Thesis Title:

River Water Level Forecasting with Adaptive ARIMA and Extreme Learning Machine Models

BY

Salem Atallah Albalawi

Master of Science (Mathematics)

Supervised by

Dr Greg Falzon

College of Science and Engineering

Flinders University

Adelaide, Australia

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Table of Contents

List of Figures

List of Tables

ABSTRACT

This dissertation delves into the predictive capabilities of two prominent modelling techniques, Extreme Learning Machine window (ELM) and the Adaptive Autoregressive Integrated Moving Average (ARIMA), for short-term forecasting of river water levels. With increasing environmental uncertainties, accurate predictions of water levels are crucial for effective water management and flood prevention. Through rigorous data processing and model training, this research employs recent river data to evaluate the performance of both models over four forecasting horizons: 1-day, 3-day, 5-day, and 7-day.

The evaluation metrics, including Root Mean Squared Errors (RMSE), Mean Absolute Deviation (MAD), and Mean Squared Errors (MSE), revealed insightful patterns about the accuracy and reliability of each model. Further, the distribution of forecast errors was analysed to understand the consistency and potential biases in predictions.

The thesis findings indicate nuanced differences in the performance of Adaptive ELM and Adaptive ARIMA, shedding light on the specific conditions and scenarios where one model may outperform the other. This comparative analysis serves as a comprehensive guide for researchers and practitioners in selecting the most suitable model for river water level forecasting under varying circumstances. The insights from this study also pave the way for future research opportunities, exploring the integration of both models or the incorporation of additional data sources to enhance forecasting accuracy.

Declaration of Originality

I certify that this thesis:

1. does not incorporate without acknowledgment any material previously submitted for a degree or diploma in any university
2. and the research within will not be submitted for any other future degree or diploma without the permission of Flinders University; and
3. to the best of my knowledge and belief, does not contain any material previously published or written by another person except where due reference is made in the text.

Signature: SALEM

Date: 30/10/2023

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Chapter 1: Introduction

1.1 BACKGROUND

1.1.1 Importance of forecasting river water levels

Accurately forecasting river water levels has profound implications for various socio-economic and environmental aspects. The immediate benefit lies in the realm of disaster preparedness. Flooding, a common and devastating natural disaster, can be better managed with an accurate forecast of river water levels (Ward et al., 2015). By predicting river swellings in advance, local authorities can implement evacuation plans, reducing economic damages and loss of life.

Moreover, these forecasts play a pivotal role in water resource management. With increasing water scarcity issues worldwide, having precise data on river water levels helps distribute and efficiently use freshwater resources (Adamowski & Karapataki, 2010). Predictive models also aid in managing dams and reservoirs, ensuring optimal electricity generation and sustainable ecological flows downstream.

1.1.2 Modern technological advancements in monitoring

The last few decades have witnessed significant technological advancements in monitoring river water levels. Traditional methods, based on reading of physical river height gauges, often manual and susceptible to human error, have now been complemented and in many cases replaced by automated sensor networks. These networks can provide real-time, high-resolution data, thus enhancing the accuracy and reliability of forecasts (Chen et al., 2014). Satellite-based remote sensing, combined with Geographic Information Systems (GIS), now offers an extensive spatial coverage, providing insights into even the most remote river basins (Tulbure et al., 2016). These tools can offer a broader context by assessing land use changes, riverbank erosion, and sediment transportation patterns. The emergence of the Internet of Things (IoT) has further revolutionized river monitoring. Sensors can now transmit data in real-time to centralized systems, enabling rapid response actions during critical situations (Gubbi et al., 2013).

1.2 PROBLEM STATEMENT

1.2.1 Current forecasting models

Hydrological forecasting has seen the development and application of several models over the years. Traditional forecasting models, like the Autoregressive Integrated Moving Average (ARIMA), have been heavily utilized in hydrological studies because of their capability to analyse and predict time series data (Hyndman & Athanasopoulos, 2018). ARIMA models rely on past data patterns to predict future trends, making them ideal for short-term forecasts. On the contrary, the recent surge in artificial intelligence research has given birth to newer forecasting models, including the Extreme Learning Machine (ELM). As a part of Single Layer Feedforward Neural Networks, ELM possesses the advantage of swiftly training large datasets and can effectively capture non-linear patterns, thus potentially improving forecast precision (Huang et al., 2012).

