my math tests (+)	47.50%	2.47	0.718
	32	SelfE5	I feel that I will be able to do well in future mathematics courses (+)	77.80%	2.91	0.695
	33	SelfE6	I feel confident when using mathematics outside of school (+)	76.20%	2.94	0.767
7. Positive perceptions of teachers' practices					**3.36**	
	34	Per1	My math teacher is friendly to us (+)	93.90%	3.46	0.7
	35	Per2	My math teacher listens carefully when we ask questions or say something (+)	93.20%	3.52	0.652
	36	Per3	My math teacher understands the problems and difficulties we experience (+)	86.80%	3.21	0.721
	37	Per4	My math teacher tries to make math lessons interesting (+)	90.40%	3.33	0.708
	38	Per7	My math teacher really wants us to enjoy learning new things (+)	93.70%	3.41	0.653
	39	Per10	My math teacher first shows step by step how to solve a specific mathematical problem, before giving us similar exercises (+)	94.40%	3.45	0.621
	40	Per11	My math teacher explains why math is important (+)	84.60%	3.21	0.771
	41	Per12	My math teacher gives us time to explore new problems and try out possible solution (+)	93%	3.42	0.682
	42	Per15	My math teacher shows us different strategies to solve problems (+)	87%	3.21	0.723
	43	Per16	My math teacher teaches us strategies to remember important math procedures (+)	96.50%	3.57	0.59
	44	Per17	My math teacher teaches us strategies to evaluate our problem solutions (+)	87.50%	3.12	0.647

5.1.3.3 Beliefs about mathematical problem-solving (MPS)

This construct consisted of two different sub-constructs. The first sub-construct included items stating that mathematical problem-solving is about understanding procedures, and it was expected this sub-construct would be a positive predictor of mathematics performance. The second sub-construct included the items stating mathematical problem-solving is about memorising procedures and formulas, and it was expected this sub-construct would be a negative predictor of mathematics performance. In the final measurement model, the first sub-construct had two items and the second construct had four items. Most of the Vietnamese students agreed with items expressing beliefs that mathematical problem-solving is about understanding procedures. Mean agreement with items representing beliefs that mathematical problem-solving is about understanding procedures was 3.43, whereas mean agreement with items representing beliefs that mathematical problem-solving is about memorising procedures was 1.99. However, there was some inconsistency in students' beliefs. For example, while 94.9% of students agreed that it is important to understand why the answers are correct rather than just getting the right answers (item MPS7), there were also 31.7% of students who agreed that finding correct answers is more important (item MPS9).

5.1.3.4 Beliefs about the importance of effort (BaE)

The results showed that the Vietnamese students valued the importance of effort in improving mathematics learning. The mean of all items representing beliefs that effort is important was 3.31. For example, most students agreed that, by trying hard, they can be smarter in mathematics (items BaE7, BaE9). There were eight out of twelve items in the final measurement model representing students' beliefs that effort is important to improve their mathematics learning, and it was expected this construct would be a positive predictor of mathematics performance.

5.1.3.5 Beliefs about mathematics ability (MA)

As shown in Table 5.4, most of the students in the sample believed that their mathematics ability could be improved if they had better study habits and strategies (over 90% of students agreed on items MA1, MA2, MA3, MA4). The mean of all items representing beliefs that mathematics ability can be improved was 3.34, and the mean of all items representing beliefs that mathematics ability is innate was 1.60. In the final measurement model, there were five out of nine items representing students' beliefs that ability can be changed by the learning process, and it was expected this construct would be a positive predictor of mathematics performance.

5.1.3.6 Self-efficacy beliefs (SelfE)

Generally, the empirical data showed that the Vietnamese students had high levels of self-efficacy beliefs. For instance, 77.8% of students agreed that they will be able to do well in future mathematics courses (item SelfE5), and 75.3% of students felt confident to ask questions in their mathematics classes (item SelfE1). The mean of all items representing high self-efficacy beliefs was 2.82. The final measurement model included four out of nine items within this construct to represent students' high self-efficacy, and it was expected this construct would be a positive predictor of mathematics performance.

5.1.3.7 Students' perceptions of teachers' practices (Per)

The Vietnamese students in the sample had positive perceptions about their teachers' practices. The mean of all items representing positive perceptions of teachers' practices was 3.36. Specifically, 93.9% of students agreed that their teachers are friendly, 93.7% of students agreed that their teachers want them to enjoy learning new things, and 96.5% of students agreed that their teachers teach them strategies to remember important mathematics procedures. In the final measurement model, there were 11 out of 17 items in this construct representing students' positive perceptions of their teachers' practices. It was expected that this construct would be a positive predictor of mathematics performance.

5.1.3.8 Gender differences in students' mathematics performance and beliefs

In order to test gender differences, independent samples *t*-tests were performed for gender based on the scores of each type of mathematics performance as well as the means of each belief construct/sub-construct obtained from the measurement model. This section presents the results of these tests. Table 5.5 presents the results of the independent samples *t*-tests for gender.

Female students tended to believe that mathematics knowledge is useful more than male students. They also tended to have a more favourable perception of their mathematics teacher than male students. Additionally, male students tended to believe more strongly that mathematical problem-solving is about memorising procedures than female students. However, the results showed that there is only slightly different in these beliefs and perception between female and male students. Female Vietnamese students tended to perform better in the district mathematics test, but worse in PISA-based test, than their male counterparts. No differences were found in other belief constructs or in curriculum-based tests.

Table 5.5. Gender differences in mathematics performance and mathematics-related beliefs

No.	Construct	Female (N = 285)		Male (N = 324)		*t*	*p*
		M	*SD*	*M*	*SD*		
1	Beliefs that mathematics knowledge is certain	1.90	0.55	1.98	0.55	- 1.83	.07
2	Beliefs that mathematics is useful	3.45	0.39	3.36	0.44	2.32	.02*
3	Positive perception of mathematics teacher	3.41	0.38	3.32	0.37	2.62	.01*
4	Beliefs that mathematical problem solving is about understanding procedures	3.46	0.48	3.39	0.52	1.69	.09
5	Beliefs that mathematical problem solving is about memorising procedures	1.93	0.52	2.03	0.52	- 2.29	.02*
6	Beliefs that mathematics ability can be changed	3.34	0.38	3.36	0.42	- 0.83	.41
7	Beliefs that effort is important	3.29	0.39	3.33	0.39	- 1.09	.28
8	Self-efficacy beliefs	2.78	0.48	2.84	0.49	- 1.61	.11
9	District mathematics exam	6.68	1.44	6.29	1.33	3.44	.00*
10	Curriculum-based test	3.06	1.12	3.13	1.10	-0.78	.43
11	PISA-based test	2.09	1.50	2.35	1.51	-2.05	.04*

*Note: * significant*

5.1.4 Assessment of mathematics performance

Students' mathematics performance was assessed based on three different tests: the district mathematics exam, the Vietnamese mathematics curriculum-based test, and the PISA-based test. This section presents the results of these assessments.

5.1.4.1 Students' performance in the district mathematics exam

This exam was designed by the Bureau of Education and Training from each district to assess all ninth grade students' mathematics learning within each district at the end of each semester. The exam covered all important aspects of mathematics in the mathematics curriculum that students were instructed in during the first semester. The range of these scores was from zero (0) to ten (10). Figure 5.1 shows the distribution of students' scores in this exam.

This variable was treated as a continuous variable because it contained all possible numbers from zero (0) to ten (10). On average, students attained high scores on the exam (mean = 6.47). The analysis from boxplots and the normality showed that this variable had no outliers and a normal distribution (value of skewness was 0.169 and value of kurtosis was -0.700).

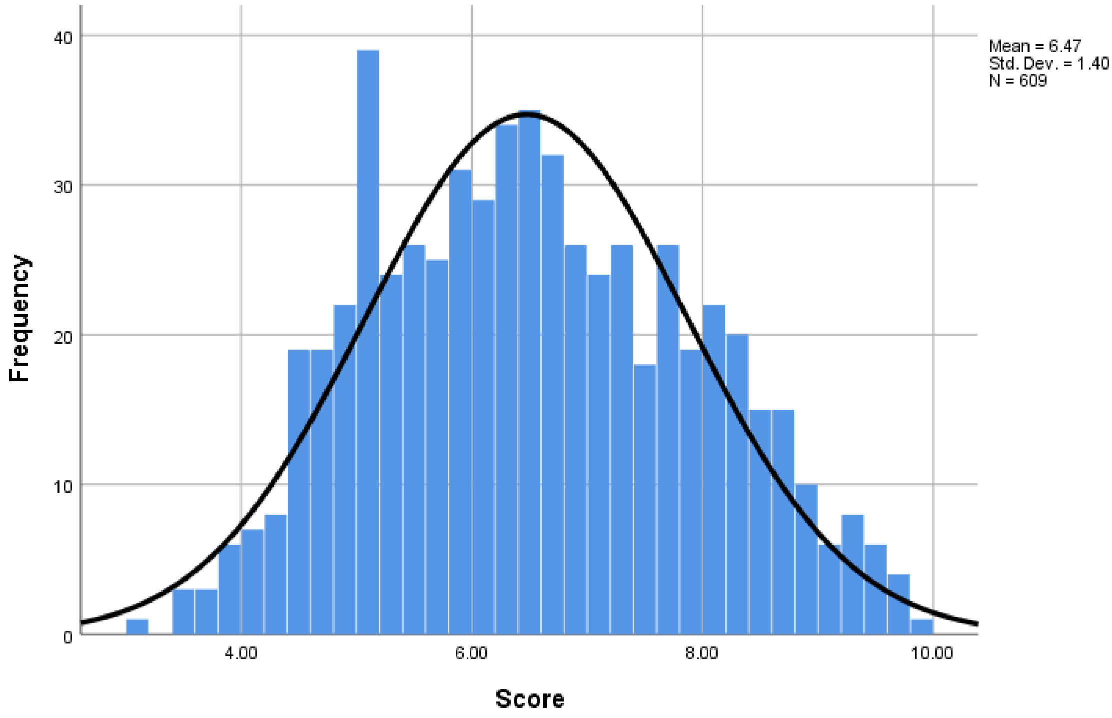


Figure 5.1: Distribution of the students' scores in the district exam

5.1.4.2 Students' performance in the curriculum-based test

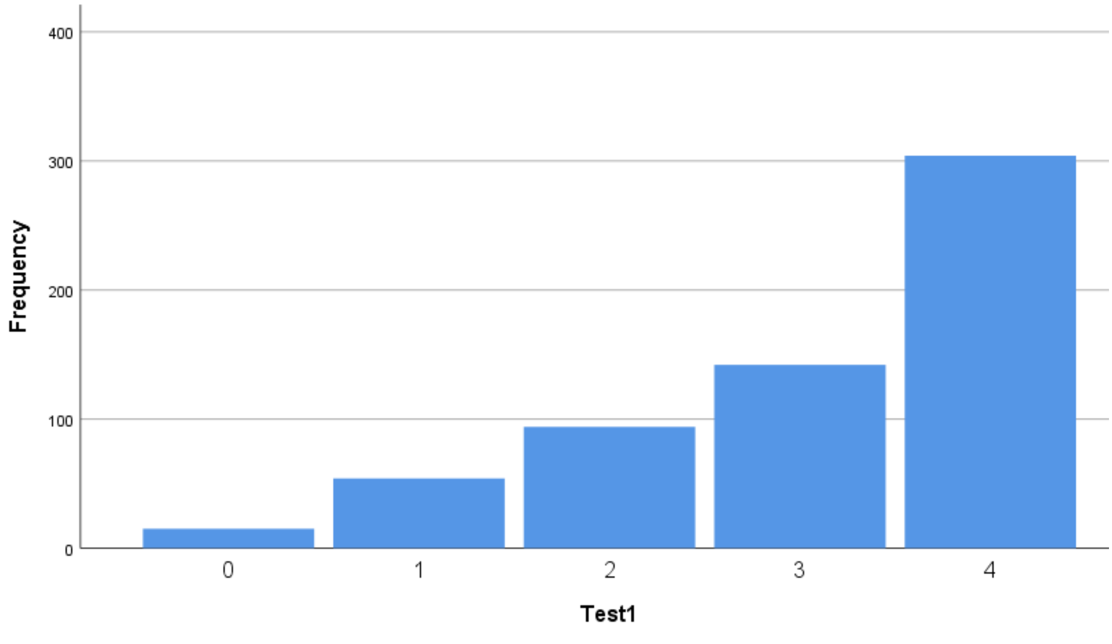


Figure 5.2: Distribution of the students' scores in the curriculum-based test

The curriculum-based performance test assessed students' ability to solve problems similar to those within the current mathematics curriculum at schools. The range of scores was zero (0) to four (4). This variable was treated as a categorical variable. There were four multiple-choice items. A correct answer was marked as one and an incorrect answer as zero. It can be seen that half of the students in the sample answered all four items correctly. Only 2.5% of students answered all four items incorrectly. Table 5.6 and Figure 5.2 show these results.

Table 5.6. Descriptive statistics for the curriculum-based and PISA-based mathematics performance test

Score	Curriculum-based test		PISA-based test	
	Frequency	**Percent**	**Frequency**	**Percent**
0	15	2.5	63	10.3
1	54	8.9	145	23.8
2	94	15.4	186	30.5
3	142	23.3	107	17.6
4	304	49.9	43	7.1
5			43	7.1
6			22	3.6
Total	**609**	**100**	**609**	**100**

5.1.4.3 Students' performance in the PISA-based test

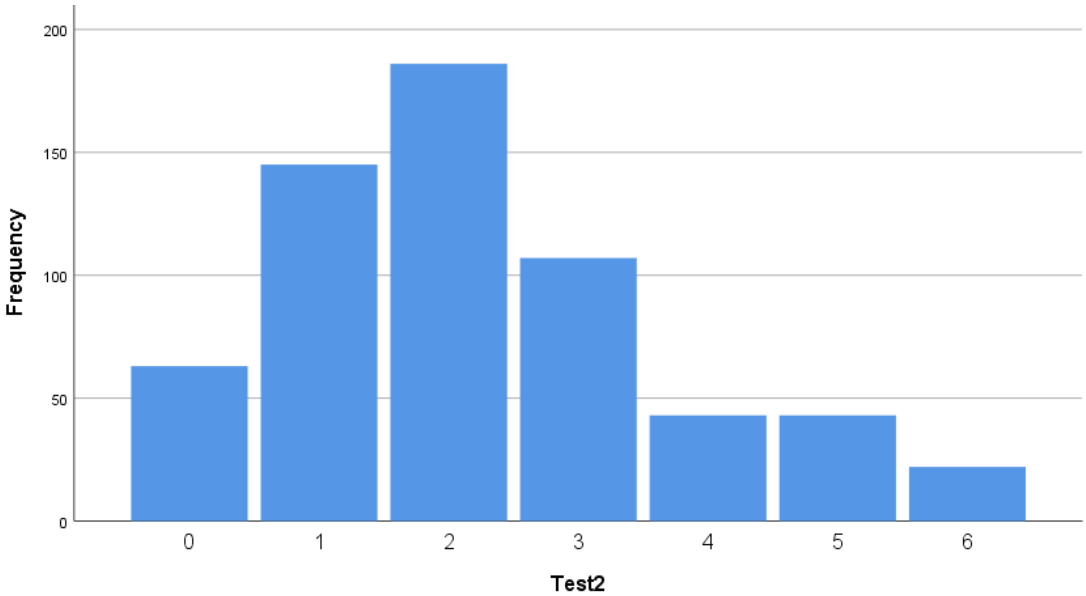


Figure 5.3: Distribution of the students' scores in the PISA-based test

This assessment of students' mathematics performance measured their ability to solve real-world problems based on problems from the PISA test. The range of scores was zero (0) to six (6) for 4 items. As shown in Table 5.6 and Figure 5.3, only 3.6% of students all items correctly to achieve a score of six, 7.1% of students got a score of five, and 10.3% of students scored zero. These results revealed that this part of the test proved difficult for the students.

Table 5.7 shows the detailed answers of the students for each item in the PISA-based mathematics performance test. Exploring the results in detail indicates that most students only did well on items 5 and 7 because these items were quite easy. For item 6, only 12.8% of students answered the problem comprehensively and correctly, whereas 52.4% students answered with an incorrect solution. For item 8, only 10.5% of students got the right answer and 82.6% of students got a completely wrong answer.

Table 5.7. Descriptive statistics for items in the PISA-based mathematics performance test

Item	Score		
	0	1	2
Item 5: Filling the blank	234 (38.4%)	375 (61.6%)	
Item 6: Solving a simple quadratic equation	319 (52.4%)	212 (34.8%)	78 (12.8%)
Item 7: Applying trigonometric ratios in right-angled triangle	165 (27.1%)	444 (72.9%)	
Item 8: solving problem with multi-steps	503 (82.6%)	42 (6.9%)	64 (10.5%)

5.1.4.4 Correlations among the three mathematics performance tests

There were significant correlations in students' performance in the three different tests as shown in Table 5.8.

Table 5.8. Correlations in students' performance in the three mathematics tests

	Score	Curriculum-based test	PISA-based test
Score	1		
Curriculum-based test	.425***	1	
PISA-based test	.479***	.475***	1

*** p < .01

It can be seen from the table above that all correlations were significant. The correlation between Score and Curriculum-based test was lowest and the correlation between Score and PISA-based test was highest.

5.1.4.5 Differences in mathematics-related beliefs between low and high performing students

One of the questions of the present research was to assess whether the beliefs of students who did well in the PISA-based test were in any way different from those who did not do well in this assessment. These students were of particular interest, because they demonstrated that they were able to transfer their mathematical knowledge to unfamiliar situations. To assess this, the sample of students was divided into two groups. The first group included students who received high scores (from 5 to 6) in the PISA- based test (high performers). This group represented students who could use their mathematics knowledge learnt in school to solve unfamiliar real-life problems. The second group included students with scores lower than 5 (low performers).

Table 5.9 shows the results of the independent samples *t*-tests that compared the mathematics-related beliefs for the groups of low and high performing students.

Table 5.9. Differences in mathematics-related beliefs between the low and high performing students

No.	Construct	Low performer (N = 544)		High performer (N = 65)		*t*	*p*
		M	*SD*	*M*	*SD*		
1	Beliefs that mathematics knowledge is certain	1.97	0.55	1.73	0.50	3.31	.01*
2	Beliefs that mathematics is useful	3.38	0.43	3.59	0.32	- 3.74	.00*
3	Positive perception of mathematics teacher	3.35	0.38	3.44	0.32	- 1.84	.07
4	Beliefs that mathematical problem solving is about understanding procedures	3.40	0.51	3.60	0.36	- 3.02	.00*
5	Beliefs that mathematical problem solving is about memorising procedures	2.02	0.52	1.68	0.48	5.17	.00*
6	Beliefs that mathematics ability can be changed	3.34	0.40	3.46	0.35	- 2.22	.03*
7	Beliefs that effort is important	3.31	0.40	3.37	0.31	- 1.31	.19
8	Self-efficacy beliefs	2.80	0.49	2.95	0.43	- 2.41	.02*

*Note: * significant*

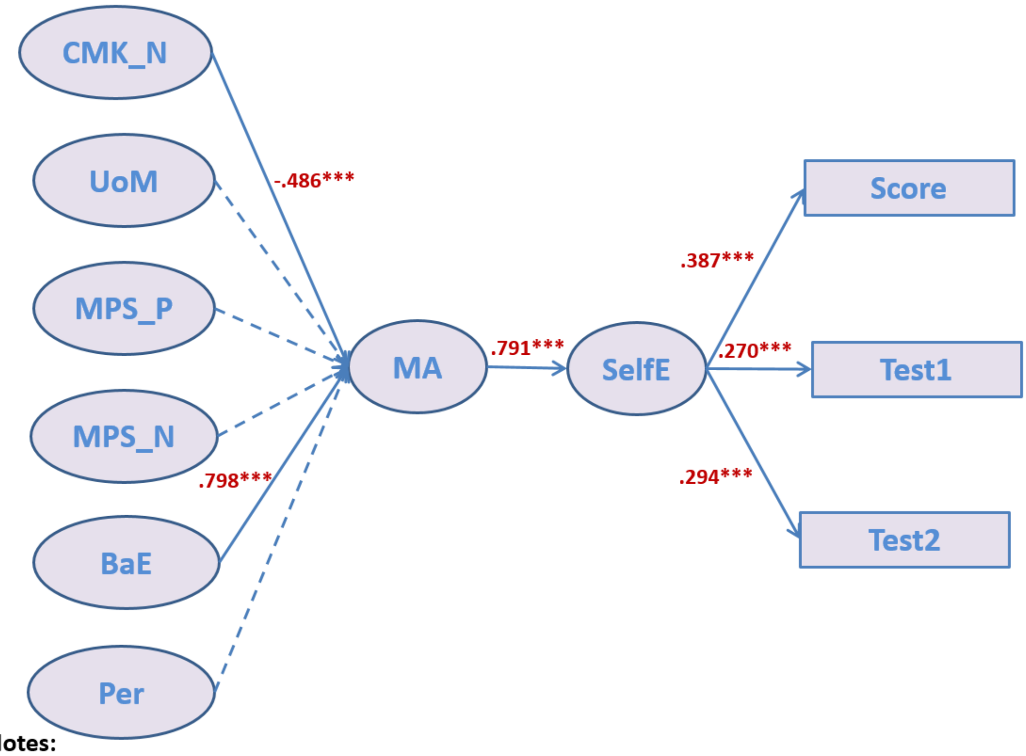
The results showed that low performers tended to believe that mathematics is certain and that mathematical problem-solving is about memorising procedures/rules more so than high performers. High performers tended to believe that mathematics is useful, that mathematical problem-solving is about understanding procedures, and that mathematics ability can be changed more strongly than low performing students. High performers also had higher belief in their self-efficacy. No differences were found in beliefs about the importance of effort and perceptions of mathematics teachers.

5.1.5 Structural models to investigate the influence of students' mathematics-related beliefs on their mathematics performance

Structural equation modelling was employed to investigate the direct and indirect effects of all the constructs validated in the measurement model on students' mathematics performance. The scores of the students in the three types of mathematics performance described earlier (district exam, curriculum-based test, PISA-based test) were used as measures of students' performance.

The hypothesised model predicted that students' beliefs about the certainty of mathematics knowledge, the usefulness of mathematics, mathematical problem-solving, the importance of effort as well as students' perceptions of teachers' practices would be direct predictors of students' beliefs about the self as a mathematics learner (beliefs about their mathematics ability and self-efficacy beliefs in mathematics). In turn, students' beliefs about the self as a mathematics learner were predicted to be direct predictors of students' mathematics performance.

The run of the proposed structural model revealed the following results of fit indices for the sample [χ^2 = 1920.564; df = 1008; χ^2/df = 1.905; RMSEA = 0.039; CFI = 0.891: TLI = 0.883; SRMR = 0.060], as shown in Figure 5.4.



Notes:

CMK_N-Math knowledge is certain; **UoM**-Math is useful; **MPS_P**-Problem solving is understanding procedures; **MPS_N**-Problem solving is memorizing procedures; **BaE**-Effort is important; **MA**-Math ability can be changed; **SelfE**-Self-efficacy; **Per**-Students' perception; **Score**-Students' score from district exam; **Test1**-curriculum-based test; **Test2**-PISA-based test.

*** $p < 0.01$, dash arrow: non-significant ($p > 0.1$)

Figure 5.4: Results of the first structural model investigating the influence of mathematics-related beliefs on students' mathematics performance

Some of fit indices, such as χ^2/df, RMSEA, and SRMR, were good, while the CFI and TLI indices were slightly lower than the cut-off values of 0.90. However, Hair et al. (2019) suggested that fit indices tend to change based on sample size, the number of observed variables, and complexity of the structural model. When researchers work with a large sample and a complex structural model, lower fit indices in comparison with cut-off values are acceptable (Hair et al., 2019).

As was predicted, the model showed direct positive effects of beliefs that mathematics ability is malleable on beliefs about self-efficacy as mathematics learners. Students' self-efficacy beliefs (SelfE) were direct positive predictors of students' mathematics performance in all three assessments. However, only beliefs that mathematics is certain and unchangeable (CMK_N) and beliefs about the importance of effort (BaE) predicted beliefs that mathematics ability can be changed through the learning process (MA). More specifically, beliefs that mathematics is certain and unchangeable predicted beliefs that mathematics ability can be changed through the learning process negatively, while beliefs that effort is important predicted beliefs that mathematics ability can be changed through the learning process positively.