1.2.2 Need for model comparisons

The plethora of available models has created a necessity to understand which models are more adept at predicting river water levels, especially in unique hydrological contexts such as Australia's diverse river systems. A detailed comparison between traditional models like ARIMA and newer counterparts such as ELM is essential. Such comparisons will not only elucidate the effectiveness of each model but also guide water resource managers and policymakers in adopting suitable methodologies for forecasting. Furthermore, understanding the intricacies of each model can lead to potential hybrid approaches that combine the strengths of both traditional and modern models (Kisi et al., 2015).

1.3 RESEARCH OBJECTIVES

This research aims to further the understanding of river water level forecasting by examining two distinct models, the adaptive ARIMA and the ELM, especially within the Australian context. The objectives underpinning this aim are:

1.3.1 Compare adaptive ARIMA and ELM models

Despite both models being employed in various hydrological studies, there remains to be a lacuna in comprehensive comparisons between them. While ARIMA, with its roots in time series analysis, has been a staple in hydrological forecasting (Hyndman & Khandakar, 2008), the ELM, a neural network-based approach, has garnered attention for its ability to train large datasets and capture non-linear trends swiftly (Huang et al., 2006). This research seeks to contrast these models, focusing on their predictive accuracy, computational efficiency, and ease of application.

1.3.2 Evaluate river water forecasting model effectiveness in an Australian context

Australia, with its unique river systems and diverse hydrological patterns, offers a distinct challenge to water level forecasting. The adaptability and precision forecasting model in such a variable context are paramount. Past studies have indicated that model effectiveness can vary based on regional specificities (Westra et al., 2014). Therefore, this research strongly emphasises testing the ARIMA and ELM models against Australian river data to ascertain their practical utility in the region.

Chapter 2: Literature Review

This chapter presents a literature review that provides a comprehensive overview of existing research on this topic, synthesizing the main theories, findings and debates in the field. By analysing and evaluating different sources, it identifies gaps in the literature and establishes the rationale for the current study. This chapter serves as the foundation for the research, guiding the development of research questions and hypotheses while demonstrating the researcher's understanding of the current state of knowledge about the topic.

2.1 AUTO-REGRESSIVE INTEGRATED MOVING AVERAGE (ARIMA) MODEL

The ARMA (Autoregressive Moving Average) model is given by

$$X_t = C + \varepsilon_t + \sum_{i=1}^{p} \varphi_t X_{\{t-1\}} + \sum_{\{i=1\}}^{q} \theta_i \varepsilon_{\{t-i\}}$$

Eq 1

Where in Equation 1 C is an additive constant (intercept term), ε_i is white noise term and ω and θ are the coefficients respectively of the auto-regressive and moving average components time series which should be estimated. The mathematical form of autoregressive component (φ_t) is shown in the following equations.

$$y_t = c + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \cdots + \beta_p y_{p-1} + \epsilon_t$$

Eq 2

Where is in Equation 2 the lag value of y_t is predictors of multiple regression the number of p is known as order of the autoregressive component.

The moving average MA(q) (θ_i) is denoted as

$$\mathrm{y_t} = \mathrm{c} + \epsilon_\mathrm{t} + \alpha_1 \epsilon_{\mathrm{t}-1} + \alpha_2 \epsilon_{\mathrm{t}-2} + \cdots + \alpha_{p\epsilon_{\mathrm{t-p}}}$$

Eq 3

Where q is the number of moving average terms.

The ARIMA (Autoregressive Integrated Moving Average) model is an extension of the ARMA model, order d differencing is performed on X_t prior to fitting the ARMA model.

$$\nabla X_t = C + \varepsilon_t + \sum_{i=1}^{p} \varphi_t \nabla X_{\{t-1\}} + \sum_{\{t=1\}}^{q} \theta_i \varepsilon_{\{t-i\}}$$

Eq 4

An automated ARIMA tool is used to identify the appropriate number of orders of ARIMA. The selection of order is made through AIC criteria. The AIC is model goodness of fit which has the mathematical form given below.

$$AIC = 2k - 2ln(\hat{L})$$

Eq 5

Where is in Equation 5 k is the number of estimated parameters and $\hat{L}$ is the maximum value of the ARIMA model likelihood function. The optimum model is chosen whose order gives a lower AIC value based on the goodness of fit measure. The ARIMA model can also be extended to include a seasonal trend term.