In view of the above results, a modified model was developed to test a more complex pattern of relationships among belief constructs and students' mathematics performance. Based on prior research showing that beliefs about the certainty of mathematics knowledge, the usefulness of mathematics, and students' perception of teacher practices may influence beliefs about mathematics problem solving (McLeod, 1989a; Schommer-Aikins, 2004; Schommer-Aikins et al., 2010), a second structural model was developed for testing. This model tested the direct effect of students' beliefs that mathematics is certain and unchangeable (CMK_N), that mathematics is useful (UoM), and students' positive perceptions of teachers' practices (Per) on beliefs that mathematical problem-solving is about understanding procedures (MPS_P), that mathematical problem-solving is about memorising procedures (MPS_N), and beliefs that effort is important (BaE). The model also tested the direct effect of these three beliefs (MPS_P, MPS_N, BaE) on beliefs that mathematics ability can be changed during the learning process (MA). The same paths among beliefs that mathematics ability can be changed during the learning process (MA), self-efficacy beliefs (SelfE) and three types of mathematics performance (district exam, curriculum-based test, PISA-based test) from the first model were also tested in the second model.

Model 2 showed good fit indices [χ^2 = 1942.239; df = 1014; χ^2/df = 1.92; RMSEA = 0.039; CFI = 0.889: TLI = 0.882; SRMR = 0.061], as depicted in Figure 5.5. The following 14 out of 16 proposed effects showed significance for the path coefficient estimations. Specifically:

(1) Beliefs that mathematics knowledge is certain and unchangeable (CMK_N) were direct positive predictors of beliefs that mathematical problem-solving is about memorising formulas and procedures.

(2) Beliefs that mathematics knowledge is certain (CMK_N) were direct positive predictors of beliefs that effort is important to improve mathematics learning (BaE).

(3) Beliefs that mathematics is useful (UoM) were direct positive predictors of beliefs that mathematical problem-solving is about understanding procedures rather than memorisation. (MPS_P).

(4) Beliefs that mathematics is useful (UoM) were direct positive predictors of beliefs about the importance of effort (BaE).

(5) Students' positive perception of teachers' practices (Per) were direct positive predictors of beliefs that mathematical problem-solving is about understanding procedures (MPS_P).

(6) Students' positive perception of teachers' practices (Per) were also positive predictors of beliefs that mathematical problem-solving is about memorising procedures (MPS_N).

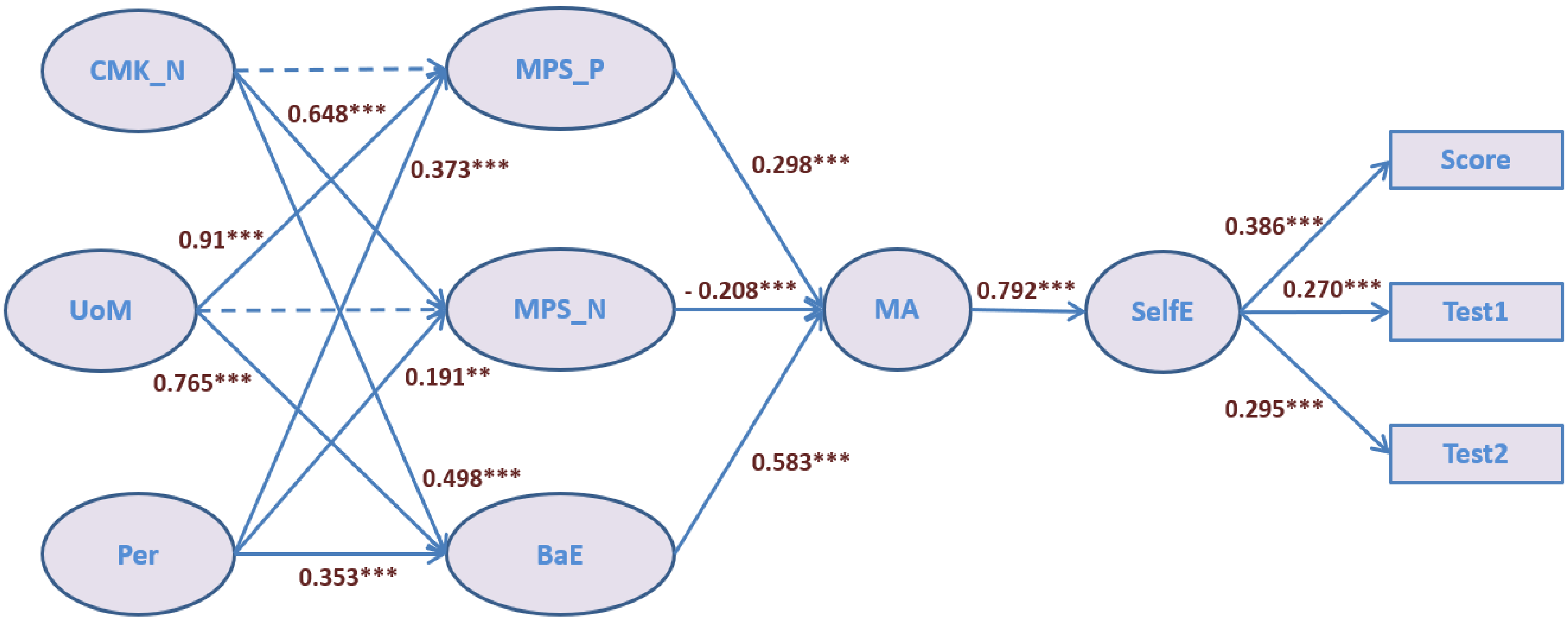
(7) Students' positive perception of teachers' practices (Per) were also positive predictors of beliefs that effort is important to improve their mathematics learning (BaE).

(8) Beliefs that mathematical problem-solving is about understanding procedures (MPS_P) and beliefs that effort is important to improve mathematics learning (BaE) were positive predictors of beliefs that mathematics ability can be changed (MA).

(9) Beliefs that mathematical problem-solving is about memorising procedures (MPS_N) were negative predictors of beliefs that mathematics ability can be changed (MA).

(10) Beliefs that mathematics ability can be changed during the learning process (MA) were positive predictors of students' self-efficacy beliefs (SelfE).

(11) Students' self-efficacy beliefs (SelfE) positively predicted the three types of mathematics performance (District exam, curriculum-based test, PISA-based test).



Notes:

CMK_N-Math knowledge is certain; **UoM**-Math is useful; **MPS_P**-Problem solving is understanding procedures; **MPS_N**-Problem solving is memorizing procedures; **BaE**-Effort is important; **MA**-Math ability can be changed; **SelfE**-Self-efficacy; **Per**-Students' perception; **Score**-Students' score from district exam; **Test1**-curriculum-based test; **Test2**-PISA-based test.

** $p < 0.05$, ***: $p < 0.01$, dash arrow: non-significant ($p > 0.1$)

Figure 5.5: Results of students' structural model 2

5.1.6 Summary of analysis of student data

This section presented the results of the analysis of students' responses to the MBQ-S and the influence of mathematics-related beliefs on their mathematics performance. First, after examining the outliers and normality of the distribution of students' data, a valid measurement model was obtained, consisting of 44 items belonging to eight different constructs of students' mathematics-related beliefs.

The influence of students' mathematics-related beliefs on their mathematics performance was investigated using structural equation modelling. Both model 1 and model 2 showed that beliefs that mathematics ability is malleable were positive predictors of beliefs in mathematics self-efficacy. Self-efficacy in mathematics was, in turn, a predictor of mathematics performance in all three assessments. Model 2 provided important information about the beliefs that predicted beliefs about mathematics ability and self-efficacy beliefs. An important predictor of beliefs that mathematics ability is changeable were beliefs in problem-solving. Specifically, beliefs that problem-solving is about understanding were positive predictors of beliefs that mathematics ability is changeable, whereas beliefs that problem-solving is about memorisation were negative predictors of the belief that mathematics ability is changeable. In turn, beliefs that mathematics knowledge is certain and beliefs about the usefulness of mathematics were important predictors of beliefs about problem-solving. Particularly, beliefs that mathematics is useful were positive predictors of the belief that mathematical problem-solving is about understanding procedures, whereas beliefs that mathematics is certain were positive predictors of beliefs that mathematics problem-solving is about memorisation. Another important construct was beliefs in the importance of effort. It appears that all Vietnamese student participants had strong beliefs that effort is important. Specifically, while beliefs that effort is important were positively predicted by beliefs that mathematics knowledge is certain, beliefs that mathematics is useful, positive perceptions of teachers' practices, these beliefs were positive predictors of beliefs that mathematics ability is changeable.

5.2 Analysis of Teacher Data

This section consists of three parts. The first part reports the results of a CFA analysis to validate the MBQ-T. The second part presents the characteristics of Vietnamese teachers' mathematics-related beliefs. The third part presents the results of a multilevel structural model that investigated the relationships between teachers' mathematics-related beliefs and students' mathematics performance.

5.2.1 Validation of the MBQ-T

5.2.1.1 Confirmatory Factor Analysis

Confirmatory factor analysis was used to validate the MBQ-T. Due to the small sample size (n=46), IBM SPSS Statistics 25.0 was used rather than Mplus 8.4. As discussed in a previous chapter, each belief construct in the MBQ-T consisted of two sub-constructs. The first sub-construct included items representing beliefs that were assumed to be positive predictors of students' mathematics performance, and the second sub-construct included items representing beliefs that were assumed to be negative predictors of students' mathematics performance.

In terms of the results from CFA analysis, Model 1 was run using all items within each sub-construct. This model showed that most items within each construct had good factor loadings, however, there were some items with low factor loadings. Specifically, beliefs that mathematics knowledge is malleable (CMKt_P), beliefs that mathematics is not useful (UoMt_N), beliefs that mathematical problem-solving is about memorising procedures (MPS_N), teachers' high self-efficacy beliefs (SelfE_P), and beliefs that students' mathematics ability can be changed during learning process (MAt_P) all had items with factor loadings over 0.4. Additionally, beliefs that mathematics knowledge is certain (CMKt_N), beliefs that mathematics is useful (UoMt_P), teachers' beliefs in transmissive teaching of mathematics (Teach_N), teachers' low self-efficacy beliefs (SelfEt_N), and beliefs about the role of students' effort in their learning (BaEt) had only one item with a factor loading of under 0.4. The rest of the sub-constructs had two items with factor loadings under 0.4; beliefs that mathematical problem-solving is about understanding procedures (MPSt_P), beliefs in constructive teaching of mathematics (Teach_P), and beliefs that students' mathematics ability is innate (MAt_N).

Model 2 was run by using only items with factor loadings of 0.4 or higher. The details of factor loadings for all sub-constructs from Model 1 and Model 2 are provided in Table 5.10.

Table 5.10. Factor loadings for each construct of MBQ-T for Model 1 and Model 2

	Total Item No.	Construct item No.	Statement	Factor loadings	
				Model 1	Model 2
I. Beliefs About the Certainty of Mathematics Knowledge (CMKt)					
1.1. Beliefs that mathematics is malleable					
CMKt_P	1	CMKt3	It is better when a math teacher shows students lots of different ways to look at the same problem (+)	0.589	0.589

	2	CMKt4	Mathematical theories are the product of creativity (+)	0.832	0.832
	3	CMKt8	Answers to questions in math change as mathematicians gather more information (+)	0.555	0.555
1.2. Beliefs that mathematics is certain and unchangeable					
CMKt_N	4	CMKt1	Most of what is true in math is already known (-)	0.449	0.500
	5	CMKt2	Math is really just knowing the right formula for the problem (-)	0.796	0.764
	6	CMKt5	In math, answers are always either right or wrong (-)	-0.184	
	7	CMKt6	There is no place for students to be creative in math class (-)	0.737	0.706
	8	CMKt7	Math problem has always only one true answer (-)	0.512	0.575
II. Beliefs About the Usefulness of Mathematics (UoMt)					
2.1. Beliefs that mathematics is useful					
UoMt_P	9	UoMt1	I teach math because I know math is useful (+)	0.534	0.517
	10	UoMt2	Knowing math will help people earn a living (+)	0.338	
	11	UoMt3	Math is a worthwhile and necessary subject (+)	0.771	0.786
	12	UoMt7	It is important to see the connections between the math students learn in class and real-world applications (+)	0.853	0.856
	13	UoMt8	Math provides the foundation for most of the principles used in science and business (+)	0.487	0.517
	14	UoMt9	Math helps us better understand the world we live in (+)	0.702	0.704
2.2. Beliefs that mathematics is not useful					
UoMt_N	15	UoMt4	Math will not be important to students in their life's work (-)	0.591	0.591
	16	UoMt5	Understanding math is important for mathematicians, economists, and scientists but not for most people (-)	0.772	0.772
	17	UoMt6	The only reason students would take a math class is because it is a requirement (-)	0.678	0.678
III. Beliefs About Mathematical Problem-Solving (MPSt)					
3.1. Beliefs that mathematical problem-solving is about understanding procedures					
MPSt_P	18	MPSt1	There are math problems that just can't be solved by following a predetermined sequence of steps (+)	0.650	0.634
	19	MPSt2	Math problems can be solved without remembering formulas (+)	0.114	
	20	MPSt3	Memorising steps is not that useful for learning to solve math problems (+)	-0.097	
	21	MPSt5	Time used to investigate why a solution to a math problem works, is time well spent (+)	0.776	0.775
	22	MPSt6	A student who doesn't understand why an answer to a math problem is correct hasn't really solved the problem (+)	0.591	0.593
	23	MPSt7	In addition to getting a right answer in math, it is important to understand why the answer is correct (+)	0.692	0.713
3.2. Beliefs that mathematical problem-solving is about memorising procedures					

MPSt_N	24	MPSt4	Learning to do math problems is mostly a matter of memorising the right steps to follow (-)	0.660	0.660
	25	MPSt8	It's not important to understand why a mathematical procedure works as long as it gives a correct answer (-)	0.676	0.676
	26	MPSt9	Getting a right answer in math is more important than understanding why the answer works (-)	0.770	0.770
	27	MPSt10	It doesn't really matter whether or not students understand a math problem as long as they can get the right answer (-)	0.811	0.811
IV. Beliefs About Mathematics Teaching (Teach)					
4.1. Beliefs in constructive teaching of mathematics					
Teach_P	28	Teach1	It is important for teachers to teach students strategies to solve math problems (+)	-0.039	0.689
	29	Teach2	Students will learn math better if teachers let them have opportunities to discuss math (+)	-0.506	0.615
	30	Teach3	Teachers should teach students ways to integrate new information with their existing knowledge of math (+)	0.836	
	31	Teach7	Teachers can help students learn math when they teach them problem-solving strategies (+)	0.775	
	32	Teach10	Students learn best when they develop their mathematical problem-solving skills (+)	-0.383	0.655
	33	Teach11	The main goal of teaching math is to help students develop learning strategies in math (+)	-0.517	0.777
4.2. Beliefs in transmissive teaching of mathematics					
Teach_N	34	Teach4	Teaching math involves mostly the transmission of math knowledge from teachers to students (-)	0.264	
	35	Teach5	Telling students the correct answers in math is the most important task for teachers (-)	0.723	0.750
	36	Teach6	The main goal of teaching math is to increase the amount of knowledge in the students' memory (-)	0.809	0.805
	37	Teach8	Repeating math knowledge in class is necessary for students learning (-)	0.584	0.581
	38	Teach9	Math learning depends mostly on how good the teacher is to tell students what they need to know about math (-)	0.711	0.712
V. Teachers' Self-Efficacy Beliefs (SelfEt)					
5.1. High self-efficacy beliefs					
SelfE_P	39	SelfEt1	I am certain I can teach math concepts effectively (+)	0.761	0.761
	40	SelfEt2	I have no difficulties answering students' math questions (+)	0.869	0.869
	41	SelfEt3	I am certain that I am a good teacher of math (+)	0.688	0.688
5.2. Low self-efficacy beliefs					
SelfEt_N	42	SelfEt4	I often wonder whether I am effective in monitoring math activities (-)	0.716	0.793
	43	SelfEt5	Sometimes I doubt whether I understand math concepts well enough to be an effective math teacher (-)	0.738	0.726

	44	SelfEt6	I wonder if I have the necessary skills to teach math (-)	0.812	0.752
	45	SelfEt7	Given a choice, I will not invite the principal to evaluate my math teaching (-)	0.508	0.528
	46	SelfEt8	Often I do not know what to do to turn students on to math (-)	0.388	
VI. Beliefs About the Role of Students' Effort in Their Learning (BaEt)					
BaEt	47	BaEt1	By trying hard, one can become smarter in math (+)	0.319	
	48	BaEt2	Studying hard can improve one's ability in math (+)	0.654	0.719
	49	BaEt3	Students can get smarter in math by trying hard (+)	0.781	0.786
	50	BaEt4	Ability in math increases when one knows the right strategies (+)	0.635	0.630
	51	BaEt5	Appropriate study skills can increase one's ability to do math (+)	0.727	0.728
	52	BaEt6	Students can get smarter in math if they know how to learn math problems (+)	0.496	0.445
VII. Beliefs About Students' Mathematics Ability (MAt)					
7.1. Beliefs that students' mathematics ability can be changed					
MAt_P	53	MAt1	Better study habits are the key to success for students who struggle in math (+)	0.738	0.738
	54	MAt2	Student who doesn't have high natural ability in math is still capable of learning difficult math material (+)	0.519	0.519
	55	MAt3	Learning good study skills can improve a students' math ability (+)	0.654	0.654
7.2. Beliefs that students' mathematics ability is innate					
MAt_N	56	MAt4	Some people are born with great math ability and some aren't (-)	0.396	
	57	MAt5	Math ability is really just something students are born with (-)	0.715	0.611
	58	MAt6	The smartest math students don't have to do many problems because they just get them quickly (-)	0.709	0.811
	59	MAt7	It is frustrating for students to have to work hard to understand a problem (-)	0.435	0.539
	60	MAt8	Students can learn new things in math, but they can't really change the math ability they were born with (-)	0.190	
	61	MAt9	Most people know at an early age whether they are good at math or not (-)	0.731	0.738

After the CFA analysis process, the regression estimates of sub-constructs for each teacher in the sample were calculated. Due to the small sample size of mathematics teachers, instead of using all items within constructs after CFA to run the multilevel analysis, these regression estimates were treated as factor scores for these sub-constructs. In the final measurement model, there were 50 out of 61 items representing 13 different sub-constructs of teachers' mathematics-related beliefs.

5.2.1.2 Reliability

CR was calculated based on the results of factor loadings within each sub-construct. Table 5.11 shows the CR of each sub-construct of teachers' mathematics-related beliefs.

Table 5.11. Construct reliability of each teachers' belief sub-construct

No	Construct	CR
1	Beliefs that mathematics knowledge is malleable (CMKt_P)	0.70
2	Beliefs that mathematics knowledge is certain and unchanging (CMKt_N)	0.73
3	Beliefs that mathematics is useful (UoMt_P)	0.84
4	Beliefs that mathematics is useful (UoMt_N)	0.72
5	Beliefs that mathematical problem-solving is about understanding procedures (MPSt_P)	0.78
6	Beliefs that mathematical problem-solving is about memorising procedures (MPSt_N)	0.82
7	Beliefs in constructive teaching of mathematics (Teach_P)	0.78
8	Beliefs in transmissive teaching of mathematics (Teach_N)	0.81
9	High self-efficacy beliefs (SelfEt_P)	0.82
10	Low self-efficacy beliefs (SelfEt_N)	0.80
11	Beliefs that students' effort is important (BaEt)	0.80
12	Beliefs that students' mathematics ability can be changed (Mat_P)	0.68
13	Beliefs that students' mathematics ability is innate (Mat_N)	0.77

Hair et al. (2019) suggested that CR should be greater than 0.60 to show good reliability. Results showed that all 13 sub-constructs had CR over 0.60. This suggested that these sub-constructs were reliable.

The next section explores what Vietnamese mathematics teachers believed within each sub-construct of teachers' mathematics-related beliefs included in the present research.

5.2.2 Characteristics of Vietnamese teachers' mathematics-related beliefs

Similar to the MBQ-S, the researcher also used IBM SPSS Statistics 25.0 to explore the data from the MBQ-T. In general, mathematics teachers had different levels of agreement for each statement in the questionnaire. The descriptive statistics of teachers' data for each belief construct is provided in Table 5.12, including the percentages of agreement for each item, the mean of each item, and each sub-construct of teachers' mathematics-related beliefs. Other information about descriptive statistics of teachers' data is provided in Appendix G.

Table 5.12. Percent and mean agreement of teachers with items representing belief constructs

	No.	Construct Name	Statement	% of agreement	Mean	SD
I. Beliefs About the Certainty of Mathematics Knowledge (CMKt)						
1.1. Beliefs that mathematics is malleable					**3.05**	
CMKt_P	1	CMKt3	It is better when a math teacher shows students lots of different ways to look at the same problem (+)	100%	3.54	0.504
	2	CMKt4	Mathematical theories are the product of creativity (+)	83.70%	3.07	0.632
	3	CMKt8	Answers to questions in math change as mathematicians gather more information (+)	53.30%	2.53	0.661
1.2. Beliefs that mathematics is certain and unchangeable					**2.15**	
CMKt_N	4	CMKt1	Most of what is true in math is already known (-)	24.40%	2.18	0.535
	5	CMKt2	Math is really just knowing the right formula for the problem (-)	15.60%	2	0.564
	6	CMKt6	There is no place for students to be creative in math class (-)	2.20%	1.78	0.471
	7	CMKt7	Math problem has always only one true answer (-)	63%	2.63	0.645
II. Beliefs About the Usefulness of Mathematics (UoMt)						
2.1. Beliefs that mathematics is useful					**3.26**	
UoMt_P	8	UoMt1	I teach math because I know math is useful (+)	77.30%	3.09	0.741
	9	UoMt3	Math is a worthwhile and necessary subject (+)	97.80%	3.37	0.532
	10	UoMt7	It is important to see the connections between the math students learn in class and real-world applications (+)	97.80%	3.46	0.622
	11	UoMt8	Math provides the foundation for most of the principles used in science and business (+)	95.70%	3.24	0.524
	12	UoMt9	Math helps us better understand the world we live in (+)	84.80%	3.13	0.653
2.2. Beliefs that mathematics is not useful					**2.19**	
UoMt_N	13	UoMt4	Math will not be important to students in their life's work (-)	8.70%	1.96	0.469
	14	UoMt5	Understanding math is important for mathematicians, economists, and scientists but not for most people (-)	40%	2.38	0.614
	15	UoMt6	The only reason students would take a math class is because it is a requirement (-)	28.30%	2.22	0.629
III. Beliefs About Mathematical Problem-Solving (MPSt)						
3.1. Beliefs that mathematical problem-solving is about understanding procedures					**3.17**	
MPSt_P	16	MPSt1	There are math problems that just can't be solved by following a predetermined sequence of steps (+)	91.30%	3.09	0.509
	17	MPSt5	Time used to investigate why a solution to a math problem works, is time well spent (+)	97.80%	3.18	0.442

	18	MPSt6	A student who doesn't understand why an answer to a math problem is correct hasn't really solved the problem (+)	77.80%	2.98	0.657
	19	MPSt7	In addition to getting a right answer in math, it is important to understand why the answer is correct (+)	100%	3.41	0.498
3.2. Beliefs that mathematical problem-solving is about memorising procedures					**2.27**	
MPSt_N	20	MPSt4	Learning to do math problems is mostly a matter of memorising the right steps to follow (-)	15.20%	2.04	0.595
	21	MPSt8	It's not important to understand why a mathematical procedure works as long as it gives a correct answer (-)	56.80%	2.55	0.548
	22	MPSt9	Getting a right answer in math is more important than understanding why the answer works (-)	45.50%	2.45	0.791
	23	MPSt10	It doesn't really matter whether or not students understand a math problem as long as they can get the right answer (-)	15.60%	2.04	0.52
IV. Beliefs About Mathematics Teaching (Teach)						
4.1. Beliefs in constructive teaching of mathematics					**3.20**	
Teach_P	24	Teach1	It is important for teachers to teach students strategies to solve math problems (+)	100%	3.37	0.488
	25	Teach2	Students will learn math better if teachers let them have opportunities to discuss math (+)	95.60%	3.24	0.529
	26	Teach10	Students learn best when they develop their mathematical problem-solving skills (+)	95.70%	3.35	0.566
	27	Teach11	The main goal of teaching math is to help students develop learning strategies in math (+)	73.90%	2.85	0.595
4.2. Beliefs in transmissive teaching of mathematics					**2.79**	
Teach_N	28	Teach5	Telling students the correct answers in math is the most important task for teachers (-)	69.60%	2.72	0.584
	29	Teach6	The main goal of teaching math is to increase the amount of knowledge in the students' memory (-)	69.60%	2.8	0.619
	30	Teach8	Repeating math knowledge in class is necessary for students learning (-)	88.40%	3.05	0.532
	31	Teach9	Math learning depends mostly on how good the teacher is to tell students what they need to know about math (-)	54.60%	2.58	0.621
V. Teachers' Self-Efficacy Beliefs (SelfEt)						
5.1. High self-efficacy beliefs					**2.56**	
SelfEt_P	32	SelfEt1	I am certain I can teach math concepts effectively (+)	60.90%	2.63	0.679
	33	SelfEt2	I have no difficulties answering students' math questions (+)	53.30%	2.56	0.624
	34	SelfEt3	I am certain that I am a good teacher of math (+)	45.50%	2.5	0.591

5.2. Low self-efficacy beliefs					**2.61**	
SelfEt_N	35	SelfEt4	I often wonder whether I am effective in monitoring math activities (-)	78.30%	2.83	0.486
	36	SelfEt5	Sometimes I doubt whether I understand math concepts well enough to be an effective math teacher (-)	43.50%	2.41	0.617
	37	SelfEt6	I wonder if I have the necessary skills to teach math (-)	56.50%	2.57	0.501
	38	SelfEt7	Given a choice, I will not invite the principal to evaluate my math teaching (-)	52.20%	2.61	0.649
VI. Beliefs About the Role of Students' Effort in Their Learning (BaEt)					**3.15**	
BaEt	39	BaEt2	Studying hard can improve one's ability in math (+)	95.70%	3.09	0.412
	40	BaEt3	Students can get smarter in math by trying hard (+)	95.70%	3.22	0.513
	41	BaEt4	Ability in math increases when one knows the right strategies (+)	100%	3.3	0.465
	42	BaEt5	Appropriate study skills can increase one's ability to do math (+)	93.30%	3.16	0.52
	43	BaEt6	Students can get smarter in math if they know how to learn math problems (+)	88.90%	2.96	0.52
VII. Beliefs About Students' Mathematics Ability (MAt)						
7.1. Beliefs that students' mathematics ability can be changed					**2.90**	
MAt_P	44	MAt1	Better study habits are the key to success for students who struggle in math (+)	93.20%	3.18	0.54
	45	MAt2	Students who do not have high natural ability in math are still capable of learning difficult math material (+)	44.40%	2.36	0.712
	46	MAt3	Learning good study skills can improve a students' math ability (+)	95.70%	3.15	0.47
7.2. Beliefs that students' mathematics ability is innate					**2.25**	
MAt_N	47	MAt5	Math ability is really just something students are born with (-)	2.20%	1.93	0.447
	48	MAt6	The smartest math students don't have to do many problems because they just get them quickly (-)	67.40%	2.67	0.701
	49	MAt7	It is frustrating for students to have to work hard to understand a problem (-)	43.20%	2.48	0.664
	50	MAt9	Most people know at an early age whether they are good at math or not (-)	4.40%	1.93	0.393

5.2.2.1 Beliefs about the Certainty of Mathematics Knowledge (CMKt)

It can be seen from Table 5.12 the Vietnamese mathematics teachers in the sample agreed with items expressing beliefs that mathematics knowledge is malleable. The mean of all items representing beliefs that mathematics is malleable was 3.05 (out of 4). All teachers agreed with the item, "It is better when a math teacher shows students lots of different ways to look at the same problem", however, the statement, "Answers to questions in math change as mathematicians gather more information" received agreement from only 53% of the teachers. Although teachers expressed less agreement with items representing representing beliefs that mathematics is certain (mean agreement of 2.15), over half of the teachers believed that a mathematics problem always has only one true answer. It appears, overall, that the teachers had some conflicting beliefs regarding the certainty, or lack thereof, of mathematical knowledge.