(Musarat et al., 2021) uses the ARIMA model to forecast the river flow level with data from the Kabul River in Swat and forecasts of the water level from 2011-2030. Model training utilised 53 years of data from 1961-2005 based on daily observations. The stationarity of the data was confirmed through the Augmented Dickey-Fuller (ADF) test and hence the ARIMA model deemed appropriate.

The author takes 480 out of 600 observations as a training set and finds the optimal seasonal ARIMA model of order (2,4) (2,2) with (d = 0) based on AIC value. Hence an ARMA model was found to be appropriate. The R-square of the final ARMA model is computed as 0.922 and MAPE as 20.11, indicating a good fit of the model to the data. According to the forecasts, the water level in the Kabul River is expected to remain fairly stable. From 2011 to 2030, there was a slight increase, reaching just over 250 cumecs. This change is relatively small compared to the recorded value of 249 cumecs in 2000. Furthermore, the water level will gradually rise between January and August, peaking at 250 cumecs in September. Once the monsoon season subsides, the water level will decrease to its lowest point of 10 cumecs from October to December, continuing until the year 2030.

2.2 AUTO-REGRESSIVE FRACTIONALLY INTEGRATED MOVING AVERAGE (ARFIMA) MODEL & MODEL ENSEMBLES

The Autoregressive Fractionally Integrated Moving Average (ARFIMA) model is a time series model that modifies the autoregressive moving average (ARMA) model to account for long-range dependence and non-stationarity. It is commonly used to analyse and forecast time series data with long memory.

This model has three parameters donated as ARFIMA(p,d,q) where p represents the order of the autoregressive component, d represents the differencing parameter for fractional integration, and q represents the order of the moving average component. The ARFIMA model has a similar form to the ARIMA model, except that the difference parameter, d, can have fractional rather than just integer values.

Papacharalampous, G., & Tyralis, H. (2020) estimate the ARFMA model through maximum likelihood estimation. They used a river flow dataset that compiles 90-year-long information from approximately 600 stations. These stations are mostly located in two continental-scale regions, specifically North America and Europe. The dataset represents various climate and catchment characteristics, making it ideal for benchmarking purposes. Comparisons shows that ARFIMA has lowest forecast Mean Square Error as compared to other models.

They also evaluate performance of Facebook's Prophet model, an additive effects time series model combining non-liner trends with daily, weekly and yearly seasonality components in addition to a 'holiday effect' (Taylor, S. J., & Letham, B. 2018). Papacharalampous, G., & Tyralis, H. (2020) also introduced a new family of hydrological time series forecasting methodologies that combine multiple individual forecasting methods. The proposed methodology is a simple and flexible approach that can be used to combine any number of individual forecasting methods. The methodology is based on a weighted average of the forecasts generated by the individual methods, where the weights are determined by a linear regression model. The regression model is trained on the historical data and is used to estimate the weights that optimize the forecast accuracy. The author aims to achieve performance improvements in the long run by comparing the proposed methodology to other commonly used forecasting methods and the results show that it has improved over the performance of the benchmark by 18.9% in terms of root mean square error (RMSE). Therefore, the forecast made by Papacharalampous, G., & Tyralis, H. (2020) allows accurate one-year ahead river flow forecasting, and that it can outperform other commonly used forecasting methods in terms of accuracy.

2.3 EMPIRICAL MODE DECOMPOSITION ARIMA (EMD/EEMD-ARIMA)

Empirical mode decomposition (EMD) and ensemble empirical mode decomposition (EEMD) is a non-stationary data processing technology. The author Wang argues that hydrological time series exhibit various time-scale characteristics that can be extracted using Empirical Mode Decomposition (EMD) as Intrinsic Mode Functions (IMFs) with different time scales. Each IMF must meet two conditions. (Wang, Qiu, & Li, 2018)

1. The number of local extreme values and zero-crossings should either be the same or differ by one across the entire dataset.
2. The mean value of the upper envelope (formed by local maxima) and the lower envelope (formed by local minima) must be zero at any given time.

The following steps areas followed to implement the EMD/EEMD for hydrological time series.