5.2.2.2 Beliefs about the Usefulness of Mathematics (UoMt)

The overwhelming majority of teachers in the sample believed that mathematics is useful. The mean agreement with items representing beliefs that mathematics is useful was 3.26. 95% of teachers believed that mathematics is a worthwhile and necessary subject, and that it is important to see the connections between the math students learn in class and real-world applications.

However, some teachers also agreed with statements expressing beliefs that mathematics is not useful for all people. For example, 40% of teachers believed that understanding mathematics is important for mathematicians, economists, and scientists but not for most people.

5.2.2.3 Beliefs about Mathematical Problem-solving (MPSt)

The Vietnamese mathematics teachers also seemed to have some conflicting beliefs regarding the nature of mathematical problem-solving. Although the teachers tended to agree with items expressing the belief that problem-solving is about understanding rather than memorisation, (mean agreement 3.17), a considerable number of teachers also agreed with items expressing beliefs that mathematical problem-solving is about memorising procedures (mean agreement 2.27). For example, most teachers agreed with items such as "Time used to investigate why a solution to a math problem works, is time well spent" (item MPSt5) and "In addition to getting a right answer in math, it is important to understand why the answer is correct" (item MPSt8). However, 56.80% of teachers also agreed with the statement "It's not important to understand why a mathematical procedure works as long as it gives a correct answer" and 45.50% agreed that "Getting a right answer in math is more important than understanding why the answer works".

5.2.2.4 Beliefs about mathematics teaching (Teach)

The results indicated that teachers had inconsistent beliefs about mathematics teaching. The mean agreement with items representing beliefs in constructive teaching was 3.20, and the mean agreement those representing beliefs in transmissive teaching was 2.79. A large percentage of teachers agreed with statements that were considered to have a negative effect on student performance. For example, more than 70% of the teachers agreed with statements such as "Telling students the correct answers in math is the most important task for teachers" (item Teach5), "the main goal of teaching math is to increase the amount of knowledge in the students' memory" (item Teach6), and "repeating math knowledge in class is necessary for students learning" (item Teach8).

5.2.2.5 Teachers' self-efficacy beliefs (SelfEt)

The mathematics teachers in the sample did not show high levels of self-efficacy in math teaching. The mean agreement with items representing high self-efficacy beliefs was only 2.56, while the mean agreement with items representing low self-efficacy beliefs was 2.61. About 50% of the teachers did not agree with the items "I have no difficulties answering students' math questions" (item SelfEt2) and "I am certain that I am a good teacher of math" (item SelfE3). Additionally, about 50% of teachers also agreed with statements such as "I often wonder whether I am effective in monitoring math activities" (item SelfEt4) and "given a choice, I will not invite the principal to evaluate my math teaching" (item SelfEt7).

5.2.2.6 Beliefs about the role of students' effort in their learning (BaEt)

Most teachers in the sample believed that students' effort is important to improve their learning. Mean agreement with items representing beliefs that effort is important was 3.15. Most teachers agreed that "studying hard can improve students' mathematics ability" (item BaEt2), and "students can get smarter in mathematics by trying hard" (item BaEt3).

5.2.2.7 Beliefs about students' Mathematics Ability (MAt)

Teachers expressed conflicting beliefs regarding the malleability of students' mathematics ability. Mean agreement with items representing beliefs that students' mathematics ability can be changed was 2.90, while the mean agreement with items representing beliefs that students' mathematics ability is innate was 2.25. Over 90% of teachers agreed with the items, "Better study habits are the key to success for students who struggle in mathematics" (item MAt1) and "Learning good study skills can improve a students' mathematics ability" (item MAt3). However, only about 45% of them also believed that "Students who do not have high natural ability in math are still capable of learning difficult math material" (item MAt2).

To sum up, it can be seen that Vietnamese mathematics teachers believed that mathematics is useful and that students' effort is important to improve mathematics learning. However, there were inconsistent beliefs regarding mathematical problem-solving, mathematics teaching, and students' mathematics ability. They also showed quite low self-efficacy beliefs.

5.2.3 SEM multilevel structural model to investigate the influence of teachers' mathematics-related beliefs on students' mathematics performance

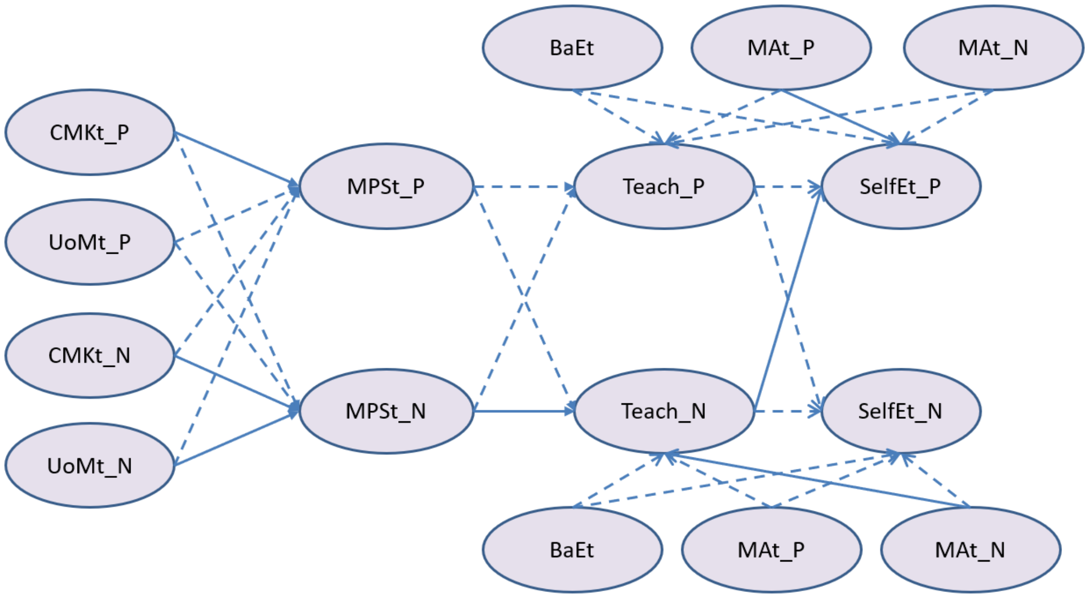
One of the purposes of the present research is to investigate the influence of teachers' mathematics-related beliefs on students' mathematics performance. SEM multilevel analysis was employed to assess this, using a process comprising two stages. In the first stage, based on the students' structural model that was found to have good fit indices, the researchers proposed a similar structural model in terms of the relationships among teachers' belief constructs. In the second step, some belief constructs were selected to run the multilevel model. This section presents the results of each stage.

5.2.3.1 SEM analysis to investigate relations between belief constructs

Structural equation modelling was used to explore the relations between the various belief constructs emerging from the analysis of the teacher data. A structural model for a similar model based on student data was also run, as previously outlined. In this model of the teachers, beliefs about the certainty of mathematics knowledge and beliefs about usefulness of mathematics were hypothesised to be predictors of beliefs about mathematical problem-solving, which in turn, these beliefs were hypothesised to be predictors of beliefs about mathematics teaching. Beliefs about mathematics teaching were hypothesised to predict teachers' self-efficacy beliefs. It was also hypothesised that teachers' beliefs about students as a mathematics learner (beliefs about the role of student effort in their learning and beliefs about students' mathematics ability) may be predictors of their beliefs about mathematics teaching and self-efficacy beliefs.

Since each belief construct consisted of two different sub-constructs, the teachers' data included responses from 46 mathematics teachers with the regression estimates of 13 beliefs sub-constructs. The results of structural equation modelling using Mplus 8.4 are depicted in Figure 5.6.

This model examined the relationships between all sub-constructs based on the proposed hypotheses for teachers' mathematics-related beliefs. The fit indices were not good for this model, [χ^2 = 71.727; df = 28; χ^2/df = 2.56; RMSEA = 0.198; CFI = 0.647: TLI = 0.282; SRMR = 0.104], due to the small sample size and multiple sub-constructs. However, this model provided some general ideas about the possible relationships between these sub-constructs.



Note: full arrow: significant (p <= 0.05); dash arrow: non-significant (p > 0.1)

Figure 5.6: Structural model of teachers' mathematics-related beliefs

It can be seen from Figure 5.6 that there were only significant paths among sub-constructs representing beliefs that were assumed to be positive predictors of students' mathematics performance and among sub-constructs representing beliefs that were assumed to be negative predictors of students' mathematics performance. Additionally, it can be seen that many paths between beliefs about the role of students' effort in their learning (BaEt) and beliefs about students' mathematics ability (both MAt_P, MAt_N) were not significantly associated with beliefs about teaching (both Teach_P, Teach_N) or teachers' self-efficacy beliefs (both SelfE_P, SelfE_N). Possible explanations for these non-significant associations include: (1) items in these sub-constructs were adapted from students' questionnaire and may not be optimal constructs for mathematics teachers; (2) there might be different relationships between these two constructs and other teachers' belief constructs that need to be further explored; and (3) the sample size of mathematics teachers was not large enough to demonstrate the proposed relationships.

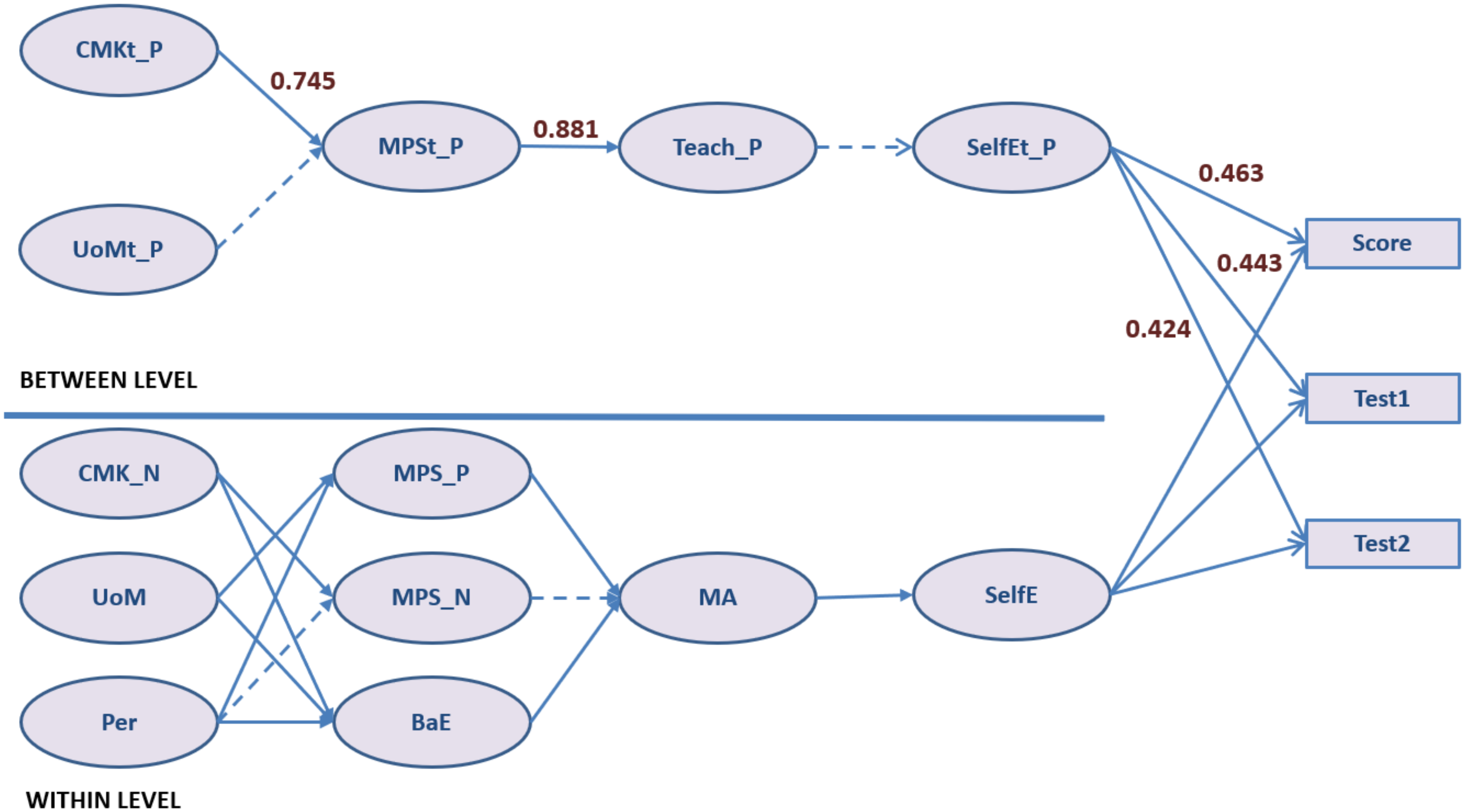
5.2.3.2 SEM multilevel analysis to explore relations between teachers' beliefs and students' mathematical performance

As mentioned in the previous chapter, of the 46 mathematics teachers in the sample, 28 teachers were teaching sixth, seventh, eighth grade students, and 18 teachers were teaching 20 ninth grade classes at the selected schools. In order to conduct SEM multilevel analysis, only data from those 18 teachers who were teaching ninth grade classes were connected with students' data because the researcher was interested in exploring whether the beliefs of these teachers could predict their students' mathematics performance.

Because of the very small number of teachers, only the sub-constructs that represented beliefs hypothesised to be positive predictors of student mathematics performance were chosen to run the multilevel model. These were: (1) beliefs that mathematics knowledge is malleable; (2) beliefs that mathematics is useful; (3) beliefs that mathematical problem solving is about understanding procedures; (4) beliefs in constructive teaching of mathematics; and, (5) high self-efficacy beliefs. Since the results from the first stage indicated that beliefs about the role of student effort (BaEt) and beliefs about students' mathematics ability (both MAt_P, MAt_N) had weak links with other belief constructs, these constructs were not included in the model.

It was hypothesised that teachers' beliefs that mathematics knowledge is malleable (CMKt_P) and beliefs that mathematics is useful (UoMt_P) would be positive predictors of beliefs that mathematical problem-solving is about understanding procedures (MPSt_P), and that beliefs that mathematical problem-solving is about understanding procedures (MPSt_P) would be positive predictors of beliefs in constructive teaching of mathematics (Teach_P). Beliefs in constructive teaching were predicted to be positive predictors of self-efficacy beliefs (SelfEt_P), and high self-efficacy beliefs (SelfEt_P) would be positive predictor of students' mathematics performance. The full model is depicted in Figure 5.7.

This model showed a good set of fit indices [χ^2 = 1309.765; df = 1033; χ^2/df = 1.268; RMSEA = 0.024; CFI = 0.912: TLI = 0.906; SRMR (student level) = 0.062, SRMR (teacher level) = 0.333]. Although SRMR (teacher level) showed a poor fit, the other fit indices satisfied the requirements of cut-off values. Figure 5.7 included two levels. The "Between level" showed the relationships among teachers' belief constructs and students' mathematics performance. The "Within level" showed the relationships among students' belief constructs and students' mathematics performance. It is worth noting that the SEM model mostly replicated the results obtained earlier about the relationships between belief constructs from student data. However, this section only focuses on the results from the teacher level.



Notes for between level:

CMKt_P-Math knowledge is malleable; **UoMt_P**-Math is useful; **MPSt_P**-Problem solving is understanding procedures; **Teach_P**-Constructive teaching; **SelfEt_P**-High self-efficacy; **Score**-Students' score from district exam; **Test1**-curriculum-based test; **Test2**-PISA-based test.

Dash arrow: non-significant ($p > 0.1$);

Figure 5.7: Structural model of students' and teachers' mathematics-related beliefs

As can be seen from Figure 5.7, results showed significant relationships among constructs of teachers' mathematics-related beliefs and students' mathematics performance. A synthesis of the key findings is as follows:

(1) Teachers' high self-efficacy beliefs were positive predictors of all three types of mathematics performance;

(2) Teachers' beliefs that mathematical problem-solving is about understanding procedures were positive predictors of their beliefs in constructive teaching of mathematics; and,

(3) Teachers' beliefs that mathematics knowledge is malleable were positive predictors of their beliefs that mathematical problem-solving is about understanding procedures.

There were non-significant path coefficients from teachers' beliefs that mathematics is useful to teachers' beliefs that mathematical problem-solving is about understanding procedures, and from teachers' beliefs in constructive teaching of mathematics to teachers' high self-efficacy beliefs.

To summarise, this section presented the results of the influence of teachers' mathematics-related beliefs on students' mathematics performance. CFA, using SPSS, was employed to assess teachers' belief constructs and to estimate the factor scores for each construct. Due to the small sample size of mathematics teachers and the complexity of the model, only five out of the thirteen teachers' belief sub-constructs (those representing beliefs assumed to be positive predictors of students' mathematics performance) were chosen to test the associations with students' mathematics performance. The results showed that teachers' high self-efficacy beliefs were positive predictors of the three measures of students' mathematics performance. Additionally, beliefs that mathematical problem-solving is about understanding procedures were positive predictors of beliefs in constructive teaching of mathematics, and beliefs that mathematics knowledge is malleable were positive predictors of beliefs that mathematical problem-solving is about understanding procedures.

5.3 Summary

In this chapter, the statistical procedures and results of the data analysis for the present research were presented. First, the preliminary data analysis resulted in the exclusion of 11 students from the original sample of 620 (n=609). Second, the results of the best measurement model for students consisted of 44 items representing eight beliefs constructs. Exploring Vietnamese students' beliefs revealed that, generally, they believed that mathematics is useful and that effort is important to improve mathematics learning. Differences in mathematics-related beliefs were found between boys and girls, as well as between high and low performers, in terms of beliefs that mathematics is useful and beliefs that mathematical problem solving is about memorising procedures.

The structural analysis of student data showed that self-efficacy beliefs positively predicted students' mathematics performance in the three types of assessments used, and beliefs about the malleability of mathematics ability were found to be predictors of beliefs in self-efficacy. Overall, beliefs that mathematics knowledge is certain, beliefs that mathematics is useful, and positive perceptions of teachers' practices were important predictors of beliefs about problem-solving and the importance of effort. Additionally, beliefs about problem-solving and beliefs about the importance of effort directly predicted beliefs about the malleability of mathematics ability.

The Vietnamese mathematics teachers expressed conflicting beliefs regarding the nature of mathematics knowledge, mathematics teaching, and mathematical problem solving, and some uncertainty in their self-efficacy as teachers. The final multilevel structural equation modelling showed that teachers' high self-efficacy beliefs directly predicted students' mathematics performance. Additionally, positive path was found between beliefs that mathematical problem-solving is about understanding procedures and beliefs in constructive teaching of mathematics, as well as between beliefs that mathematics knowledge is malleable and beliefs that mathematical problem-solving is about understanding procedures.

CHAPTER 6 DISCUSSION AND CONCLUSIONS

This chapter consists of three main sections. The first section discusses the results of the research relevant to the mathematics-related beliefs of the Vietnamese students and the influence of these beliefs on their mathematics performance. The second section discusses the results of the research concerning beliefs of the Vietnamese mathematics teachers and the influence of these beliefs on their students' mathematics performance. The last section discusses educational implications of the research for the teaching and learning of mathematics in Vietnam and directions for future research.

6.1 Vietnamese students' mathematics-related beliefs and the influence of these beliefs on their mathematics performance

The research revealed a complex pattern of relations amongst various mathematics-related beliefs of the Vietnamese students. Some of these beliefs were found to be direct or indirect positive predictors of mathematics performance while others were negative predictors. Overall, self-efficacy was found to be a direct predictor of the students' mathematics performance, while beliefs that mathematics ability can be changed and improved were a direct predictor of self-efficacy in mathematics. The structural equation model showed a positive path that connected beliefs about effort and perceptions of teachers to self-efficacy. More specifically, the positive perceptions of teachers by the students were found to be a positive predictor of beliefs in the importance of effort, leading to beliefs in the changeability of mathematics ability and self-efficacy. Another positive path led from beliefs about the nature of mathematics and mathematical problem-solving to self-efficacy and mathematics performance. These were the beliefs that mathematics is useful and that mathematics problem solving is about understanding rather memorisation of rules. Negative predictors of the beliefs of self-efficacy and that mathematics ability can be changed were the beliefs that mathematics is certain and unchangeable and that problem-solving is about memorisation of rules and procedures.

Overall, the Vietnamese students in the present sample expressed strong agreement with many beliefs that were positive predictors of mathematics performance, such as high self-efficacy beliefs, beliefs that mathematics ability can be changed, beliefs in the importance of effort and positive perceptions of their teachers' practices. They also agreed with statements indicating that mathematics is useful but tended to agree more strongly with statements indicating that mathematics, as a discipline, is absolute and certain. The students seemed to be conflicted with respect to their beliefs regarding whether mathematical problem-solving is more about memorisation of rules or about understanding of rules.

In this section, the mathematics-related beliefs that were positive predictors of mathematics performance will first be discussed, followed by a discussion of the mathematics-related beliefs that were negative predictors of mathematics performance. First, some issues related to mathematics performance will be discussed.

Mathematics performance. Mathematics performance was assessed by examining students' scores in three tests. One was a district mathematics test which was given to all ninth grade students in that particular district in Vietnam and which tested their mathematical knowledge based on the curriculum taught for that year. The other two tests were designed specifically for the purposes of the present research. One of these was a curriculum-based test and the other was a test which used problems from the OECD PISA exam. The results showed that the students' performance in the district exam correlated positively with their performance in the two tests designed for the purposes of the present study. Overall, the students did well on the curriculum-based test but they had quite poor performance on the PISA-based test. This was an expected finding given that the problems taken from the OECD PISA exam were unfamiliar to the students. In interviews conducted after the testing, many of the students reported that they thought they did well only on the curriculum-based test and that they were not familiar with the real-world problems used in the PISA exam. Many of them stated that this was the first time they have seen such problems in mathematics. The finding that the students did poorly in this test indicated that few of them were able to transfer their mathematics knowledge to new situations that were different from the school contexts in which they were instructed. The SEM model investigated predicted students' performance in all three tests.

Relations between self-efficacy, mathematics ability and mathematics performance. Two mathematics-related beliefs were found to be strong predictors of mathematics performance. These were the beliefs in high self-efficacy in mathematics and beliefs that mathematics ability can be changed and improved. In the SEM model, beliefs that mathematics ability can be changed was a direct positive predictor of self-efficacy in mathematics which, in turn, was a positive and direct predictor of mathematics performance. The finding regarding the importance of self-efficacy beliefs, and the mediating role that these beliefs play between several other mathematics-related beliefs and mathematics performance, is consistent with findings from prior research (Nasser & Birenbaum, 2005; Zarch & Kadivar, 2006), and indicates the importance of these beliefs for mathematics teaching and learning (Bandura, 1977; Liu & Koirala, 2009; Muis & Foy, 2010; Ünlü & Ertekin, 2017).

The present research showed that, overall, the Vietnamese students held high self-efficacy beliefs about mathematics (M = 2.82). Over 75% of students agreed with items stating that they were confident to ask questions in their mathematics classes and to use mathematics outside of school. However, it is also worth noting that only about 47% of the students were certain that they can do well in their mathematics tests, although they seemed to be more confident that they would do well in future mathematics courses. The finding that Vietnamese students had high self-efficacy beliefs that were significant positive predictors of their mathematics performance is consistent with the results of prior research with Vietnamese students by Sezgin (2017). In addition, this finding is also supported by the results of PISA 2012, which showed that Vietnamese students' self-efficacy beliefs were strongly associated with mathematics performance, and that Vietnamese students reported higher levels of confidence to solve mathematics problems (both pure and applied) compared to other countries in the European and Asian regions (OECD, 2013). These findings may contribute to explaining why Vietnamese students performed well on an international assessment such as PISA.

Another positive predictor of mathematics performance through the mediation of self-efficacy beliefs was the beliefs that mathematics ability is malleable and can be changed during the process of education. The Vietnamese students expressed overwhelming agreement with statements indicating that they can improve their mathematics ability, and rejected statements indicating that mathematics ability is innate. Over 90% (M = 3.34) agreed that learning good study skills can improve mathematics ability and that better study habits can make a difference. The relation between beliefs in the malleability of mathematics ability and self-efficacy is consistent with Bandura's (1993) finding that if students believe that ability is fixed or innate, they tend to have much lower levels of self-efficacy in problem-solving, and they tend to get lower academic performance. The finding is also consistent with Bonne and Johnston's (2016) results that self-efficacy beliefs are positively and significantly correlated with incremental beliefs of ability and negatively correlated with entity beliefs of ability. Dweck and Leggett (1988) also found that students who believe that the ability to learn is fixed at birth will display helpless behaviour, while those with a strong belief that the ability to learn can improve will persist and try different strategies. Additionally, the positive relationship between beliefs that mathematics ability is malleable and mathematics performance is also consistent with prior research, which found negative relationships between beliefs in innate ability and achievement (e.g., Schommer, 1993).

Prior research has shown that students in Western countries tend to more strongly believe that ability is innate and cannot be changed than their counterparts from Asian countries (Uttal, 1997). This might also explain why research with students from Asian countries shows significant relationships between beliefs that mathematics ability can be changed and mathematics performance; a result not always found in research with students from Western countries (e.g., Schommer-Aikins & Duell, 2013; Schommer-Aikins et al., 2005). It is easy to understand how beliefs that mathematics ability can be changed can have an important influence on mathematics performance. It is only if students believe that their mathematics ability can be improved that they will try harder to learn mathematics. Both prior research and the present research confirmed relationship between beliefs in the malleability of mathematics ability and effort, as will be discussed in the next sub-section.

Relations between students' beliefs about the importance of effort and teacher perception with mathematics ability, self-efficacy beliefs, and mathematics performance. The SEM model showed that students' positive perception of their teachers were positive predictors of beliefs in the importance of effort. Beliefs in the importance of effort were, in turn, positive predictors of beliefs in the malleability of mathematics ability, self-efficacy, and mathematics performance. Although this pattern of relations has not been previously reported in the literature, it is consistent with previous findings, such as Muis and Foy's (2010) argument that students' beliefs about the importance of effort have an indirect effect on mathematics performance through their beliefs in achievement goals and self-efficacy.

The Vietnamese students overwhelmingly agreed with statements that effort is important in improving mathematics learning (M = 3.31). This finding is in concert with the results of Sangcap's (2010) research with Asian students. Leung (2001) also showed that students in Eastern countries tend to work harder, and put more effort in their learning, compared to students from Western countries. Using PISA data, Parandekar and Sedmik (2016) investigated the profiles of Vietnamese students and compared them with students from seven other developing countries. The results showed that Vietnamese students were more likely to emphasise effort and spent more study time studying mathematics out of school.

The above findings are consistent with mathematics education practices in Vietnam where the importance of effort in school achievement is stressed from an early age. Parents express high expectations of their children's performance, and mathematics is always considered one of the most important school subjects in Vietnam. As a result, parents tend to encourage their children to make a great deal of effort and try hard to do well in mathematics. Effort is also highly valued by Vietnamese teachers. As will be discussed later, the Vietnamese teachers in the present sample agreed with statements that effort is important to improve mathematics learning (M = 3.15).

The positive relations between the Vietnamese students' beliefs in the importance of effort, the beliefs that mathematics ability can be improved, and mathematics performance is especially noteworthy. It is possible that this positive association is also related to the practices of learning mathematics in Vietnam where students are highly encouraged to try hard to improve their performance. If students get better results in mathematics after trying hard, they will tend to believe their mathematics ability can be changed during the process of learning. This association is consistent with the finding that students in Asian countries believed more in the role of effort than students in Western countries, as shown in the work of Uttal (1997), Mason (2003) with Italian students, and Hemmings and Kay (2010) with Australian students.

Most of the Vietnamese students had a positive perception of their teachers' practices (M = 3.36). A possible explanation of this finding could be that Vietnamese students tend to be respectful of their teachers and consider them as role models, and their teachers tend to act accordingly. The finding that Vietnamese students seem to have a more positive perception of their teachers in comparison to their peers in other countries might be expained by the differences in the role of teachers in different cultures. As Leung (2001) pointed out, while "the role of the teacher is to cater for the needs of the individual student [in Western culture], role modelling of the teacher is traditionally of great importance in the Chinese culture" (Leung, 2001, p. 44).

The results of the SEM model indicated that students' perception of their teachers' practices indirectly predicted mathematics performance in a positive way through beliefs that effort is important, beliefs that mathematics ability can be changed, and self-efficacy beliefs. This result is consistent with the findings of Op't Eynde and De Corte's (2003) work of high correlations between students' positive perceptions of their mathematics teachers and their confidence in their mathematical capacities. It is also consistent with a host of other related results. Murdock and Miller (2003) found that students' perceptions about teachers influence their self-efficacy beliefs and their values; Op't Eynde et al. (2006a) found that students with positive beliefs about their teachers feel more confident about mathematics learning; Tarmizi and Tarmizi (2010) showed that students' beliefs about the role and the functioning of their teacher were strongly and positively associated with their beliefs about being mathematically competent; Wang et al. (2019) found that students' perceptions about teacher praise was related with their perceived mathematics achievement; and You et al. (2016) found that students' perceptions of teachers' motivational behaviours indirectly predicted mathematics achievement through students' mathematics self-efficacy and intrinsic motivation.

In the case of the Vietnamese mathematics education context, the results of the present research are consistent with Hoang's (2007) findings that the more frequently teachers present their students with strategies to solve mathematics problems, the better students perform in mathematics, and that students' active learning strategies are significantly associated with mathematics performance.

Beliefs about the usefulness of mathematics, beliefs about mathematics problem-solving, and mathematics performance. Another pattern of positive relations was found between beliefs in the usefulness of mathematics and beliefs that mathematical problem-solving is about understanding rather than memorisation. Both of these beliefs were, in turn, positive predictors of mathematics performance through the mediation of beliefs in the malleability of mathematics ability and self-efficacy. Although the relationship between these beliefs and mathematics performance has only, to date, been explored in the present research, it is consistent with the results of Köller (2001), who found indirect links between these beliefs. Op't Eynde and De Corte (2003) also found that the more students valued mathematics, the more confident they were in mathematics learning. This result helps explain the non-significant relationships between beliefs about usefulness of mathematics and mathematics performance in prior research, such as Pajares and Miller (1994) and Arikan et al. (2016).

The Vietnamese students overwhelmingly agreed with statements expressing beliefs that mathematics is useful (*M* = 3.43). This finding is consistent with prior research by Palmer (1994) who found that Vietnamese students tended to find that mathematics is more useful than their Autralian peers. This could be explained by the finding that Vietnamese mathematics teachers also tended to agree highly with statements indicating that mathematics is useful (*M* = 3.26), a result consistent with Bryan et al. (2007) who investigaged the value of mathematics amongst teachers in four regions (Mainland China, Hong Kong, Australia, and the US).

It is worth noting that results also showed beliefs about the usefulness of mathematics directly and positively predicted beliefs about the importance of effort. In other words, the more students believed that mathematics is useful, the more likely they were to believe that effort is important to improve their mathematics learning. Again, this finding may also explain why Vietnamese students valued the importance effort.

Beliefs about the nature of mathematical problem-solving. Beliefs about the nature of problem-solving consisted of two sub-constructs, namely, beliefs that mathematical problem-solving is about understanding procedures and beliefs that mathematical problem-solving is about memorising procedures. The results showed that both of these two sub-constructs predicted mathematics performance through beliefs about mathematics ability and self-efficacy, but their predictions were in contrasting directions. While beliefs that mathematical problem-solving is about memorising procedures was a negative predictor of mathematics performance, beliefs that mathematical problem-solving is about understanding procedures was a positive predictor of mathematics performance. These results are consistent with findings of Yuanita and Zulnaidi (2018) and Mason (2003), who also showed that beliefs about problem-solving directly or indirectly predicted mathematical performance. The indirect effect of beliefs about problem-solving on mathematics performance may explain why these beliefs were not found to be direct predictors of mathematics performance by Schommer-Aikins et al. (2005) and Schommer-Aikins and Duell (2013).

There were only two items in the validated MBQ-S that tested agreement with beliefs that mathematics problem-solving is about understanding procedures, and the Vietnamese students in the sample overwhelmingly agreed with them (*M*= 3.43). However, 45% of the Vietnamese students also agreed with the statement that “learning to do math problems is mostly a matter of memorising the right steps to follow” and 32% agreed with the statement that “getting a right answer in math is more important than understanding why the answer works”. There emerged some inconsistencies in students’ beliefs in this area that can be understood if one considers the nature of teaching in Vietnam. Most mathematics teachers still apply transmissive teaching approaches, focusing on helping students memorise mathematical formulas and solve mathematical problems out of context, rather than constructive teaching approaches where students have more opportunities to explore and discover (T. T. Nguyen et al., 2020). This observation aligned with Op't Eynde and De Corte (2003) who also emphasised the orientation toward achievement and grading in the school mathematical context.

The presence of beliefs in the importance of memorisation for problem-solving has been observed in prior research, such as Schoenfeld (1992) who showed that many students believe that problem-solving in mathematics is about memorisation. In addition, research has also showed that students from Asian countries are more inclined to stress the role of repetition and rote memory in learning, as opposed to understanding, compared to students from Western countries (e.g., Dahlin & Watkins, 2000). The result of the structural equation model of the present research, which found that beliefs that problem-solving is about memorisation is a negative predictor of beliefs in mathematical ability and indirectly of mathematical performance (while beliefs that problem-solving is about understanding are a positive predictor), add to previous research and indicate the importance of these beliefs in the mathematical belief system of students.

Beliefs about the certainty of mathematics knowledge, mathematical problem solving and mathematics performance. The results of the present research showed that students' beliefs that mathematics knowledge is certain were direct predictors of beliefs that mathematical problem-solving is about memorising procedures and, therefore, indirect negative predictors of mathematics performance. In other words, the more students believed that mathematics knowledge is certain, the more they believed that mathematical problem-solving is about memorising procedures and the less they believed that mathematics ability can be changed, the lower their levels self-efficacy, and the lower the mathematics performance they achieved.

This finding is consistent with previous research in which beliefs that mathematics is certain and absolute were found to predict mathematics performance negatively and directly or through mediators (Hofer, 2000; Köller, 2001; Schommer, 1990, 1998). It is also consistent with Buehl and Alexander's (2005) results that students who believed more in the certainty of mathematics knowledge had lower levels of motivation and task performance, and Cano's (2005) work, which showed that the more students believe knowledge is certain, the more they approach learning through surface strategies (rather than deep strategies), and the lower they perform on academic achievement. It is logical that the more students think that mathematics knowledge is an unchanging, closed system, the more they are likely to also think that mathematics problem-solving is about memorising certain procedures and rules with little need for understanding and creativity. This makes this epistemological belief important to consider, especially in the context of the Vietnamese transition to a competence-based curriculum in mathematics.

An interesting finding of the research was the positive path leading from beliefs that mathematics is certain to beliefs in the importance of effort. It appears that the more the Vietnamese students believed that mathematics knowledge is certain, the more they tended to agree with statements indicating that effort is important to improve mathematics learning. Although there is no similar result in the literature, this result may be related to the characteristics of mathematics teaching and learning in Vietnam. As discussed previously, Vietnamese students are taught mathematics with a traditional teaching approach that emphasises that effort is needed for students to acquire the body of this certain mathematics knowledge which consists mostly of memorising rules. This explanation is consistent with Bryan et al.'s (2007) and Leung's (2001) findings about the characteristics of mathematics teaching and learning in Eastern countries.

Gender differences in Vietnamese students' mathematics-related beliefs and mathematics performance. Testing gender differences in mathematics performance showed that female students performed better in the district mathematics exam than male students, although male students were found to perform better than female students in the PISA-based test. This result is consistent with other findings regarding gender differences in the literature. Prior research on gender differences in achievement using meta-analysis showed a stable female advantage compared with male, both in language courses and mathematics (Voyer & Voyer, 2014). However, research has also shown contrasting gender differences amongst high performers in mathematics. An analysis of gender differences in the results of the 2012 PISA (OECD, 2014) exam showed that boys outperformed girls in mathematics in most countries (including Vietnam). According to the report, girls tended to "report less perseverance, less motivation to learn mathematics, less belief in their own mathematics skills, and higher levels of anxiety about mathematics" (OECD, 2014, p. 4). This finding is also consistent with Darmawan's (2016) findings that boys tended to outperform girls in mathematics in the Asian region (including Vietnam).

What were the beliefs of the girls in the present sample that resulted in their better overall performance in the district mathematics exam? The results of the research showed that female students valued the usefulness of mathematics in their daily lives more so than male students and perceived their mathematics teachers more positively. One possible explanation of this finding might be that the sample of teachers was mostly female. As a result, the female students' beliefs may be more aligned with their teachers' beliefs than boys. The female students also tended to agree more with statements that mathematics is about understanding, in contrast to their male counterparts who tended to believe mathematical problem-solving is about memorising procedures. This latter finding is consistent with Mason's (2003) and Prendergast et al.' s (2018) work with secondary school students. The finding that female students valued the usefulness of mathematics in their daily lives more than male students is inconsistent with Fennema and Sherman's (Fennema, 1989; Fennema & Sherman, 1977) work with students in Western countries, but agrees with Sangcap's (2010) findings with students in Asian countries. This result is also consistent with Kiwanuka et al.'s (2017) study with seventh grade students in Uganda which showed that girls had higher scores on items investigating the usefulness of mathematics compared to boys.

Differences in Vietnamese students' mathematics-related beliefs between low and high performers. High performers in the present research were students who received the highest scores in the PISA-based test (scoring at least 5 out of a total score of 6). Only 65 of the total sample of 609 students were included in this group of high performers. Their high performance in the PISA-based test was interpreted to indicate that these students were able to transfer their mathematics knowledge to solve unfamiliar problems based on real-world situations.

What were the mathematics related beliefs of the high mathematics performers in the PISA-based test compared to their peers? The results of the present research showed that the high performers expressed higher agreement compared to the low performing students in a cluster of beliefs found to be positive predictors of mathematics performance. For example, they were more likely to express agreement with beliefs indicating that mathematics is not certain and absolute, that it is useful, that problem-solving is about understanding rather than memorisation, and that mathematics ability can be changed. High performers also had higher self-efficacy beliefs and perceived their teachers' practices more positively than low performers.

The above results are consistent with prior research. For example, Op't Eynde and De Corte (2003, cited in Andrews et al., 2011) found that high performers believed that mathematics is useful more than low performers. These findings are also consistent with Hannula and Laakso's (2011) findings that, in general, high performers are likely to be more positive than low performers in terms of expectations and affect (e.g., beliefs). Hoang (2007) investigated the relationships between different aspects and mathematics performance of twelve-year-old students in Vietnam and found that students who more regularly discuss real-world problems tend to perform better in mathematics.

The differences between high and low performers in their mathematics related beliefs further support the finding of positive relations between certain mathematics-related beliefs and mathematics performance. Beliefs in the usefulness of mathematics, the changeability of mathematics, the importance of understanding rather than memorisation, and the possibility to improve mathematics ability are the beliefs that were more strongly expressed by students who were higher performers than their peers who performed less well in the PISA test. These findings indicate that more attention needs to be paid in the Vietnamese mathematics curriculum to stress the creativity and changeability of mathematics knowledge and the importance of understanding rather than memorisation in problem-solving.

To summarise, the results of the present research confirmed a number of findings from prior research, such as the relationships between self-efficacy beliefs and beliefs about mathematics ability with mathematics performance. Additionally, the results provided empirical evidence to support some of the assumptions of prior research (Depaepe et al., 2016; Kloosterman, 1996; McLeod, 1992; Schommer-Aikins et al., 2010) regarding a number of complex relations among mathematics-related beliefs and mathematics performance, such as beliefs about the certainty of mathematics knowledge, the usefulness of mathematics, mathematical problem-solving, and students' perceptions of teachers' practices. The present research confirmed the important role of beliefs about effort in mathematics teaching and learning in Vietnam with significant links found among these beliefs and other types of mathematics-related beliefs, and mathematics performance. The Vietnamese students in the sample expressed high agreement with a number of beliefs found to be positive predictors of mathematics performance, such as self-efficacy beliefs, beliefs in the importance of effort, beliefs in the usefulness of mathematics and positive perceptions of their teachers. One area where the beliefs of the Vietnamese students could be improved has to do with beliefs that mathematics knowledge is not certain and absolute, and the related beliefs that mathematical problem-solving is about understanding rather than memorization.

6.2 Vietnamese teachers' mathematics-related beliefs and the influence of these beliefs on students' mathematics performance

The Vietnamese mathematics teachers expressed strong agreement with statements indicating that effort is important for student learning but provided inconsistent responses for many of the other beliefs investigated. For example, they expressed agreement with beliefs considered to be both positive and negative predictors of student performance, such as beliefs about the certainty of mathematics knowledge, mathematical problem-solving, and mathematics teaching. They also expressed relatively low levels of self-efficacy. The multilevel analysis that investigated the influence of teachers' beliefs on student performance showed that teacher self-efficacy was a significant positive predictor of student performance. Additionally, teachers' beliefs that mathematics knowledge is not certain were direct positive predictors of beliefs that mathematical problem-solving is about understanding procedures which, in turn, were direct positive predictors of beliefs in constructive teaching. These results will be discussed in greater detail below.

Teachers' mathematics-related beliefs

Most of the Vietnamese teachers agreed with statements that effort is important to improve mathematics learning (M = 3.15). This finding is consistent with teaching practices in Vietnam and expectations regarding student learning. Teachers expect students to exhibit high performance in tests and examinations, which may be because student grades are used to assess teacher performance (Parandekar & Sedmik, 2016). Therefore, they encourage students to study hard to achieve high grades. The results also supported research showing that teachers in Eastern countries are more likely to stress the importance of effort compared to their peers in the West (*studying hard* versus *pleasurable learning*, Leung, 2001).

The teachers expressed conflicting beliefs in most of the other belief constructs investigaged. For example, regarding beliefs about the certainty of mathematics knowledge most teachers seemed to agree overall that mathematics knowledge is not certain (M = 3.05), yet, only about half of them agreed that "answers to questions in mathematics change as mathematicians gather more information", while over 60% of them believed that a mathematics problem has always only one true answer. Agreement with beliefs that mathematics knowledge is certain and absolute are consistent with prior research which showed that mathematics teachers in Asian countries tend to emphasise the content of mathematics knowledge, while their peers in Western countries tend to focus more on the process of doing mathematics (Leung, 2001).

The Vietnamese teachers tended to agree with statements expressing beliefs that mathematics is useful (M = 3.26), yet many of them also expressed agreement with the statement that "understanding mathematics is important for mathematicians, economists, and scientists but not for most people". Ambiguity about the usefulness of mathematics is consistent with a tendency of Asian teachers to view mathematics as developing abstract thinking skills, and that "developing abstract thinking in students is one of the objectives of teaching mathematics" (Bryan et al., 2007, p. 330). As a result, teachers focus little on applying mathematics to real-world situations. Bryan et al.'s (2007) work showed that, although mathematics teachers in four regions (Mainland China, Hong Kong, Australia, and the US) all valued mathematics, those from Eastern cultures tended to emphasise the usefulness of mathematics in daily life less than their Western peers.

Vietnamese teachers also provided conflicting data about mathematical problem-solving. Although most of them expressed agreement with beliefs indicating that mathematical problem-solving is about understanding procedures (*M* = 3.17), nearly 57% of them also agreed that "it's not important to understand why a mathematical procedure works as long as it gives a correct answer", and over 45% of them agreed that getting a right answer in mathematics is more important than understanding why the answer works. Again, beliefs that mathematical problem-solving is more about the memorisation of procedures support the results of prior research showing that mathematics teachers in Asian countries tended to emphasise rote learning, while their Western peers tended to focus more on meaningful learning (Leung, 2001).

Another area where the teachers exhibited conflicting beliefs concerned the changeability of mathematics ability. Although the teachers tended to believe that mathematics abiity can be changed during the learning process (*M* = 2.90), only about half of them believed that students who do not have high natural ability in mathematics are still capable of learning difficult math material. A possible explanation for this finding is that, from the mathematics teachers' perspective, due to large student numbers in classes, they may see a bigger picture where some students can understand difficult mathematics materials and solve difficult mathematics problems whereas others cannot. If teachers hold these beliefs, it may negatively influence their teaching and, as a consequence, it may have negative effects on the process of mathematics teaching and learning, especially with a new competence-based curriculum based currently being implemented in Vietnam.

The Vietnamese mathematics teachers also held inconsistent beliefs about mathematics teaching. Many teachers expressed agreement with beliefs in transmissive teaching (*M* = 2.79). This is consistent with other findings indicating that teachers in Eastern countries tend to emphasise the provision of knowledge to students (Bryan et al., 2007) and with Ly and Brew's (2010) work which showed that Vietnamese pre-service mathematics teachers tended to emphasise contructive teaching less than their Australian peers. However, the empirical evidence from the present research also revealed that the Vietnamese teachers simultaneously expressed agreement with statements indicating beliefs in constructive teaching (M = 3.20). Specifically, they expressed beliefs in statements expressing the importance of developing strategies and skills to solve mathematics problems as well as integrating new information with students' existing knowledge. Agreement with these beliefs may be related to the fact that the teachers participated in training programs organised by Ministry of Education and Training about teaching methods consistent with the new competence-based mathematics curiculum. As a result, they had been exposed to theories and practices on student-centred approaches in teaching mathematics. Agreement on constructive teaching might also be the result of conformity with socially desirable norms.

The Vietnamese mathematics teachers expressed relatively low levels of self-efficacy beliefs (M = 2.56). Only about half of them agreed with statements indicating that they have no difficulties answering students' mathematics questions and that they are certain that they are good mathematics teachers. Over 78% of them agreed that they often wonder whether they are effective in monitoring mathematics activities.

The low self-confidence of the teachers in the present sample does not align with results of prior research showing that Vietnamese pre-service mathematics teachers had high levels of self-efficacy (M = 4.65, Ly & Brew, 2010). The results are also inconsistent with the finding that these teachers' students expressed relatively high levels of self-efficacy in their mathematics abilities. A possible explanation for the low levels of self-efficacy in the teacher sample is that the teachers were experiencing a transitional period of curriculum reform during which they had been exposed to new information about mathematics teaching and learning without having enough time to fully integrate it with their existing belief systems and teaching practices. As discussed earlier, many teachers expressed conflicting beliefs about mathematics teaching, indicating agreement with statements indicating beliefs both in transmissive and constructive teaching. It is possible that their conflicting beliefs about mathematics learning and teaching made them to feel less confident in their teaching.

In summary, the observed inconsistency in teachers' beliefs about the nature of mathematics and mathematics teaching in the present research may have occurred because Vietnamese teachers may have been exposed to new information about the teaching and learning of mathematics which was inconsistent with their existing methods of transmissive teaching as well as with certain traditional beliefs about mathematics (e.g., mathematics knowledge is certain, problem-solving is about memorisation), and which they may not have completely understood. The inconsistency in teachers' beliefs in the present research confirms recent research revealing teachers' conflicting beliefs, as in Vosniadou et al.'s (2020) and Darmawan et al.'s (2020) work which showed inconsistency in the beliefs of Australian pre-service teachers about self-regulated learning and constructive and transmissive teaching.

The influence of the Vietnamese teachers' mathematics-related beliefs on their students' mathematics performance

The results of the first structural model using the data of all the mathematics teachers in the sample provided some general ideas about the relationships among the 13 belief sub-constructs included in the MBQ-T. Due to the fact that this model did not have good fit indices, many paths were not significant, and the sample of mathematics teachers was small, results must be viewed with caution. However, these results revealed some possible significant associations between some types of beliefs. Five belief sub-constructs for the multilevel analysis were chosen based on these results.

The final multilevel structural equation model with these five belief constructs was run 18 teachers and their students. The results showed that the teachers' self-efficacy beliefs directly and positively predicted student mathematics performance in all the three types of assessments investigated (district mathematical exam, curriculum-based test, and PISA-based assessment). This finding supports results of previous studies, such as those by Chang (2015), Rutherford et al. (2017), and Perera & John (2020), which also showed direct associations between teachers' self-efficacy beliefs and students' mathematics performance and provide further evidence for the importance of these beliefs of teachers for student achievement.

Prior research has shown positive relationships between beliefs in constructivist teaching, mathematical problem solving, and self-efficacy in teachers. For example, Peterson et al. (1989) showed the positive influence of teachers' constructivist beliefs on the performance of their students in mathematical problem-solving tasks. Polly et al. (2013) found a significant positive relationship between teachers' beliefs in constructive teaching and students' problem-solving achievement and Voss et al. (2013) found that the constructivist beliefs of teachers were positively related to students' mathematics performance. However, the present research showed a non-significant path from beliefs about constructive teaching of mathematics to teacher self-efficacy and to students' performance.This may be because of the small number of teachers in the sample or because the mathematics teachers held inconsistent beliefs about mathematics teaching, expressing beliefs in both transmissive and constructive teaching of mathematics. Further research with a larger number of teachers is required to investigate these relationships.

Due to the non-significant link between beliefs in constructive teaching and high self-efficacy, the results showed no evidence that teachers' beliefs about mathematical problem-solving predicted students' mathematics performance. However, an important result was that teachers' beliefs that mathematical problem-solving is about understanding procedures positively predicted their beliefs in constructive teaching of mathematics. In other words, the more mathematics teachers believed that mathematical problem-solving is about understanding procedures, the more they believed in constructive teaching. This result is consistent with findings from many studies in the literature showing links between teacher' mathematics-related beliefs and their teaching. For example, Cross (2009) argued that teachers' beliefs about the nature of mathematics had strong links with beliefs about mathematics pedagogy. This finding is also consistent with Liljedahl et al.'s (2007) finding that teachers' beliefs about the nature of mathematics influence their views on mathematics teaching and learning and Saadati et al.'s (2019) result that both traditional beliefs (i.e., mathematics as fixed procedures), and reformed beliefs (i.e., mathematics as a process of inquiry), significantly predicted teacher-centred practices and student-centred practices respectively.

Although there have been some studies investigating teachers' epistemological beliefs, such as Chrysostomou and Philippou (2010) who found that teachers' epistemological beliefs predict their self-efficacy, little is known about teachers' beliefs about the certainty of mathematics knowledge and the association of these beliefs with students' mathematics performance. However, the results of the present research showed that teachers' beliefs that mathematics knowledge is malleable directly and positively predicted their beliefs that mathematical problem-solving is about understanding procedures. This is an important finding that has not previously been reported in the research literature. This finding provides empirical support to the arguments regarding relationships between epistemological beliefs about mathematics and beliefs about mathematics teaching and learning made in prior research (Depaepe et al., 2016; Schommer-Aikins et al., 2010).

To summarise, there were three important findings of the present research regarding teachers' mathematics related beliefs. First, the teachers were conflicted about many of their beliefs about mathematics, such as beliefs regarding the certainty of mathematics, the nature of mathematical problem-solving, the malleability of students' mathematics ability, and the nature of mathematics teaching. This is probably because they had been exposed to new information about mathematics learning and teaching not compatible with their teaching practices but did not have enough time to integrate it within their belief systems. Second, the multivariate model showed that teachers' self-efficacy directly predicted students' mathematics performance, providing further support to similar findings in the existing literature. Finally, the results also revealed some important interrelations among teachers' beliefs, such as beliefs that mathematics knowledge is not certain, beliefs that mathematical problem-solving is about understanding procedures, and beliefs about constructive teaching of mathematics. The interrelationships found in the present research constitute new contributions to the research literature and provide empirical support to arguments or hypotheses expressed in prior research about the association among epistemological beliefs about mathematics, beliefs about the nature of mathematics, beliefs about mathematics teaching and learning, and beliefs about the self as mathematics teachers with students' mathematics performance (Depaepe et al., 2016; Kloosterman, 1996; McLeod, 1992; Schommer-Aikins et al., 2010).

6.3 Implications for the teaching and learning of mathematics in Vietnam

The Vietnamese students were found to express agreement with a number of beliefs that the SEM model showed to be positive predictors of mathematics achievement, such as that mathematics is useful, effort is important to improve mathematics learning, and mathematics ability is malleable. The students also had high self-efficacy beliefs and positive perceptions of their teachers' practices, which the SEM model also showed to be positive predictors of mathematics performance. Encouraging and further developing these beliefs is important for the improvement of students' mathematics learning in the future.

The SEM model also revealed some beliefs which were negative predictors of mathematics performance, such as beliefs that mathematics knowledge is certain and that mathematical problem-solving is about memorization rather than understanding. The investigation of the differences between the students who were high and low mathematics performers also showed that the students who performed well in the PISA-based test tended to believe less in the certainty of mathematics knowledge, innate ability, and memorisation in mathematics learning. This suggests that interventions to influence the development in these beliefs in the positive direction, can potentially improve mathematics self-efficacy and mathematics performance in all students and especially the transfer of mathematics knowledge to unfamiliar and real-world situations. Such intervention programs will navigate well with the requirements from the new competence-based curriculum in Vietnam where teachers are required to focus more on constructive teaching and on developing students' competencies rather than on the transmission of mathematics knowledge. These recommendations are consistent with the results of research by Mason and Scrivani (2004), which showed educational interventions using novel learning environments can be successful in improving the mathematics-related beliefs of students.

The findings of the research have implication also in the case of the Vietnamese mathematics teachers. The present research found that the Vietnamese teachers had inconsistent beliefs about the certainty of mathematics knowledge, mathematical problem-solving and mathematics teaching, and that they also had relative low levels of self-efficacy. As discussed in the previous section, a significant potential factor for this inconsistency might be that the Vietnamese teachers were experiencing a transitional period of curriculum reform from a content-based to competence-based approach. These findings indicate that there should be better teacher professional development programs in Vietnam where the focus should be not only on developing the teachers' knowledge and skills of teaching, but also on strengthening and developing their beliefs about mathematics learning and instruction. Although beliefs about mathematics are difficult to change some effective strategies to change teachers' non-availing beliefs have been reported in the literature (e.g., Lawson et al., 2019). It would be ideal if these changes could be made in pre-service teacher training courses by developing suitable curriculum and indicating the relationships between different beliefs and their influence on teachers' practices, as well as on students' motivation and performance. Prior research has shown that it is easier to influence the beliefs of pre-service and early career teachers compared to experienced teachers. For instance, Anderson and Piazza (1996) found that pre-service teachers' beliefs may be changed based on different instructional pedagogy and classroom environment, which supports the approach of constructivism. Blömeke et al. (2015) also investigated the development of early career teachers' beliefs. They found that their beliefs about the nature of mathematics were more dynamic three years after completing their initial teacher education courses. With regard to the Vietnamese education context, such interventions can be especially significant once the mathematics curriculum reform has been implemented because, as Handal and Herrington (2003) indicated, teachers' mathematics-related beliefs and curriculum are important to ensure the success of the implementation of a new curriculum, and that the "mismatch between curriculum goals and teachers' belief systems is a factor that affects current curriculum change in mathematics education” (Handal & Herrington, 2003, p. 62).

6.4 Significance, limitations of the research and future directions

Significance of the research

The results of the present research provided for the first time data from a relatively large sample of Vietnamese students and a smaller sample of Vietnamese teachers about their mathematics-related beliefs, enabling comparisons between the beliefs of these students and teachers and their peers from Asian and Western countries. The results further confirmed that mathematics-related beliefs play an important role in the processes of mathematics teaching and learning.

Two mathematics-related beliefs questionnaires were constructed and validated, adding to the existing literature. One important advantage of these questionnaires was the inclusion for each belief construct of a sub-construct investigating beliefs hypothesized to be positive predictors of student mathematics performance and another, investigating beliefs that were hypothesized to be negative predictors. The results showed that particularly the Vietnamese teachers often provided inconsistent responses, expressing agreement both with beliefs found to be positive predictors of students’ mathematics performance and with beliefs found to be negative predictors. This finding highlights the importance of the provision of further professional development to teachers, especially in the current educational context in Vietnam as it transitions to a more competence-based curriculum and instruction.

The present study paid more attention to the investigation of relations among many different belief constructs than previous research. Different categories of mathematics-related beliefs were investigated, such as epistemological beliefs about mathematics, beliefs about the nature of mathematics, beliefs about mathematics teaching and learning, and beliefs about self as a mathematics learner. The structural equation modeling tested a host of predictions about the possible relations amongst the various belief constructs and their relations to student mathematics performance. The results obtained from the final SEM model helped to illuminate and explain the absence of some significant relationships between specific belief constructs and mathematics performance in some prior research. Overall, the results confirmed the importance of the self-efficacy beliefs of both students and their teachers for student mathematics performance. A number of other belief constructs were found to be indirect positive predictors of students' mathematics performance, such as beliefs in the importance of effort, the usefulness of mathematics, students' positive perceptions of their teachers, the importance of considering problem solving as depending on understanding rather than on memorization and on considering mathematical ability as something that can be changed and improved. An important path was also revealed, indicating that beliefs that mathematics knowledge is certain was a positive predictor of beliefs that mathematics problem solving is about memorization and that both were indirectly negative predictors of mathematics performance.

The results of the research have particular importance within the Vietnamese educational context and help to further accentuate the importance of understanding the belief systems of students and teachers and of the influences they exert on students' mathematics learning.

Limitations of the research and future directions

The present research had some limitations. Mathematics-related beliefs consist of many multi-faceted constructs. Within the scope of the present research, it was possible to investigate only some of these belief constructs. In the final validated questionnaires, some (sub-) constructs were not optimally represented consisting of only two or three items. One sub-construct was removed from the students' final structural model because it did not fit the data well. Further research is needed to enrich, improve and validate the present instruments.

The sample of the teachers was very small and only five belief (sub-) constructs were included in the final structural model that investigated the influence of teachers' beliefs on students' mathematical performance. Furthermore, considering that beliefs are not measured only by closed questionnaires, further research is needed using interviews and alternative methodologies to provide a more complete picture of the Vietnamese students' and teachers' mathematics-related beliefs and their relationships to students' mathematics performance.

6.5 Conclusions

The purpose of the present research was to investigate the influence of students' and teachers' mathematics-related belief system on students' mathematics performance. Three following research questions were comprehensively answered based on the empirical data.

Research Question 1: What are the emergent profiles of Vietnamese students' and teachers' mathematics-related beliefs? And how do these profiles of students differ among female and male students, and among low and high performers of problem-solving?

Research Question 2: How do students' mathematics-related beliefs influence their mathematics performance?

Research Question 3: How do teachers' mathematics-related beliefs influence their students' mathematics performance?

With respect to the first research question, the results showed that the Vietnamese students and teachers tended to believe that effort is important to improve mathematics learning. The Vietnamese students believed that mathematics is useful and mathematics ability can be changed. They also had high self-efficacy beliefs and positive perceptions of their teachers' practices. The Vietnamese teachers held some conflicting beliefs regarding the certainty of mathematics, the usefulness of mathematics, mathematical problem-solving, students' mathematics ability, and mathematics teaching, and they had relatively low self-efficacy beliefs. It was hypothesized that the presence of opposing beliefs in the case of the teachers may be due to the fact that the teachers were experiencing curriculum reform in Vietnam.

The present research also found that high mathematics performers tended to agree more strongly than their peers who were low mathematics performers that mathematics is useful, mathematics ability can be changed, that mathematics knowledge is not certain, and that mathematical problem-solving is about understanding rather than memorization. In terms of gender differences, the present research found that the girls performed better than the boys in the assessments used. However, the boys performed better than the girls in the group of high performing students, based on their scores in the PISA based assessment. The girls in the sample were more likely to believe that mathematics is useful and tended to have better perceptions of their mathematics teacher than boys. In contrast, boys tended to believe that mathematical problem-solving is about memorising procedures more than did girls.

With regards to the second research question, the structural equation modelling showed that 14 out of 16 path coefficients from the student structural model were significant. The results revealed that, in general, students' mathematics-related beliefs had hierarchical relationships to each other and to mathematics performance. Specifically, the results showed that students' beliefs that mathematics ability can be changed directly and positively predicted self-efficacy beliefs which, in turn, were direct positive predictors of their mathematics performance. Additionally, the findings of the present research showed that there were interrelationships among the different types of beliefs investigated, namely, among beliefs that mathematics knowledge is certain, beliefs that mathematics is useful, beliefs that mathematical problem-solving is about understanding procedures, beliefs that mathematical problem-solving is about memorising procedures, and students' perceptions of their teachers' practices. Finally, the findings also confirmed that the Vietnamese students expressed high agreement with beliefs that effort is important to improve mathematics learning.

With respect to the third research question, the results of the multilevel SEM showed hierarchical relationships among teachers' beliefs and students' mathematics performance. Specifically, teachers' high self-efficacy beliefs in their mathematics teaching directly predicted students' mathematics performance. It was also shown that beliefs that mathematical problem-solving is about understanding procedures directly predicted teachers' beliefs about constructive teaching of mathematics, and that teachers' beliefs that mathematics knowledge is not certain directly and positively predicted their beliefs that mathematical problem-solving is about understanding procedures.

APPENDICES

Appendix A: Research Ethics Approval

APPROVAL NOTICE

Project No.:	**8181**
Project Title:	The Influence of Students' and Teachers' Mathematics-Related Beliefs on Students' Mathematics Performance
Principal Researcher:	Mr Xuan Cuong Dang
Email:	dang0107@flinders.edu.au

Approval Date:	26 October 2018	Ethics Approval Expiry Date:	**28 February 2021**

The above proposed project has been **approved** on the basis of the information contained in the application, its attachments and the information subsequently provided with the addition of the following comment(s):

Additional information required following commencement of research:

1. Permissions
   Please ensure that copies of the correspondence granting permission to conduct the research from the Nghe An Department of Education and Training; and permissions from the School Principals are submitted to the Committee *on receipt.* Please ensure that the SBREC project number is included in the subject line of any permission emails forwarded to the Committee. Please note that data collection should not commence until the researcher has received the relevant permissions (item D8 and Conditional approval response – number 14).

Appendix B: Letter of Introduction for Department of Education and Training

College of Education, Psychology and Social Work
Education Building
GPO Box 2100
Adelaide SA 5001
Tel: +61 8 8201 7800
stella.vosniadou@flinders.edu.au
https://www.flinders.edu.au
CRICOS Provider No. 00114A

LETTER OF INTRODUCTION – PERMISSION REQUEST

(Department of Education and Training)

Dear Sir/Madam,

This letter is to introduce Xuan Cuong Dang who is a PhD student in the College of Education, Psychology and Social Work at Flinders University. He is undertaking research leading to the production of a thesis or other publications on the subject of the influence of students' and teachers' mathematics-related beliefs on students' mathematics performance.

The mathematics-related beliefs of students and teachers have been found to have significant effects on students' mathematics performance. This research has so far been conducted in Western countries but only a few studies, incomplete studies, exist in Asian countries. Furthermore, there is no research on this topic in Vietnam. The present research will investigate in great detail the beliefs that Vietnamese students and teachers have about mathematics and will determine the effects that these beliefs have on students' performance. The results of the research will help educators, researchers and stakeholders in Vietnam to have a better understanding of factors that affect mathematics performance and will be invaluable in providing suggestions for changes in curricula and instruction that will improve the teaching and learning of mathematics in Vietnamese schools.

I would like to have your permission for Xuan Cuong Dang to be in contact secondary schools in your province to ask their participation in this project. The teachers will be asked to complete a questionnaire and the students will complete a questionnaire and also take a mathematics test. Some teachers and students will be also invited to participate in the interview sessions. No more than 1.5 hours on all activities would be required in each school.

The information provided will be treated in the strictest confidence and none of the participants will be individually identifiable in the resulting thesis, report or other publications.

Any enquiries you may have concerning this project should be directed to me at the address given above or stella.vosniadou@flinders.edu.au.

Yours sincerely

Stella Vosniadou

Strategic Professor, College of Education, Psychology and Social Work

This research project has been approved by the Flinders University Social and Behavioural Research Ethics Committee (Project number 8181). For more information regarding ethical approval of the project the Executive Officer of the Committee can be contacted by telephone on 8201 3116, by fax on 8201 2035 or by email human.researchethics@flinders.edu.au

College of Education, Psychology and Social Work
Education Building
GPO Box 2100
Adelaide SA 5001
Tel: +61 8 8201 7800
stella.vosniadou@flinders.edu.au
https://www.flinders.edu.au
CRICOS Provider No. 00114A

Appendix C: Letter of Introduction for School Principals

LETTER OF INTRODUCTION – PERMISSION REQUEST

(School Principals)

Dear Sir/Madam,

This letter is to introduce Xuan Cuong Dang who is a PhD student in the College of Education, Psychology and Social Work at Flinders University. He is undertaking research leading to the production of a thesis or other publications on the subject of the influence of students' and teachers' mathematics-related beliefs on students' mathematics performance.

The mathematics-related beliefs of students and teachers have been found to have significant effects on students' mathematics performance. This research has so far been conducted in Western countries but only a few studies, incomplete studies, exist in Asian countries. Furthermore, there is no research on this topic in Vietnam. The present research will investigate in great detail the beliefs that Vietnamese students and teachers have about mathematics and will determine the effects that these beliefs have on students' performance. The results of the research will help educators, researchers and stakeholders in Vietnam to have a better understanding of factors that affect mathematics performance and will be invaluable in providing suggestions for changes in curricula and instruction that will improve the teaching and learning of mathematics in Vietnamese schools.

I would like to have your permission for Xuan Cuong Dang to be in contact the 9th grade teachers and students in your school to ask their participation in this project. The teachers will be asked to complete a questionnaire and the students will complete a questionnaire and also take a mathematics test. Some teachers and students will be also invited to participate in the interview sessions. No more than 1.5 hours on all activities would be required.

The information provided will be treated in the strictest confidence and none of the participants will be individually identifiable in the resulting thesis, report or other publications.

Any enquiries you may have concerning this project should be directed to me at the address given above or stella.vosniadou@flinders.edu.au.

Yours sincerely

Stella Vosniadou

Strategic Professor, College of Education, Psychology and Social Work

This research project has been approved by the Flinders University Social and Behavioural Research Ethics Committee (Project number 8181). For more information regarding ethical approval of the project the Executive Officer of the Committee can be contacted by telephone on 8201 3116, by fax on 8201 2035 or by email human.researchethics@flinders.edu.au

Appendix D: Information Sheet for Mathematics Teachers

Teacher Information Sheet

Title: The Influence of Students' and Teachers' Mathematics-related Beliefs on Students' Mathematics Performance

Principal Researcher: Xuan Cuong Dang

Associate Researchers: Professor Stella Vosniadou and Dr Mun Yee Lai

Location: Flinders University

1. Introduction

You are invited to take part in the research project: *The Influence of Students' and Teachers' Mathematics-related Beliefs on Students' Mathematics Performance.*

This Information sheet provides information about the research project. Knowing what is involved will help you decide if you want to take part in the research. Please read this information carefully. Ask questions about anything that you don't understand or want to know more about.

2. What is the purpose of this research?

The purpose of the present research is to investigate Vietnamese students' and teachers' mathematics-related beliefs and understand how these beliefs influence students' mathematics performance.

3. Why is this research necessary?

Research in Western countries has shown that students' beliefs about mathematics influence mathematics performance. This research has not been done in Vietnam. The present research project will investigate students' and teachers' beliefs about mathematics and will determine whether they have an effect on students' mathematics performance. The research has implications about the teaching on mathematics and can result in recommendations that will improve students' mathematics performance in Vietnam.

4. What does participation in this research involve?

If you choose to participate in this research, we will ask you to answer a questionnaire. The questionnaire includes questions about your mathematics-related beliefs. The questionnaire will take about 30 minutes to complete.

5. Other relevant information about the research project

Participation in this research will begin only once you have read this information sheet and agree to participate in this research.

Participation in this research project is voluntary. If you do not wish to take part, you do not have to. If you do consent to participate, you may withdraw at any time, but please notify the researcher before you withdraw.

6. Who is organising and funding the research?

This research project is being conducted by Xuan Cuong Dang, PhD students, Professor Stella Vosniadou, and Dr Mun Yee Lai at Flinders University.

7. Who has reviewed the research project?

The research project has been reviewed and approved by the Social and Behavioural Research Ethics Committee of Flinders University, Australia and by the Department of Education and Training of the Nghe An province.

8. Further information and who to contact

If you would like to have any further information concerning this project or if you have any problems which may be related to your involvement in the project, you can contact the principal researcher:

Name	Xuan Cuong Dang
Position	PhD student at Flinders University
Email	dang0107@flinders.edu.au

Appendix E: Information Sheet for Parents

Parent Information Sheet

(Questionnaire and Mathematics Test)

Title: The Influence of Students' and Teachers' Mathematics-related Beliefs on Students' Mathematics Performance

Principal Researcher: Xuan Cuong Dang

Associate Researchers: Professor Stella Vosniadou and Dr Mun Yee Lai

Location: Flinders University

1. Introduction

Your child is invited to take part in the research project: *The Influence of Students' and Teachers' Mathematics-related Beliefs on Students' Mathematics Performance.*

This Information Sheet provides information about the research project. Knowing what is involved will help you decide if you allow your child to take part in the research. Please read this information carefully. Ask questions about anything that you don't understand or want to know more about.

2. What is the purpose of this research?

The purpose of the present research is to investigate Vietnamese students' and teachers' mathematics-related beliefs and understand how these beliefs influence students' mathematics performance.

3. Why is this research necessary?

Research in Western countries has shown that students' beliefs about mathematics influence mathematics performance. This research has not been done in Vietnam. The present research project will investigate students' and teachers' beliefs about mathematics and will determine whether they have an effect on students' mathematics performance. The research has implications about the teaching on mathematics and can result in recommendations that will improve students' mathematics performance in Vietnam.

4. What does participation in this research involve?

If your child participates in this research, we will ask him/her to answer a questionnaire and to take a mathematics test. The questionnaire will take about 30 minutes to complete and the mathematics test will take 45 minutes to complete.

5. Other relevant information about the research project

Participation in this research project is voluntary. If you do not wish your child to take part, you do not have to. If you do consent to participate, your child may withdraw at any time.

If you do decide NOT to allow your child to participate in the research project please sign the Consent Form that has been sent to you.

6. Who is organising and funding the research?

This research project is being conducted by Xuan Cuong Dang, PhD students, Professor Stella Vosniadou, and Dr Mun Yee Lai at Flinders University.

7. Who has reviewed the research project?

The research project has been reviewed and approved by the Social and Behavioural Research Ethics Committee of Flinders University, Australia and by the Department of Education of the Nghe An province.

8. Further information and who to contact

If you want any further information concerning this project or if you have any problems which may be related to your child's involvement in the project, you can contact the principal research:

Name	Xuan Cuong Dang
Position	PhD student at Flinders University
Email	dang0107@flinders.edu.au

Appendix F: Descriptive Statistics of MBQ-S

	Total item No.	Construct item No.	Statement	N	Missing	Skewness	Kurtosis
I. Beliefs about the Certainty of Mathematics Knowledge (CMK)							
	1	CMK1	Most of what is true in math is already known (-)	608	1	0.401	-0.116
	2	CMK2	Math is really just knowing the right formula for the problem (-)	604	5	0.31	-0.308
	3	CMK3	I prefer a math teacher who shows students lots of different ways to look at the same problem (+)	607	2	-1.495	1.921
	4	CMK4	Mathematical theories are the product of creativity (+)	592	17	-0.689	0.74
	5	CMK5	In math, the answers are always either right or wrong (-)	594	15	-0.192	-1.007
	6	CMK6	There is no place for students to be creativity in math class (-)	607	2	1.211	1.062
	7	CMK7	Math problem has always only one true answer (-)	604	5	0.027	-1.183
	8	CMK8	Answers to questions in math change as mathematicians gather more information (+)	606	3	-0.166	-0.654
II. Beliefs About Usefulness of Mathematics (UoM)							
	9	UoM1	I study math because I know how useful it is (+)	608	1	-0.841	0.388
	10	UoM2	Knowing math will help me earn a living (+)	598	11	-0.618	0.2
	11	UoM3	Math is a worthwhile and necessary subject (+)	593	16	-1.557	2.73
	12	UoM4	Math will not be important to me in my life’s work (-)	606	3	-1.028	0.263
	13	UoM5	Math is of no relevance to my life (-)	592	17	1.442	1.22
	14	UoM6	Studying math is a waste of time (-)	606	3	2.881	9.426
	15	UoM7	Understanding math is important for mathematicians, economists, and scientists but not for most people (-)	608	1	0.964	0.289
	16	UoM8	The only reason I would take a math class is because it is a requirement (-)	607	2	0.667	-0.381
	17	UoM9	It is important to see the connections between the math I learn in class and real-world applications (+)	605	4	-0.912	0.708
	18	UoM10	Math provides the foundation for most of the principles used in science and business (+)	603	6	-0.659	0.211
	19	UoM11	Math helps us better understand the world we live in (+)	606	3	-0.553	0.053

III. Beliefs About Mathematical Problem-Solving (MPS)							
	20	MPS1	There are math problems that just can't be solved by following a predetermined sequence of steps (+)	588	21	-0.533	0.024
	21	MPS2	Math problems can be solved without remembering formulas (+)	603	6	0.985	0.439
	22	MPS3	Memorizing steps is not that useful for learning to solve math problem (+)	604	5	0.707	-0.491
	23	MPS4	Learning to do math problems is mostly a matter of memorizing the right steps to follow (-)	605	4	0.034	-0.772
	24	MPS5	Time used to investigate why a solution to a math problem works is time well spent (+)	594	15	-0.788	1.225
	25	MPS6	A student who doesn't understand why an answer to a math problem is correct hasn't really solved the problem (+)	609	0	-0.234	-0.568
	26	MPS7	In addition to getting a right answer in math, it is important to understand why the answer is correct (+)	606	3	-1.633	2.544
	27	MPS8	It's not important to understand why a mathematical procedure works as long as it gives a correct answer (-)	608	1	0.496	0.033
	28	MPS9	Getting a right answer in math is more important than understanding why the answer works (-)	605	4	0.254	-0.682
	29	MPS10	It doesn't really matter that you understand a math problem as long as you can get the right answer (-)	608	1	1.263	1.174
IV. Beliefs About the Importance of Effort (BaE)							
	30	BaE1	Math problems that take a long time don't bother me (+)	605	4	-0.433	0.154
	31	BaE2	I feel I can do math problems that take a long time to complete (+)	609	0	-0.257	0.189
	32	BaE3	I find I can do hard math problems if I just keep trying (+)	609	0	-0.426	0.126
	33	BaE4	If I can't do a math problem in a few minutes, I probably can't do it at all (-)	605	4	0.716	-0.049
	34	BaE5	If I can't solve a math problem quickly, I quit trying (-)	607	2	0.656	-0.022
	35	BaE6	I'm not very good at solving math problems that take a long time to figure out (-)	599	10	-0.326	-0.17
	36	BaE7	By trying hard, one can become smarter in math (+)	605	4	-0.768	0.281
	37	BaE8	Studying hard can improve one's ability in math (+)	609	0	-0.55	0.241
	38	BaE9	I can get smarter in math by trying hard (+)	607	2	-1.223	1.45
	39	BaE10	Ability in math increases when one knows the right strategies (+)	609	0	-0.659	0.353

	40	BaE11	Appropriate study skills can increase one's ability to do math (+)	608	1	-0.564	0.002
	41	BaE12	I can get smarter in math if I know how to solve math problems (+)	608	1	-0.718	0.602
V. Beliefs About Mathematics Ability (MA)							
	42	MA1	When I'm having trouble in math class, better study habits can make a big difference (+)	608	1	-0.567	0.387
	43	MA2	I'm confident I could learn math if I had better study strategies (+)	609	0	-0.864	0.699
	44	MA3	When I don't understand something, I keep asking questions (+)	609	0	-0.555	-0.059
	45	MA4	Learning good study skills can improve my math ability (+)	609	0	-0.786	0.753
	46	MA5	Math is like a foreign language to me and even if I work hard, I'll never really understand it (-)	608	1	1.235	1.02
	47	MA6	I knew at an early age what my math ability was (-)	602	7	0.42	-0.358
	48	MA7	It is frustrating when I have to work hard to understand a problem (-)	603	6	0.81	0.401
	49	MA8	I can learn new things in math, but I can't really change the math ability I was born with (-)	592	17	0.215	-0.894
	50	MA9	I'm just not a math student (-)	604	5	1.051	0.903
VI. Students' Self-Efficacy Beliefs (SelfE)							
	51	SelfE1	I feel confident enough to ask questions in my mathematics class (+)	608	1	-0.425	-0.064
	52	SelfE2	I am certain that I can do well in my math tests (+)	609	0	0.052	-0.251
	53	SelfE3	I can complete all of the assignments in my mathematics course (+)	608	1	0.043	-0.369
	54	SelfE4	I will be able to use mathematics in my future career when needed (+)	607	2	-0.962	0.69
	55	SelfE5	I feel that I will be able to do well in future mathematics courses (+)	608	1	-0.496	0.545
	56	SelfE6	I feel confident when using mathematics outside of school (+)	609	0	-0.486	0.07
	57	SelfE7	I am afraid to give an incorrect answer during my mathematics class (-)	604	5	0.073	-0.766
	58	SelfE8	I worry that I will not be able to get a good grade in my mathematics course (-)	604	5	-0.253	-0.547
	59	SelfE9	I feel stressed when listening to mathematics teachers in class (-)	603	6	0.713	-0.105
VII. Students' Perceptions of Teachers' Practices (Per)							
	60	Per1	My math teacher is friendly to us (+)	605	4	-1.431	2.422
	61	Per2	My math teacher listens carefully when we ask questions or say something (+)	607	2	-1.255	1.291

	62	Per3	My math teacher understands the problems and difficulties we experience (+)	607	2	-0.702	0.426
	63	Per4	My math teacher tries to make math lessons interesting (+)	602	7	-0.935	0.878
	64	Per5	My math teacher does not really care how we feel in class (-)	608	1	0.943	-0.063
	65	Per6	My math teacher appreciates it when we have tried hard, even if our results are not so good (+)	603	6	-0.55	-0.542
	66	Per7	My math teacher really wants us to enjoy learning new things (+)	607	2	-0.967	1.19
	67	Per8	My math teacher thinks mistakes are bad (-)	602	7	0.26	-0.495
	68	Per9	My math teacher thinks mistakes are okay as long as we are learning something from them (+)	606	3	-0.469	-0.674
	69	Per10	My math teacher first shows step by step how to solve a specific mathematical problem, before giving us similar exercises (+)	609	0	-0.841	0.507
	70	Per11	My math teacher explains why math is important (+)	604	5	-0.775	0.226
	71	Per12	My math teacher gives us time to really explore new problems and try out possible solution procedure (+)	600	9	-1.143	1.496
	72	Per13	My math teacher wants us to understand the content of this math course, not just memorize it (+)	607	2	-1.058	-0.158
	73	Per14	My math teacher lets us do a lot of group work in this math class (+)	604	5	-0.34	-0.453
	74	Per15	My math teacher shows us different strategies to solve problems (+)	608	1	-0.726	0.504
	75	Per16	My math teacher teaches us strategies to remember important math procedures (+)	606	3	-1.256	1.79
	76	Per17	My math teacher teaches us strategies to evaluate our problem solutions (+)	600	9	-0.457	0.652

Appendix G: Descriptive Statistics of MBQ-T

	Total Item No.	Construct item No.	Statement	N	Missing	Skewness	Kurtosis
I. Beliefs About the Certainty of Mathematics Knowledge (CMKt)							
	1	CMKt1	Most of what is true in math is already known (-)	45	1	0.162	0.237
	2	CMKt2	Math is really just knowing the right formula for the problem (-)	45	1	0.000	0.386
	3	CMKt3	It is better when a math teacher shows students lots of different ways to look at the same problem (+)	46	0	-0.181	-2.059
	4	CMKt4	Mathematical theories are the product of creativity (+)	43	3	-0.052	-0.364
	5	CMKt5	In math the answers are always either right or wrong (-)	46	0	-0.008	-0.257
	6	CMKt6	There is no place for students to be creative in math class (-)	45	1	-0.678	0.127
	7	CMKt7	Math problem has always only one true answer (-)	46	0	-0.513	0.318
	8	CMKt8	Answers to questions in math change as mathematicians gather more information (+)	45	1	-0.128	-0.084
II. Beliefs About the Usefulness of Mathematics (UoMt)							
	9	UoMt1	I teach math because I know math is useful (+)	44	2	-0.148	-1.121
	10	UoMt2	Knowing math will help people earn a living (+)	46	0	-0.112	-0.056
	11	UoMt3	Math is a worthwhile and necessary subject (+)	46	0	0.089	-1.028
	12	UoMt4	Math will not be important to students in their life's work (-)	46	0	-0.158	1.899
	13	UoMt5	Understanding math is important for mathematicians, economists, and scientists but not for most people (-)	45	1	0.184	-0.062
	14	UoMt6	The only reason students would take a math class is because it is a requirement (-)	46	0	0.364	0.584
	15	UoMt7	It is important to see the connections between the math students learn in class and real world applications (+)	46	0	-1.276	3.467
	16	UoMt8	Math provides the foundation for most of the principles used in science and business (+)	46	0	0.241	-0.128
	17	UoMt9	Math helps us better understand the world we live in (+)	46	0	-0.135	-0.583

III. Beliefs About Mathematical Problem-Solving (MPSt)							
	18	MPSt1	There are math problems that just can't be solved by following a predetermined sequence of steps (+)	46	0	0.164	1.010
	19	MPSt2	Math problems can be solved without remembering formulas (+)	46	0	0.822	0.783
	20	MPSt3	Memorizing steps is not that useful for learning to solve math problems (+)	46	0	0.474	-0.243
	21	MPSt4	Learning to do math problems is mostly a matter of memorizing the right steps to follow (-)	46	0	0.654	2.229
	22	MPSt5	Time used to investigate why a solution to a math problem works, is time well spent (+)	45	1	0.877	0.946
	23	MPSt6	A student who doesn't understand why an answer to a math problem is correct hasn't really solved the problem (+)	45	1	0.022	-0.560
	24	MPSt7	In addition to getting a right answer in math, it is important to understand why the answer is correct (+)	46	0	0.365	-1.954
	25	MPSt8	It's not important to understand why a mathematical procedure works as long as it gives a correct answer (-)	44	2	-0.632	-0.739
	26	MPSt9	Getting a right answer in math is more important than understanding why the answer works (-)	44	2	0.156	-0.286
	27	MPSt10	It doesn't really matter whether or not students understand a math problem as long as they can get the right answer (-)	45	1	0.069	0.974
IV. Beliefs About Mathematics Teaching (Teach)							
	28	Teach1	It is important for teachers to teach students strategies to solve math problems (+)	46	0	0.559	-1.767
	29	Teach2	Students will learn math better if teachers let them have opportunities to discuss math (+)	45	1	0.211	-0.175
	30	Teach3	Teachers should teach students ways to integrate new information with their existing knowledge of math (+)	46	0	-1.597	7.388
	31	Teach4	Teaching math involves mostly the transmission of math knowledge from teachers to students (-)	46	0	1.402	2.939
	32	Teach5	Telling students the correct answers in math is the most important task for teachers (-)	46	0	-0.581	0.674

	33	Teach6	The main goal of teaching math is to increase the amount of knowledge in the students' memory (-)	46	0	0.143	-0.422
	34	Teach7	Teachers can help students learn math when they teach them problem solving strategies (+)	46	0	-1.305	5.377
	35	Teach8	Repeating math knowledge in class is necessary for students learning (-)	43	3	0.055	0.800
	36	Teach9	Math learning depends mostly on how good the teacher is to tell students what they need to know about math (-)	45	1	-0.011	-0.202
	37	Teach10	Students learn best when they develop their mathematical problem solving skills (+)	46	0	-0.128	-0.684
	38	Teach11	The main goal of teaching math is to help students develop learning strategies in math (+)	46	0	0.048	-0.168
V. Teachers' Self-Efficacy Beliefs (SelfEt)							
	39	SelfEt1	I am certain I can teach math concepts effectively (+)	46	0	-0.276	0.106
	40	SelfEt2	I have no difficulties answering students' math questions (+)	45	1	0.075	-0.232
	41	SelfEt3	I am certain that I am a good teacher of math (+)	44	2	0.709	-0.420
	42	SelfEt4	I often wonder whether I am effective in monitoring math activities (-)	46	0	-0.451	0.666
	43	SelfEt5	Sometimes I doubt whether I understand math concepts well enough to be an effective math teacher (-)	46	0	0.050	-0.192
	44	SelfEt6	I wonder if I have the necessary skills to teach math (-)	46	0	-0.272	-2.016
	45	SelfEt7	Given a choice, I will not invite the principal to evaluate my math teaching (-)	46	0	0.596	-0.567
	46	SelfEt8	Often I do not know what to do to turn students on to math (-)	44	2	0.142	1.693
VI. Beliefs about the role of students' Effort in their learning (BaEt)							
	47	BaEt1	By trying hard, one can become smarter in math (+)	46	0	0.108	2.491
	48	BaEt2	Studying hard can improve one's ability in math (+)	46	0	0.654	2.904
	49	BaEt3	Students can get smarter in math by trying hard (+)	46	0	0.313	0.086
	50	BaEt4	Ability in math increases when one knows the right strategies (+)	46	0	0.879	-1.285
	51	BaEt5	Appropriate study skills can increase one's ability to do math (+)	45	1	0.215	0.500

	52	BaEt6	Students can get smarter in math if they know how to learn math problems (+)	45	1	-1.082	4.860
VII. Beliefs about Students' Mathematics Ability (MAt)							
	53	MAt1	Better study habits are the key to success for students who struggle in math (+)	44	2	0.138	0.176
	54	MAt2	Student who doesn't have high natural ability in math is still capable of learning difficult math material (+)	45	1	-0.252	-0.400
	55	MAt3	Learning good study skills can improve a students' math ability (+)	46	0	0.516	1.058
	56	MAt4	Some people are born with great math ability and some aren't (+)	46	0	-0.354	0.615
	57	MAt5	Math ability is really just something students are born with (-)	45	1	1.269	11.136
	58	MAt6	The smartest math students don't have to do many problems because they just get them quickly (-)	46	0	-0.660	0.537
	59	MAt7	It is frustrating for students to have to work hard to understand a problem (-)	44	2	0.585	-0.018
	60	MAt8	Students can learn new things in math, but they can't really change the math ability they were born with (-)	45	1	0.213	-0.036
	61	MAt9	Most people know at an early age whether they are good at math or not (-)	45	1	-0.637	3.747

REFERENCES

Abedalaziz, N., & Akmar, S. (2012). Epistemology Beliefs About Mathematical Problem Solving Among Malaysian Students. *OIDA International Journal of Sustainable Development, 5*(1), 59-74.

Ambrose, R., Clement, L., Philipp, R., & Chauvot, J. (2004). Assessing Prospective Elementary School Teachers' Beliefs About Mathematics and Mathematics Learning: Rationale and Development of a Constructed-Response-Format Beliefs Survey. *School Science and Mathematics, 104*(2), 56-69. doi:10.1111/j.1949-8594.2004.tb17983.x

Ampadu, E. (2012). Students' Perceptions of their Teachers' Teaching of Mathematics: The Case of Ghana. *International Online Journal of Educational Sciences, 4*(2), 351-358.

Anderson, D. S., & Piazza, J. A. (1996). Changing Beliefs: Teaching and Learning Mathematics in Constructivist Preservice Classrooms. *Action in Teacher Education, 18*(2), 51-62. doi:10.1080/01626620.1996.10462833

Anderson, J., White, P., & Sullivan, P. (2005). Using a schematic model to represent influences on, and relationships between, teachers' problem-solving beliefs and practices. *Mathematics Education Research Journal, 17*(2), 9-38. doi:10.1007/BF03217414

Andrews, P. (2015). The Curricular Importance of Mathematics: A Comparison of English and Hungarian Teachers' Espoused Beliefs. *Journal of Curriculum Studies, 39*(3), 317-338.

Andrews, P., & Diego-Mantecón, J. (2015). Instrument Adaptation in Cross-Cultural Studies of Students' Mathematics-Related Beliefs: Learning from Healthcare Research. *Compare: A Journal of Comparative and International Education, 45*(4), 545-567. doi:10.1080/03057925.2014.884346

Andrews, P., Diego-Mantecón, J., Vankúš, P., & Op 't Eynde, P. (2011). Construct Consistency in the Assessment of Students' Mathematics-related Beliefs: A Three-way Cross-sectional Pilot Comparative Study. *Acta Didactica Universitatis Comenianae Mathematics*(11), 1-25.

Areepattamannil, S., & Kaur, B. (2013). *Mathematics Teachers' Perceptions of Their Students' Mathematical Competence: Relations to Mathematics Achievement, Affect, and Engagement in Singapore and Australia*. Paper presented at the Annual Meeting of the Mathematics Education Research Group of Australasia (MERGA), Melbourne, Victoria, Australia.

Arikan, S., van de Vijver, F. J. R., & Yagmur, K. (2016). Factors Contributing to Mathematics Achievement Differences of Turkish and Australian Students in TIMSS 2007 and 2011. *EURASIA Journal of Mathematics, Science & Technology Education, 12*(8), 2039-2059. doi:10.12973/eurasia.2016.1268a

Arredondo, D. E., & Rucinski, T. T. (1996). *Epistemological Beliefs of Chilean Educators and School Reform Efforts*. Paper presented at the Tercer Encuentro National de Enfoques Cognitivos Actuales en Educacion, Santiago, Chile, November 7-8.

Ayebo, A., & Mrutu, A. (2019). An Exploration of Calculus Students' Beliefs about Mathematics. *International Electronic Journal of Mathematics Education, 14*(2), 385-392.

Bandura, A. (1977). Self-efficacy: Toward a Unifying Theory of Behavioral Change. *Psychological Review, 84*(2), 191–215.

Bandura, A. (1993). Perceived Self-Efficacy in Cognitive Development and Functioning. *Educational Psychologist, 28*(2), 117-148.

Bandura, A. (2006). Guide for Creating Self-Efficacy Scales. In F. Pajares & T. Urdan (Eds.), *Self-Efficacy Beliefs of Adolescents* (pp. 307–337): Information Age Publishing.
Barkatsas, A. T., & Malone, J. (2005). A Typology of Mathematics Teachers' Beliefs about

Teaching and Learning Mathematics and Instructional Practices. *Mathematics Education Research Journal, 17*(2), 69-90.
Baxter Magolda, M. B. (2002). Epistemological Reflection: The Evolution of Epistemological Assumptions from Age 18 to 30. In B. K. Hofer & P. R. Pintrich (Eds.), *Personal Epistemology: The Psychology of Beliefs about Knowledge and Knowing* (pp. 89-102). New York: Routledge.
Bendixen, L. D., & Feucht, F. C. (2010). *Personal Epistemology in the Classroom: Theory, Research, and Implications for Practice*: Cambridge University Press.
Bendixen, L. D., Schraw, G., & Dunkle, M. E. (1998). Epistemic Beliefs and Moral Reasoning. *The Journal of Psychology, 132*(2), 187-200. doi:10.1080/00223989809599158
Bentler, P. M. (1990). Comparative Fit Indexes in Structural Models. *Psychological Bulletin, 107*(2), 238–246.
Berkaliev, Z., & Kloosterman, P. (2009). Undergraduate Engineering Majors' Beliefs About Mathematics. *School Science and Mathematics, 109*(3), 175-182.
Beswick, K. (2005). The Beliefs/Practice Connection in Broadly Defined Contexts. *Mathematics Education Research Journal, 17*(2), 39-68. doi:10.1007/BF03217415
Beswick, K. (2012). Teachers' Beliefs about School Mathematics and Mathematicians' Mathematics and Their Relationship to Practice. *Educational Studies in Mathematics: An International Journal, 79*, 127-147. doi:10.1007/s10649-011-9333-2
Beswick, K. (2019). The Role of Knowledge and Beliefs in Helping Learners to Progress Their Mathematical Understanding. *Journal of Mathematics Teacher Education, 22*, 125–128. doi:10.1007/s10857-019-09432-5
Blömeke, S., Felbrich, A., Müller, C., Kaiser, G., & Lehmann, R. (2008). Effectiveness of Teacher Education. *ZDM Mathematics Education, 40*, 719-734. doi:10.1007/s11858-008-0096-x
Blömeke, S., Hoth, J., Döhrmann, M., Busse, A., Kaiser, G., & König, J. (2015). Teacher Change during Induction: Development of Beginning Primary Teachers' Knowledge, Beliefs and Performance. *International Journal of Science and Mathematics Education, 13*(2), 287-308.
Bonne, L., & Johnston, M. (2016). Students' Beliefs about Themselves as Mathematics Learners. *Thinking Skills and Creativity, 20*, 17-28. doi:10.1016/j.tsc.2016.02.001
Broadbooks, W. J., Elmore, P. B., Pedersen, K., & Bleyer, D. R. (1981). A Construct Validation Study of the Fennema-Sherman Mathematics Attitudes Scales. *Educational and Psychological Measurement, 41*(2), 551-557. doi:10.1177/001316448104100238
Brown, T. A. (2006). *Confirmatory Factor Analysis for Applied Research*. New York: The Guilford Press.
Bryan, C. A., Wang, T., Perry, B., Wong, N.-Y., & Cai, J. (2007). Comparison and Contrast: Similarities and Differences of Teachers' Views of Effective Mathematics Teaching and Learning from Four Regions. *ZDM Mathematics Education, 39*(4), 329-340.
Buehl, M. M. (2003). *At the Crossroads of Epistemological and Motivation: Modeling the Relations between Students' Domain-specific Epistemological Beliefs, Achievement Motivation, and Task Performance.* (Doctoral dissertation), Retrieved from Proquest Digital Dissertations (3107202)
Buehl, M. M., & Alexander, P. A. (2001). Beliefs About Academic Knowledge. *Educational Psychology Review, 13*(4), 385-418. doi:10.1023/A:1011917914756

Buehl, M. M., & Alexander, P. A. (2005). Motivation and Performance Differences in Students' Domain-Specific Epistemological Belief Profiles. *American Educational Research Journal, 42*(4), 697-726. doi:10.3102/00028312042004697

Buehl, M. M., Alexander, P. A., & Murphy, P. K. (2002). Beliefs about Schooled Knowledge: Domain Specific or Domain General? *Contemporary Educational Psychology, 27*(3), 415-449. doi:10.1006/ceps.2001.1103

Buehl, M. M., & Beck, J. S. (2015). Beliefs and Teachers' Practices. In H. Fives & M. G. Gill (Eds.), *International handbook of research on teachers' beliefs* (pp. 336-352): Routledge.

Byrne, B. M. (2012). *Structural Equation Modeling With Mplus: Basic Concepts, Applications, and Programming*: Routledge.

Callejo, M. L., & Vila, A. (2009). Approach to Mathematical Problem Solving and Students' Belief Systems: Two Case Studies. *Educational Studies in Mathematics, 72*, 111-126. doi:10.1007/s10649-009-9195-z

Campbell, P. F., Nishio, M., Smith, T. M., Clark, L. M., Conant, D. L., Rust, A. H., . . . Choi, Y. (2014). The Relationship Between Teachers' Mathematical Content and Pedagogical Knowledge, Teachers' Perceptions, and Student Achievement. *Journal for Research in Mathematics Education, 45*(4), 419-459.

Cano, F. (2005). Epistemological Beliefs and Approaches to Learning: Their Change through Secondary School and Their Influence on Academic Performance. *British Journal of Educational Psychology, 75*, 203-221. doi:10.1348/000709904X22683

Chang, Y.-L. (2015). Examining Relationships among Elementary Mathematics Teacher Efficacy and Their Students' Mathematics Self-efficacy and Achievement. *EURASIA Journal of Mathematics, Science & Technology Education, 11*(6), 1307-1320.

Chrysostomou, M., & Philippou, G. (2010). Teachers' Epistemological Beliefs and Efficacy Beliefs about Mathematics. *Procedia - Social and Behavioral Sciences, 9*, 1509-1515. doi:10.1016/j.sbspro.2010.12.357

Cooney, T. (2003). *Mathematics Teacher Education in Rural Communities: Developing a Foundation for Action*. Paper presented at the ACCLAIM Research Symposium, McArthur, OH.

Creswell, J. W. (2014). *Educational Research: Planning, Conducting and Evaluating Quantitative and Qualitative Research* (Fourth ed.). Boston: Pearson.

Cross, D. I. (2009). Alignment, Cohesion, and Change: Examining Mathematics Teachers' Belief Structures and Their Influence on Instructional Practices. *Journal of Mathematics Teacher Education, 12*(5), 325-346. doi:10.1007/s10857-009-9120-5

Cross, D. I. (2015). Dispelling the Notion of Inconsistencies in Teachers' Mathematics Beliefs and Practices: A 3-year Case Study. *Journal of Mathematics Teacher Education, 18*(2), 173-201. doi:10.1007/s10857-014-9276-5

Dahlin, B., & Watkins, D. (2000). The Role of Repetition in the Processes of Memorising and Understanding: A Comparison of the Views of German and Chinese Secondary School Students in Hong Kong. *British Journal of Educational Psychology, 70*(1), 65-84.

Darmawan, I. G. N. (2016). Assessing the Quality and Equity of Student Performance in Five Southeast Asian Countries. In L. M. Thien, N. A. Razak, J. P. Keeves, & I. G. N. Darmawan (Eds.), *What Can PISA 2012 Data Tell Us? Performance and Challenges in Five Participating Southeast Asian Countries* (pp. 159-180). The Netherlands: Sense Publishers.

Darmawan, I. G. N., Vosniadou, S., Lawson, M. J., Van Deur, P., & Wyra, M. (2020). The Development of An Instrument to Test Pre-service Teachers' Beliefs Consistent and Inconsistent with Self-regulation Theory. *The British journal of educational psychology*, e12345. doi:10.1111/bjep.12345

De Corte, E. (2015). Mathematics-related Beliefs of Ecuadorian Students of Grades 8–10. *International Journal of Educational Research, 72*, 1-13.
De Corte, E., Op't Eynde, P., & Verschaffel, L. (2002). Knowing What to Believe: The Relevance of Students' Mathematical Beliefs for Mathematics Education. In B. K. Hofer & P. R. Pintrich (Eds.), *Personal Epistemology: The Psychology of Beliefs about Knowledge and Knowing* (pp. 297-320). New York: Routledge.
Debellis, V. A. (1996). *Interactions between Affect and Cognition during Mathematical Problem-solving: A two-year Case Study of Four Elementary School Children.* (Doctoral dissertation), Retrieved from Proquest Digital Dissertations (9630716)
DeBellis, V. A., & Goldin, G. A. (1997). The Affective Domain in Mathematical Problem Solving. In E. Pehkonen (Ed.), *Proceedings of the 21st Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 209-216). University of Helsinki: Lahti, Finland: PME Program Committee Publisher.
Debellis, V. A., & Goldin, G. A. (2006). Affect and Meta-Affect in Mathematical Problem Solving: A Representational Perspective. *Educational Studies in Mathematics, 63*(2), 131-147.
Depaepe, F., De Corte, E., & Verschaffel, L. (2016). Mathematical Epistemological Beliefs. In J. A. Greene, W. A. Sandoval, & I. Bråten (Eds.), *Handbook of Epistemic Cognition* (pp. 147-164). London: Routledge.
Diego-Mantecón, J., Blanco, T., Chamoso, J., & Cáceres, M. (2019). An Attempt to Identify the Issues Underlying the Lack of Consistent Conceptualisations in the field of Student Mathematics-related Beliefs. *PLoS One, 14*(11), e0224696. doi:10.1371/journal.pone.0224696
Dowker, A., Sarkar, A., & Looi, C. Y. (2016). Mathematics Anxiety: What Have We Learned in 60 Years? *Frontiers in Psychology, 7*. doi:10.3389/fpsyg.2016.00508
Dweck, C. S. (1986). Motivational Processes Affecting Learning. *American Psychologist, 41*(10), 1040–1048.
Dweck, C. S. (2000). *Self-theories: Their Role in Motivation, Personality, and Development*. New York: Psychology Press.
Dweck, C. S., & Leggett, E. (1988). A Social-Cognitive Approach to Motivation and Personality. *Psychological Review, 95*(2), 256-273.
Elby, A., Macrander, C., & Hammer, D. (2016). Epistemic Cognition in Science. In J. A. Greene, W. A. Sandoval, & I. Bråten (Eds.), *Handbook of Epistemic Cognition.* (pp. 113-127). London: Routledge.
Ernest, P. (1989a). The Impact of Beliefs on the Teaching of Mathematics. In P. Ernest (Ed.), *Mathematics Teaching: The State of the Art*. London: Falmer Press.
Ernest, P. (1989b). The Knowledge, Beliefs and Attitudes of the Mathematics Teacher: A Model. *Journal of Education for Teaching, 15*(1), 13-33.
Espinosa, A. (2018). It's not What You Think: Perceptions Regarding the Usefulness of Mathematics May Hinder Performance. *Journal of Numerical Cognition, 4*(1), 235–242.
Felbrich, A., Kaiser, G., & Schmotz, C. (2012). The Cultural Dimension of Beliefs: An Investigation of Future Primary Teachers' Epistemological Beliefs Concerning the Nature of Mathematics in 15 Countries. *The International Journal on Mathematics Education, 44*, 355-366. doi:10.1007/s11858-012-0418-x
Fennema, E. (1989). The Study of Afect and Mathematics: A Proposed Generic Model for Research. In D. B. McLeod & V. M. Adams (Eds.), *Affect and Mathematical Problem Solving: A New Perspective* (pp. 205-219). New York: Springer-Verlag.
Fennema, E., & Sherman, J. (1976). Fennema-Sherman Mathematics Attitude Scale. *Journal for Research in Mathematics Education, 7*(5), 324-326.

Fennema, E., & Sherman, J. (1977). Sex-Related Differences in Mathematics Achievement, Spatial Visualization and Affective Factors. *American Educational Research Journal, 14*(1), 51-71. doi:10.3102/00028312014001051

Ferla, J., Valcke, M., & Cai, Y. (2009). Academic Self-efficacy and Academic Self-concept: Reconsidering Structural Relationships. *Learning and Individual Differences, 19*(4), 499-505. doi:10.1016/j.lindif.2009.05.004

Fidell, L. S., & Tabachnick, B. G. (2014). *Using Multivariate Statistics* (Sixth ed.): Pearson.

Finch, W. H., & Bolin, J. E. (2017). *Multilevel Modeling Using Mplus*: Taylor & Francis Group, LLC.

Fives, H., & Gill, M. G. (2015). *International Handbook of Research on Teachers' Beliefs*. New York: Routledge.

Francis, D. C., Rapacki, L., & Eker, A. (2015). The Individual, the Context, and Practice: A Review of the Research on Teachers' Beliefs Related to Mathematics. In H. Fives & M. G. Gill (Eds.), *International Handbook of Research on Teachers' Beliefs* (pp. 336-352). New York: Routledge.

Furinghetti, F., & Pehkonen, E. (2002). Rethinking Characterizations of Beliefs. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A Hidden Variable in Mathematics Education?* (pp. 39-57): Springer Netherlands.

Garofalo, J. (1989). Beliefs and Their Influence on Mathematical Performance. *The Mathematics Teacher, 82*(7), 502-505.

Goldin, G. A. (2002). Affect, Meta-Affect, and Mathematical Belief Structures. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A Hidden Variable in Mathematics Education?* (pp. 59-72): Springer Netherlands.

Goldin, G. A., Hannula, M. S., Heyd-Metzuyanim, E., Jansen, A., Kaasila, R., Lutovac, S., . . . Zhang, Q. (2016). *Attitudes, Beliefs, Motivation and Identity in Mathematics Education: An Overview of the Field and Future Directions*: Springer Open.

Goldin, G. A., Rösken, B., & Törner, G. (2009). Beliefs - No longer a Hidden Variable in Mathematical Teaching and Learning Processes. In J. Maaß & W. Schloeglmann (Eds.), *Beliefs and Attitudes in Mathematics Education: New Research Results* (pp. 1-18). Rotterdam: Sense.

Grootenboer, P., & Marshman, M. (2016). *Mathematics, Affect and Learning: Middle School Students' Beliefs and Attitudes About Mathematics Education*. Singapore: Springer.

Gurría, A. (2016). *PISA 2015 Results in Focus*. Retrieved from OECD:

Hae-Young, K. (2013). Statistical Notes for Clinical Researchers: Assessing Normal Distribution (2) using Skewness and Kurtosis. *Restorative Dentistry & Endodontics, 38*(1), 52-54. doi:10.5395/rde.2013.38.1.52

Hailikari, T., Nevgi, A., & Komulainen, E. (2008). Academic Self-beliefs and Prior Knowledge as Predictors of Student Achievement in Mathematics: A Structural Model. *Educational Psychology, 28*(1), 59-71. doi:10.1080/01443410701413753

Hair, J. F., Black, W. C., Babin, B. J., & Anderson, R. E. (2019). *Multivariate Data Analysis* (8th ed.): Cengage, Annabel Ainscow.

Handal, B., & Herrington, A. (2003). Mathematics Teachers' Beliefs and Curriculum Reform. *Mathematics Education Research Journal, 15*(1), 59-69.

Hannula, M. S. (2012). Exploring New Dimensions of Mathematics-related Affect: Embodied and Social Theories. *Research in Mathematics Education, 14*(2), 137-161. doi:10.1080/14794802.2012.694281

Hannula, M. S., & Laakso, J. (2011). The Structure of Mathematics Related Beliefs, Attitudes and Motivation among Finnish Grade 4 and Grade 8 Students. In B. Ubuz (Ed.), *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 9-16). Ankara, Turkey: PME.

Hannula, M. S., Leder, G. C., Morselli, F., Vollstedt, M., & Zhang, Q. (2019). *Affect and Mathematics Education: Fresh Perspectives on Motivation, Engagement, and Identity*: Springer Open.

Hannula, M. S., Op't Eynde, P., Schlöglmann, W., & Wedege, T. (2007). Affect and Mathematical Thinking. In D. Pitta Pantazi & G. Philippou (Eds.), *Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education*. Department of Education: University of Cyprus.

Harrington, D. (2008). *Confirmatory Factor Analysis*: Oxford University Press.

Heck, R. H., & Thomas, S. L. (2015). *An Introduction to Multilevel Modeling Techniques: MLM and SEM Approaches using Mplus* (3rd ed.): Routledge.

Hemmings, B., & Kay, R. (2010). Prior Achievement, Effort, and Mathematics Attitude as Predictors of Current Achievement. *The Australian Educational Researcher, 37*(2), 41-58.

Hoang, T. H. (2007). *Learning and Instruction in Mathematics: A Study of Achievement in Saigon, Vietnam.* International Journal for Mathematics Teaching and Learning. Retrieved from http://www.cimt.org.uk/journal/hoang.pdf

Hofer, B. K. (2000). Dimensionality and Disciplinary Differences in Personal Epistemology. *Contemporary Educational Psychology, 25*(4), 378-405. doi:10.1006/ceps.1999.1026

Hofer, B. K. (2001). Personal Epistemology Research: Implications for Learning and Teaching. *Educational Psychology Review, 13*, 353-383. doi:10.1023/A:1011965830686

Hofer, B. K. (2002). Personal Epistemology as a Psychological and Educational Construct: An Introduction. In B. K. Hofer & P. R. Pintrich (Eds.), *Personal Epistemology: The Psychology of Beliefs about Knowledge and Knowing*. New York: Routledge.

Hofer, B. K. (2004). Introduction: Paradigmatic Approaches to Personal Epistemology. *Educational Psychologist, 39*(1), 1-3. doi:10.1207/s15326985ep3901_1

Hofer, B. K. (2006). Beliefs About Knowledge and Knowing: Integrating Domain Specificity and Domain Generality: A Response to Muis, Bendixen, and Haerle (2006). *Educational Psychology Review, 18*, 67-76. doi:10.1007/s10648-006-9000-9

Hofer, B. K., & Pintrich, P. R. (1997). The Development of Epistemological Theories: Beliefs About Knowledge and Knowing and Their Relation to Learning. *Review of Educational Research, 67*(1), 88-140. doi:10.3102/00346543067001088

Hoffman, B., & Schraw, G. (2009). The Influence of Self-efficacy and Working Memory Capacity on Problem-solving Efficiency. *Learning and Individual Differences, 19*(1), 91-100. doi:10.1016/j.lindif.2008.08.001

Hu, S. (2018). *Affect, Motivation, and Engagement in the Context of Mathematics Education: Testing a Dynamic Model of Interactive Relationships.* (Doctoral dissertation, University of Kentucky, Kentucky), Retrieved from https://uknowledge.uky.edu/edp_etds/71/

Hunt, T. E., Clark-Carter, D., & Sheffield, D. (2015). Exploring the Relationship Between Mathematics Anxiety and Performance: An Eye-Tracking Approach. *Applied Cognitive Psychology, 29*, 226-231. doi:10.1002/acp.3099

Iannone, P., & Simpson, A. (2019). The Relation between Mathematics Students' Discipline-Based Epistemological Beliefs and Their Summative Assessment Preferences. *International Journal of Research in Undergraduate Mathematics Education, 5*(2), 147-162. doi:10.1007/s40753-019-00086-5

IBM Corp. (Released 2017). IBM SPSS Statistics for Windows, Version 25.0. Armonk, NY: IBM Corp.

Ibrahim, A.-W. (2014). The Students' Perception of Teachers' Classroom Effectiveness on Their Self-Concepts in Lagos Metropolis. *Journal of Teaching and Teacher Education, 2*(2), 133-141.

Jehng, J. J., Johnson, S. D., & Anderson, R. C. (1993). Schooling and Students' Epistemological Beliefs about Learning. *Contemporary Educational Psychology, 18*(1), 23-35. doi:10.1006/ceps.1993.1004

Justicia-Galiano, M. J., Martín-Puga, M. E., Linares, R., & Pelegrina, S. (2017). Math Anxiety and Math Performance in Children: The Mediating Roles of Working Memory and Math Self-concept. *British Journal of Educational Psychology, 87*(4), 573-589. doi:10.1111/bjep.12165

Kabiri, M., & Kiamanesh, A. R. (2004). *The Role of Self-Efficacy, Anxiety, Attitudes and Previous Math Achievement in Students' Math Performance*. Paper presented at the 3rd International Biennial SELF Research Conference, Berlin.

Kadijevich, D. (2008). TIMSS 2003: Relating Dimensions of Mathematics Attitude to Mathematics Achievement. *Zbornik: Institut za Pedagoška Istraživanja, 40*(2), 327-346. doi:10.2298/ZIPI0802327K

Kelloway, E. K. (2015). *Using Mplus for Structural Equation Modeling: A Researcher's Guide* (Second ed.): SAGE Publications.

Khine, M. S. (2008). *Knowing, Knowledge and Beliefs: Epistemological Studies across Diverse Cultures* (1st ed.). Dordrecht: Springer Netherlands.

Khine, M. S. (2013). *Application of Structural Equation Modeling in Educational Research and Practice*: Sense Publishers.

Kitchener, K. S. (1983). Cognition, Metacognition, and Epistemic Cognition: A Three-level Model of Cognitive Processing. *Human Development, 4*, 222-232.

Kiwanuka, H. N., Van Damme, J., Van Den Noortgate, W., Anumendem, D. N., Vanlaar, G., Reynolds, C., & Namusisi, S. (2017). How do Student and Classroom Characteristics Affect Attitude toward Mathematics? A Multivariate Multilevel Analysis. *School Effectiveness and School Improvement, 28*(1), 1-21. doi:10.1080/09243453.2016.1201123

Kline, R. B. (2011). *Principles and Practice of Structural Equation Modeling* (Third ed.). New York: The Guilford Press.

Kloosterman, P. (1988). Self-confidence and Motivation in Mathematics. *Journal of Educational Psychology, 80*(3), 345–351.

Kloosterman, P. (1996). Students' Beliefs about Knowing and Learning Mathematics: Implications for Motivation. In M. Carr (Ed.), *Motivation in Mathematics* (pp. 131-156). Cresskill, NJ: Hampton Press.

Kloosterman, P. (2002). Beliefs about Mathematics and Mathematics Learning in the Secondary School: Measurement and Implications for Motivation. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A Hidden Variable in Mathematics Education?* (pp. 247-270). Dordrecht: Kluwer Academic Publishers.

Kloosterman, P., & Stage, F. K. (1992). Measuring Beliefs About Mathematical Problem Solving. *School Science and Mathematics, 92*(3), 109-115.

Köller, O. (2001). Mathematical World Views and Achievement in Advanced Mathematics in Germany: Findings from TIMSS Population 3. *Studies in Educational Evaluation, 27*(1), 65-78. doi:10.1016/S0191-491X(01)00014-1

Kurniatia, I., & Suryab, E. (2017). Student's Perception of their Teacher Teaching Style's. *International Journal of Sciences: Basic and Applied Research, 33*(2), 91-98.

Lawson, M., Vosniadou, S., Deur, P., Wyra, M., & Jeffries, D. (2019). Teachers' and Students' Belief Systems About the Self-Regulation of Learning. *Educational Psychology Review, 31*(1), 223-251. doi:10.1007/s10648-018-9453-7

Le, C. L. V. (2009). Education Reform in Lower Secondary Education in Vietnam. In Y. Hirosato & Y. Kitamura (Eds.), *The Political Economy of Educational Reforms and Capacity Development in Southeast Asia: Cases of Cambodia, Laos and Vietnam* (Vol. 13, pp. 217-236). Dordrecht, Netherlands: Springer.

Le, N. S. (2015). Dạy học Toán ở Trường Phổ thông theo Định hướng Phát triển Năng lực [Teaching Mathematics at Schools to Develop Students' Competencies]. *Tạp chí Giáo dục, 350*, 31-33.

Leder, G. C. (2019). Mathematics-Related Beliefs and Affect. In M. Hannula, G. C. Leder, F. Morselli, M. Vollstedt, & Q. Zhang (Eds.), *Affect and Mathematics Education: Fresh Perspectives on Motivation, Engagement, and Identity* (pp. 15-35): Springer Open.

Leder, G. C., & Grootenboer, P. (2005). Affect and Mathematics Education. *Mathematics Education Research Journal, 17*(2), 1-8. doi:10.1007/BF03217413

Leder, G. C., Pehkonen, E., & Törner, G. (2002). *Beliefs: A Hidden Variable in Mathematics Education?* The Netherlands: Springer.

Leder, G. C., Pehkonen, E., & Törner, G. (2002). Chapter 1: Setting the Scene. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A Hidden Variable in Mathematics Education?* (pp. 1-12). The Netherlands: Springer.

Leung, F. K. (2001). In Search of an East Asian Identity in Mathematics Education. *An International Journal, 47*(1), 35-51. doi:10.1023/A:1017936429620

Li, M., & Yu, P. (2010). Study on the Inconsistency between a Pre-service Teacher's Mathematics Education Beliefs and Mathematics Teaching Practice. *Journal of Mathematics Education, 3*(2), 40-57.

Liljedahl, P. (2019). *Mathematical Problem Solving: Current Themes, Trends, and Research*: Cham: Springer.

Liljedahl, P., Rolka, K., & Rösken, B. (2007). Affecting Affect: The Re-education of Preservice Teachers' Beliefs about Mathematics and Mathematics Learning and Teaching. In M. Strutchens & W. Martin (Eds.), *69th NCTM Yearbook – The Learning of Mathematics* (pp. 319-330): National Council of Teachers of Mathematics.

Liu, X., & Koirala, H. (2009). *The Effect of Mathematics Self-Efficacy on Mathematics Achievement of High School Students*. Paper presented at the Northeastern Educational Research Association (NERA) Annual Conference, Connecticut.

Lodewyk, K. R. (2007). Relations among Epistemological Beliefs, Academic Achievement, and Task Performance in Secondary School Students. *Educational Psychology, 27*(3), 307-327. doi:10.1080/01443410601104080

Ly, B. H., & Brew, C. (2010). Philosophical and Pedagogical Patterns of Beliefs among Vietnamese and Australian Mathematics Preservice Teachers: A Comparative Study. *Australian Journal of Teacher Education, 35*(2), 67-87.

Ma, X. (1999). A Meta-Analysis of the Relationship between Anxiety toward Mathematics and Achievement in Mathematics. *Journal for Research in Mathematics Education, 30*(5), 520-540.

Ma, X., & Kishor, N. (1997). Assessing the Relationship between Attitude toward Mathematics and Achievement in Mathematics: A Meta-Analysis. *Journal for Research in Mathematics Education, 28*(1), 26-47. doi:10.2307/749662

Manzano-Sanchez, H., Outley, C., Gonzalez, J. E., & Matarrita-Cascante, D. (2018). The Influence of Self-Efficacy Beliefs in the Academic Performance of Latina/o Students in the United States: A Systematic Literature Review. *Hispanic Journal of Behavioral Sciences, 40*(2), 176–209.

Mason, L. (2003). High School Students' Beliefs about Math, Mathematical Problem Solving, and Their Achievement in Math: A Cross-sectional Study. *Educational Psychology, 23*, 73–85.

Mason, L., Boscolo, P., Tornatora, M., & Ronconi, L. (2013). Besides Knowledge: A Cross-sectional Study on The Relations between Epistemic Beliefs, Achievement Goals, Self-beliefs, and Achievement in Science. *Instructional Science, 41*(1), 49-79. doi:10.1007/s11251-012-9210-0

Mason, L., & Scrivani, L. (2004). Enhancing Students' Mathematical Beliefs: An Intervention Study. *Learning and Instruction, 14*(2), 153-176. doi:10.1016/j.learninstruc.2004.01.002

McLeod, D. B. (1989a). Beliefs, Attitudes, and Emotions: New Views of Affect in Mathematics Education. In D. B. McLeod & V. M. Adams (Eds.), *Affect and Mathematical Problem Solving: A New Perspective* (pp. 245-255). New York: Springer-Verlag.
McLeod, D. B. (1989b). The Role of Affect in Mathematical Problem Solving. In D. B. McLeod & V. M. Adams (Eds.), *Affect and Mathematical Problem Solving: A New Perspective* (pp. 20-36). New York: Springer-Verlag.
McLeod, D. B. (1992). Research on Affect in Mathematics Education: A Reconceptualization. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 575–596). New York: Macmillan.
McLeod, D. B., & Adams, V. M. (1989). *Affect and Mathematical Problem Solving: A New Perspective*. New York: Springer-Verlag.
Meiyue, J., Feng, D., & Yanmin, G. (2008). A Comparative Study on Chinese Teachers' and Students' Beliefs about Mathematics, Mathematics Teaching and Learning in Middle School. *Research in Mathematical Education, 12*(3), 235-249.
Midgley, C., Feldlaufer, H., & Eccles, J. S. (1989). Student/Teacher Relations and Attitudes toward Mathematics before and after the Transition to Junior High School. *Child Development, 60*, 981–992.
Minister of Education and Training. (2018a). Chương trình Giáo dục Phổ thông môn Toán - Thông tư số 32/2018/TT-BGDĐT ngày 26 tháng 12 năm 2018 của Bộ trưởng Bộ Giáo dục và Đào tạo về việc Ban hành Chương trình Giáo dục Phổ thông [Mathematics Education Curiculum - Decree No. 32/2018/TT-BGDĐT dated 26th December 2018 isued General Education Curiculum in Vietnam].
Minister of Education and Training. (2018b). Thông tư số 32/2018/TT-BGDĐT ngày 26 tháng 12 năm 2018 của Bộ trưởng Bộ Giáo dục và Đào tạo về việc Ban hành Chương trình Giáo dục Phổ thông [Decree No. 32/2018/TT-BGDĐT dated 26th December 2018 isued General Education Curiculum in Vietnam].
Mosvold, R., & Fauskanger, J. (2014). Teachers’ Beliefs about Mathematical Knowledge for Teaching Definitions. *International Electronic Journal of Mathematics Education, 8*(2-3), 43–61.
Muis, K. R. (2004). Personal Epistemology and Mathematics: A Critical Review and Synthesis of Research. *Review of Educational Research, 74*(3), 317-377.
Muis, K. R. (2008). Epistemic Profiles and Self-Regulated Learning: Examining Relations in the Context of Mathematics Problem Solving. *Contemporary Educational Psychology, 33*(2), 177-208. doi:10.1016/j.cedpsych.2006.10.012
Muis, K. R., Bendixen, L. D., & Haerle, F. (2006). Domain-Generality and Domain-Specificity in Personal Epistemology Research: Philosophical and Empirical Reflections in the Development of a Theoretical Framework. *Educational Psychology Review, 18*(1), 3-54. doi:10.1007/s10648-006-9003-6
Muis, K. R., & Foy, M. J. (2010). The Effects of Teachers’ Beliefs on Elementary Students’ Beliefs, Motivation, and Achievement in Mathematics. In L. D. Bendixen & F. C. Feucht (Eds.), *Personal Epistemology in the Classroom: Theory, Research, and Implications for Practice* (pp. 435-469): Cambridge University Press.
Mulhern, F., & Rae, G. (1998). Development of a Shortened form of the Fennema-Sherman Mathematics Attitudes Scales. *Educational and Psychological Measurement, 58*(2), 295-306. doi:10.1177/0013164498058002012
Murdock, T. B., & Miller, A. D. (2003). Students' Motivational Identity: Variable- Centered and Person - Centered Analytic Approaches. *The Elementary School Journal, 103*(4), 383-399.
Muthén, L. K., & Muthén, B. O. (1998-2017). *Mplus User’s Guide* (Eighth ed.). Los Angeles, CA: Muthén & Muthén.

Namkung, J. M., Peng, P., & Lin, X. (2019). The Relation Between Mathematics Anxiety and Mathematics Performance Among School-Aged Students: A Meta-Analysis. *Review of Educational Research, 89*(3), 459-496. doi:10.3102/0034654319843494
Nasser, F., & Birenbaum, M. (2005). Modeling Mathematics Achievement of Jewish and Arab Eighth Graders in Israel: The Effects of Learner-Related Variables. *Educational Research and Evaluation, 11*(3), 277-302. doi:10.1080/13803610500101108
Neber, H., & Schommer-Aikins, M. (2002). Self-regulated Science Learning with Highly Gifted Students: The Role of Cognitive, Motivational, Epistemological, and Environmental Variables. *High Ability Studies, 13*(1), 59-74. doi:10.1080/13598130220132316
Nguyen, H. C. (2012). Giải quyết Vấn đề trong Môn Toán – Xu hướng Nghiên cứu và Thực tiễn Dạy học [Problem solving in mathematics - Study trends and teaching practice]. *Tạp chí Khoa học giáo dục, 87*, 6-9, 46.
Nguyen, T. L. P. (2014). Đề xuất Cấu trúc và Chuẩn Đánh giá Năng lực Giải quyết vấn đề trong Chương trình Giáo dục Phổ thông mới [Propose Structure and Assessment Standards of Problem Solving in New Curriculum]. *Tạp chí Khoa học giáo dục, 111*.
Nguyen, T. L. P. (2015). Đánh giá Năng lực Giải quyết Vấn đề ở Trường Phổ thông [Assessing Problem Solving Capability at Schools]. *Tạp chí Khoa học giáo dục, 112*.
Nguyen, T. L. P., & Dang, X. C. (2015). Xây dựng Công cụ Đánh giá Năng lực Giải quyết Vấn đề ở Trường Phổ thông [Develop Instruments to Measure Problem Solving Capability at Schools]. *Tạp chí Khoa học giáo dục, 113*.
Nguyen, T. T., Trinh, T. P. T., Ngo, H. T. V., Hoang, N. A., Tran, T., Pham, H. H., & Bui, V. N. (2020). Realistic Mathematics Education in Vietnam: Recent Policies and Practices. *International Journal of Education and Practice*.
Nguyen, T. T., Trinh, T. P. T., & Tran, T. (2019). Realistic Mathematics Education (RME) and Didactical Situations in Mathematics (DSM) in the Context of Education Reform in Vietnam. *Journal of Physics: Conference Series, 1340*(012032).
OECD. (2006). *PISA Released Items - Mathematics*. OECD. Retrieved from https://www.oecd.org/pisa/38709418.pdf
OECD. (2013). *PISA 2012 Results: Ready to Learn: Students' Engagement, Drive and Self-Beliefs (Volume III)*: OECD Publishing.
OECD. (2014). *PISA 2012 Results: What Students Know and Can Do – Student Performance in Mathematics, Reading and Science (Volume I, Revised edition, February 2014)*: OECD Publishing.
OECD. (2019). *PISA 2018 Results (Volume I): What Students Know and Can Do*: OECD Publishing.
Op't Eynde, P., & De Corte, E. (2003). *Student's Mathematics-related Belief Systems: Design and Analysis of a Questionnaire*. Paper presented at the Annual Meeting of the American Educational Research Association, Chicago, April 21-25.
Op't Eynde, P., De Corte, E., & Verschaffel, L. (2002). Framing Students' Mathematics-Related Beliefs. A Quest for Conceptual Clarity and a Comprehensive Categorization. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A Hidden Variable in Mathematics Education?* (pp. 13-38): Springer Netherlands.
Op't Eynde, P., De Corte, E., & Verschaffel, L. (2006a). Beliefs and Metacognition: An Analysis of Junior High Students' Mathematics-related Beliefs. In M. Veenman & A. Desoete (Eds.), *Metacognition in Mathematics Education*. New York: Nova Science.
Op't Eynde, P., De Corte, E., & Verschaffel, L. (2006b). Epistemic Dimensions of Students' Mathematics-related Belief Systems. *International Journal of Educational Research, 45*(1–2), 57-70. doi:10.1016/j.ijer.2006.08.004
Pajares, F. (1992). Teachers' Beliefs and Educational Research: Cleaning up a Messy Construct. *Review of Educational Research, 62*(3), 307–332.

Pajares, F. (2006). Self-Efficacy During Childhood and Adolescence: Implications for Teachers and Parents. In F. Pajares & T. Urdan (Eds.), *Self-Efficacy Beliefs of Adolescents* (pp. 339-367): Information Age Publishing.

Pajares, F., & Graham, L. (1999). Self-Efficacy, Motivation Constructs, and Mathematics Performance of Entering Middle School Students. *Contemporary Educational Psychology, 24*(2), 124-139. doi:10.1006/ceps.1998.0991

Pajares, F., & Miller, M. D. (1994). Role of Self-Efficacy and Self-Concept Beliefs in Mathematical Problem Solving: A Path Analysis. *Journal of Educational Psychology, 86*(2), 193-203. doi:10.1037/0022-0663.86.2.193

Pajares, F., & Miller, M. D. (1995). Mathematics Self-Efficacy and Mathematics Performances: The Need for Specificity of Assessment. *Journal of Counseling Psychology, 42*(2), 190-198. doi:10.1037/0022-0167.42.2.190

Palmer, R. G. (1994). *Attitudes towards Mathematics of Vietnamese and non-Vietnamese Senior Secondary Female Students.* (Bachelor of Education with Honours, Edith Cowan University, Australia), Retrieved from https://ro.ecu.edu.au/cgi/viewcontent.cgi?article=1637&context=theses_hons

Parandekar, S. D., & Sedmik, E. K. (2016). *Unraveling a Secret: Vietnam's Outstanding Performance on the PISA Test: Policy Research Working Paper*. Retrieved from World Bank Group:

Pehkonen, E. (1995). *Pupils' View of Mathematics: Initial Report for an International Comparison Project.* Retrieved from University of Helsinki, Department of Teacher Education:

Pepin, B., & Roesken-Winter, B. (2015). *From Beliefs to Dynamic Affect Systems in Mathematics Education : Exploring a Mosaic of Relationships and Interactions*: Springer.

Perera, H. N., & John, J. E. (2020). Teachers' Self-efficacy Beliefs for Teaching Math: Relations with Teacher and Student Outcomes. *Contemporary Educational Psychology*, 101842. doi:10.1016/j.cedpsych.2020.101842

Perry, W. G. J. (1968). *Patterns of Development in Thought and Values of Students in a Liberal Arts College: A Validation of a Scheme*. Retrieved from Cambridge, MA: Bureau of Study Counsel:

Peterson, P. L., Fennema, E., Carpenter, T. P., & Loef, M. (1989). Teacher's Pedagogical Content Beliefs in Mathematics. *Cognition and Instruction, 6*(1), 1-40.

Pham, X. C., & Pham, T. H. C. (2018). Teaching Mathematics at Primary Schools from the Perspective of Freudenthal's Theory of Realistics Mathematics Education. *Vietnam Journal of Education, 2*.

Phan, H. P. (2008). Multiple Regression Analysis of Epistemological Beliefs, Learning Approaches, and Self-regulated Learning. *Electronic Journal of Research in Educational Psychology, 16*(1), 157-184.

Philipp, R. A. (2007). Mathematics Teachers' Beliefs and Affect. In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning (Vol. 1)* (pp. 257-315). USA: National Council of Teachers of Mathematics.

Polly, D., McGee, J. R., Wang, C., Lambert, R. G., Pugalee, D. K., & Johnson, S. (2013). The Association between Teachers' Beliefs, Enacted Practices, and Student Learning in Mathematics. *The Mathematics Educator, 22*(2), 11-30.

Prendergast, M., Breen, C., Bray, A., Faulkner, F., Carroll, B., Quinn, D., & Carr, M. (2018). Investigating Secondary Students Beliefs about Mathematical Problem-solving. *International Journal of Mathematical Education in Science and Technology, 49*(8), 1203-1218. doi:10.1080/0020739X.2018.1440325

Presser, S., Couper, M., Lessler, J., Martin, E., Martin, J., Rothgeb, J., & Singer, E. (2004). Methods for Testing and Evaluating Survey Questions. *Public Opinion Quarterly, 68*(1), 109–130.

Richardson, F. C., & Suinn, R. M. (1972). The Mathematics Anxiety Rating Scale: Psychometric Data. *Journal of Counseling Psychology, 19*(6), 551-554. doi:10.1037/h0033456

Roesken, B., Hannula, M., & Pehkonen, E. (2011). Dimensions of Students' Views of Themselves as Learners of Mathematics. *The International Journal on Mathematics Education, 43*(4), 497-506. doi:10.1007/s11858-011-0315-8

Rott, B., Törner, G., Peters-Dasdemir, J., & Möller, A. S. (2018). *Views and Beliefs in Mathematics Education The Role of Beliefs in the Classroom* (1st ed.). Cham: Springer International Publishing.

Rutherford, T., Long, J. J., & Farkas, G. (2017). Teacher Value for Professional Development, Self-efficacy, and Student Outcomes within a Digital Mathematics Intervention. *Contemporary Educational Psychology, 51*, 22-36. doi:10.1016/j.cedpsych.2017.05.005

Saadati, F., Cerda, G., Giaconi, V., Reyes, C., & Felmer, P. (2019). Modeling Chilean Mathematics Teachers' Instructional Beliefs on Problem Solving Practices. *International Journal of Science and Mathematics Education, 17*(5), 1009-1029. doi:10.1007/s10763-018-9897-8

Sangcap, P. G. A. (2010). Mathematics-related Beliefs of Filipino College Students: Factors Affecting Mathematics and Problem Solving Performance. *Procedia - Social and Behavioral Sciences, 8*, 465–475.

Schoenfeld, A. H. (1983). Beyond the Purely Cognitive: Belief Systems, Social Cognitions, and Metacognitions As Driving Forces in Intellectual Performance. *Cognitive Science, 7*(4), 329-363.

Schoenfeld, A. H. (1985a). *Mathematical Problem Solving*. New York: Academic Press.

Schoenfeld, A. H. (1985b). *Students' Beliefs About Mathematics and Their Effects on Mathematical Performance: A Questionnaire Analysis*. Paper presented at the Annual Meeting of the American Educational Research Associaion (69th,Chicago, IL, March 31-April 4).

Schoenfeld, A. H. (1988). When Good Teaching Leads to Bad Results: The Disasters of "well-taught" Mathematics Courses. *Educational Psychologist, 23*(2), 145–166.

Schoenfeld, A. H. (1989). Exploration of Students' Mathematical Beliefs and Behavior. *Journal for Research in Mathematics Education, 20*(4), 338–355.

Schoenfeld, A. H. (1992). Learning to Think Mathematically: Problem Solving, Metacognition, and Sense Making in Mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334–370). New York: Macmillan.

Schoenfeld, A. H. (2011). *How We Think: A Theory of Goal-oriented Decision Making and Its Educational Applications*. New York: Routledge.

Schoenfeld, A. H. (2015). What Counts, When? Reflection on Beliefs, Affect, Attitude, Orientations, Habits of Mind, Grain Size, Time Scale, Context, Theory, and Method. In B. Pepin & B. Roesken-Winter (Eds.), *From Beliefs to Dynamic Affect Systems in Mathematics Education: Exploring a Mosaic of Relationships and Interactions (Advances in Mathematics Education)* (pp. 395-404). Switzerland: Springer International Publishing.

Schommer-Aikins, M. (2002). An Evolving Theoretical Framework for an Epistemological Belief System. In B. K. Hofer & P. R. Pintrich (Eds.), *Personal Epistemology: The Psychology of Beliefs about Knowledge and Knowing* (pp. 103-118). New York: Routledge.

Schommer-Aikins, M. (2004). Explaining the Epistemological Belief System: Introducing the Embedded Systemic Model and Coordinated Research Approach. *Educational Psychologist, 39*(1), 19-29.

Schommer-Aikins, M. (2008). Applying the Theory of an Epistemological Belief System to the Investigation of Students' and Professors' Mathematical Beliefs. In M. S. Khine (Ed.), *Knowing, Knowledge and Beliefs Epistemological Studies across Diverse Culture* (1st ed., pp. 303-323). Dordrecht: Springer Netherlands.

Schommer-Aikins, M., Bird, M., & Bakken, L. (2010). Manifestations of an Epistemological Belief System in Preschool to Grade Twelve Classrooms. In L. D. Bendixen & F. C. Feucht (Eds.), *Personal Epistemology in the Classroom: Theory, Research, and Implications for Practice* (pp. 31-54): Cambridge University Press.
Schommer-Aikins, M., & Duell, O. K. (2013). Domain Specific and General Epistemological Beliefs: Their Effects on Mathematics. *Revista de Investigacion Educativa, 31*(2), 317-330. doi:10.6018/rie.31.2.170911
Schommer-Aikins, M., Duell, O. K., & Barker, S. (2003). Epistemological Beliefs Across Domains Using Biglan's Classification of Academic Disciplines. *Journal of the Association for Institutional Research, 44*(3), 347-366. doi:10.1023/A:1023081800014
Schommer-Aikins, M., Duell, O. K., & Hutter, R. (2005). Epistemological Beliefs, Mathematical Problem-Solving Beliefs, and Academic Performance of Midle School Students. *The Elementary School Journal, 105*(3), 289-304.
Schommer, M. (1990). Effects of Beliefs about the Nature of Knowledge on Comprehension. *Journal of Educational Psychology, 82*(3), 498-504.
Schommer, M. (1993). Epistemological Development and Academic Performance Among Secondary Students. *Journal of Educational Psychology, 85*(3), 406-411. doi:10.1037/0022-0663.85.3.406
Schommer, M. (1994). An Emerging Conceptualization of Epistemological Beliefs and Their Role in Learning. In R. Garner & P. Alexander (Eds.), *Beliefs About Text and About Text Instruction* (pp. 25-39). Hillsdale, NJ: Erlbaum.
Schommer, M. (1998). The Role of Adults' Beliefs about Knowledge in School, Work, and Everyday Life. In M. C. Smith & T. Pourchot (Eds.), *Adult Learning and Development: Perspectives from Educational Psychology* (pp. 127-143). Mahwah, NJ: Lawrence Erlbaum Associates.
Schommer, M., Crouse, A., & Rhodes, N. (1992). Epistemological Beliefs and Mathematical Text Comprehension: Believing It is Simple does not Make It so. *Journal of Educational Psychology, 84*(4), 435–443.
Schommer, M., & Walker, K. (1995). Are Epistemological Beliefs Similar Across Domains? *Journal of Educational Psychology, 87*(3), 424-432.
Schraw, G., Dunkle, M. E., & Bendixen, L. D. (1995). Cognitive Processes in well-defined and ill-defined Problem Solving. *Applied Cognitive Psychology, 9*(6), 523-538. doi:10.1002/acp.2350090605
Schreiber, J. B., Nora, A., Stage, F. K., Barlow, E. A., & King, J. (2006). Reporting Structural Equation Modeling and Confirmatory Factor Analysis Results: A Review. *The Journal of Educational Research, 99*(6), 323-338.
Schumacker, R. E., & Lomax, R. G. (2004). *A Beginner's Guide to Structural Equation Modeling* (Second Ed.): Lawrence Erlbaum Associates.
Sezgin, G. (2017). *Factors Affecting Mathematics Literacy of Students Based on PISA 2012: A Cross-Cultural Examination.* (Master of Arts, Ġhsan Doğramacı Bilkent University, Turkey), Retrieved from http://repository.bilkent.edu.tr/handle/11693/33192
Silver, E. A. (1985). *Teaching and Learning Mathematical Problem Solving: Multiple Research Perspectives*. Mahwah, NJ: Lawrence Erlbaum Associates.
Stacey, K., & Turner, R. (2015). *Assessing Mathematical Literacy: The PISA Experience*. Cham: Springer International Publishing.
Stage, F. K., & Kloosterman, P. (1995). Gender, Beliefs, and Achievement in Remedial College-Level Mathematics. *The Journal of Higher Education, 66*(3), 294-311.
Stipek, D., & Gralinski, J. (1996). Children's Beliefs about Intelligence and School Performance. *Journal of Educational Psychology, 88*(3), 397-407. doi:10.1037/0022-0663.88.3.397

Streefland, L. (1991). *Fractions in Realistic Mathematics Education: A Paradigm of Developmental Research (Mathematics Education Library)*: Springer.
Tarmizi, R. A., & Tarmizi, M. A. A. (2010). Analysis of Mathematical Beliefs of Malaysian Secondary School Students. *Procedia - Social and Behavioral Sciences, 2*, 4702–4706.
Taylor, P., Fraser, B. J., & Fisher, D. L. (1993). *Monitoring the Development Constructivist Learning Environments*. Paper presented at the Annual Convention of the National Science Teachers Association, Kansas City.
Thompson, A. G. (1992). Teachers' Beliefs and Conceptions: A Synthesis of the Research. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 127-146). New York: Macmillan.
Trakulphadetkrai, N. V. (2012). Relationship between Classroom Authority and Epistemological Beliefs as Espoused by Primary School Mathematics Teachers from the Very High and Very Low Socio-economic Regions in Thailand. *Journal of International and Comparative Education, 1*(2), 71-89. doi:10.14425/00.45.75
Underhill, R. (1988). Focus on Research into Practice in Diagnostic and Prescriptive Mathematics: Mathematics Learners' Beliefs: A Review. *Focus on Learning Problems in Mathematics, 10*(1), 55–69.
Ünlü, M., & Ertekin, E. (2017). A Structural Equation Model for Factors Affecting Eighth Graders' Geometry Achievement. *Kuram ve Uygulamada Egitim Bilimleri, 17*(5), 1815-1846. doi:10.12738/estp.2017.5.0545
Uttal, D. H. (1997). Beliefs about Genetic Influences on Mathematics Achievement: A Cross-cultural Comparison. *Genetica, 99*, 165–172.
Van Zoest, L. R., Jones, G. A., & Thornton, C. A. (1994). Beliefs about Mathematics Teaching Held by Pre-service Teachers Involved in a First Grade Mentorship Program. *Mathematics Education Research Journal, 6*(1), 37-55. doi:10.1007/BF03217261
Vandecandelaere, M., Speybroeck, S., Vanlaar, G., De Fraine, B., & Van Damme, J. (2012). Learning Environment and Students' Mathematics Attitude. *Studies in Educational Evaluation, 38*(3-4), 107-120. doi:10.1016/j.stueduc.2012.09.001
VanSledright, B., & Maggioni, L. (2016). Epistemic Cognition in History. In J. A. Greene, W. A. Sandoval, & I. Bråten (Eds.), *Handbook of Epistemic Cognition.* (pp. 128-146). London: Routledge.
Vosniadou, S., Lawson, M. J., Wyra, M., Van Deur, P., Jeffries, D., & Darmawan, I. G. N. (2020). Pre-service teachers' beliefs about learning and teaching and about the self-regulation of learning: A conceptual change perspective. *International Journal of Educational Research, 99*. doi:10.1016/j.ijer.2019.101495
Voss, T., Kleickmann, T., Kunter, M., & Hachfeld, A. (2013). Mathematics Teachers' Beliefs. In J. Baumert (Ed.), *Cognitive Activation in the Mathematics Classroom and Professional Competence of Teachers: Results from the COACTIV Project* (2013 ed., Vol. 8, pp. 249-272). Boston, MA: Springer.
Voyer, D., & Voyer, S. D. (2014). Gender Differences in Scholastic Achievement: A Meta-Analysis. *Psychological Bulletin, 140*(4), 1174–1204.
Wang, G., Zhang, S., & Cai, J. (2019). Chinese High School Students' Mathematics-Related Beliefs and Their Perceived Mathematics Achievement: A Focus on Teachers' Praise. *Eurasia Journal of Mathematics, Science and Technology Education, 15*(7), em1713. doi:10.29333/ejmste/105875
Wang, J., & Wang, X. (2012). *Structural Equation Modeling: Applications Using Mplus*: Higher Education Press.

Wheeler, D. L. W. (2007). *The Development and Construct Validation of the Epistemological Beliefs for Mathematics.* (Doctoral dissertation), Retrieved from Proquest Digital Dissertations (3298008)

Wikoff, R. L., & Buchalter, B. D. (1986). Factor Analysis of Four Fennema-Sherman Mathematics Attitude Scales. *International Journal of Mathematical Education in Science and Technology, 17*(6), 703-706.

Wilkins, J. L. M., & Ma, X. (2003). Modeling Change in Student Attitude Toward and Beliefs About Mathematics. *The Journal of Educational Research, 97*(1), 52-63. doi:10.1080/00220670309596628

Woolfolk-Hoy, A., Woolfolk-Hoy, K., & Davis, H. A. (2009). Teachers' Self-Efficacy Beliefs. In K. R. Wentzel & A. Wigfield (Eds.), *Handbook of Motivation at School* (pp. 627-653). New York: Routledge.

World Bank (Producer). (2020a). GNI per capita, Atlas method (current US$) - East Asia & Pacific.

World Bank (Producer). (2020b). GNI per capita, Atlas method (current US$) - Euro area.

Xu, Z., & Jang, E. E. (2017). The Role of Math Self-efficacy in the Structural Model of Extracurricular Technology-related Activities and Junior Elementary School Students' Mathematics Ability. *Computers in Human Behavior, 68*, 547-555. doi:10.1016/j.chb.2016.11.063

You, S., Dang, M., & Lim, S. (2016). Effects of Student Perceptions of Teachers' Motivational Behavior on Reading, English, and Mathematics Achievement: The Mediating Role of Domain Specific Self-Efficacy and Intrinsic Motivation. *Journal of Research and Practice in Children's Services, 45*(2), 221-240. doi:10.1007/s10566-015-9326-x

Yuanita, P., & Zulnaidi, H. (2018). The Effectiveness of Realistic Mathematics Education Approach: The Role of Mathematical Representation as Mediator between Mathematical Belief and Problem Solving. *PLoS One, 13*(9), e0204847. doi:10.1371/journal.pone.0204847

Yusuf, M. (2011). The Impact of Self-efficacy, Achievement Motivation, and Self-regulated Learning Strategies on Students' Academic Achievement. *Procedia - Social and Behavioral Sciences, 15*, 2623-2626. doi:10.1016/j.sbspro.2011.04.158

Zan, R., Brown, L., Evans, J., & Hannula, M. S. (2006). Affect in Mathematics Education: An Introduction. *Educational Studies in Mathematics, 63*(2), 113-121. doi:10.1007/s10649-006-9028-2

Zarch, M. K., & Kadivar, P. (2006). *The Role of Mathematics Self-efficacy and Mathematics Ability in the Structural Model of Mathematics Performance*. Paper presented at the 9th WSEAS International Conference on Applied Mathematics, May 27-29, Istanbul, Turkey.