Step 1: In the observed average N-day hydrological time series denoted by $\{x(t) \epsilon X: t = 1, 2, ..., N\}$, all of the local extreme points are identified. Which are then connected using the maxima and minima of a fitted cubic spline line to create the upper envelope $x_{max}(t)$ and lower envelope $x_{min}(t)$.

Step 2: the average of upper and lower envelope m(t) is then calculated as

$$m(t) = \frac{1}{2}(x_{max}(t) + x_{min}(t))$$

Eq 6

Step 3: A new time series, h(t) is obtained by subtracting the mean average envelope m(t) from the observed time series x(t) as shown in Equation 7.

$$h(t) = x(t) - m(t)$$

Eq 7

Step 4: Repeat Steps 1-3 until *h(t)* doesn't satisfy the above two requirements of the IMF.

Step 5: The residual $r_1(t)$ is obtain by subtracting $l_1(t)$ from the observed series *x(t)* as shown below.

$$r_1(t) = x(t) - L_1(t)$$

Eq 8

Step 6: Repeat Steps 1-5 using the residual component $r_1(t)$ as the input time series (instead of *x(t)).*

This process is iteratively applied to the set of residuals r2(t), r3(t), … rn-1(t) to generates a series of IMFs $l_1(t), l_2(t), ..., l_n(t)$, and ultimately a residual series $r_{n(t)}$ that is either a monotonic function or a function with only one extreme value. The original time series can then be expressed using the equation 9.

$$x(t) = \sum_{i=1}^{n} c(t)_i + r_n(t)$$

Eq 9

Where is in Equation 9 $c(t)_i$ represented the sum of the IMF components and the final residual series $r_{n(t)}$. This decomposition allows for the identification of different scales and potential trends within the streamflow data. Unlike previous methods, EMD operates in the temporal space rather than the frequency domain, making it an empirical, direct, and adaptable approach for data analysis.

Wang (2018) utilised the ten-day streamflow data series with a EMD decomposition containing several components at different time scales followed by a one-step ahead ARIMA forecasting model. The developed model is used to forecast the flow level of yellow river located in China. This hybrid EMD-ARIMA model was model was trained on 10-day average stream flow data from 2007-2012 followed and then used to forecast the five-years of average water flow levels (2013-2018) for the Yellow River in China. The results shows that EMD-ARIMA (MAPE 0.186) more perform accurately as compared to simple ARIMA (MAPE) (Wang 2018).

2.4 ARTIFICIAL NEURAL NETWORKS (ANN)

The ANN is a neural machine learning model that consists of multiple neurons organized into layers, and these layers are interconnected with each other, forming a complete neural network. The key components of the ANN are:

1. An input node layers consisting of I nodes. These nodes receive a vector of input attributes. (Such a river water level time series) denoted as $x = [x_1, x_2, \ldots, x_I]$.
2. Hidden layers, each layer of weights is connected to the preceding and subsequent layers in the network. For instance, the first hidden layer can contain I nodes with each node connected to each of the input layer attributes and also connected to all nodes in the next layer. These weights can be represented as a set of vectors ($w = [w_1, w_2, \ldots, w_I]$) for each layer, with each set of weights determining the strength of the connections between ANN layers. In addition, an additive constant can be supplied to each weight to alter the overall mean value of the layer connection ($b = [b_1, b_2, \ldots, b_I]$).

3. Output Layer. Post the hidden layer(s), the network weights are further processed to obtain an output. In the case of a ANN regression model, this will usually a numerical attribute (such as water level) or alternatively a numerical input to the next hidden layer. Nonlinear processing is incorporated though the use of an activation function, which links the total synaptic input to the neuron's output activation. The total synaptic input (u) is computed by taking the inner product of the input and weight vectors, as described in the equation 10:

$$u = \sum_{i=1}^{l} w_i x_i + b_i$$

Eq 10

The resulting synaptic input is then processed by a pre-determined output activation function (denoted Φ). The resulting output, denoted as y, is calculated as $y = \Phi(u)$, represents the activation function of the neuron. A range of activation functions such as the sigmoid activation function 10 can be selected to govern the level of non-linear response present in the network. The sum of these output activation function values then provides the final output response of the network (e.g., the predicted river weight level by the network).

There are many hydrological features that can be used as an input attribute to the ANN node such as precipitation, soil moisture, temperature, lagged (previous) flow levels etc. to forecast the stream flow level.

Yonaba, H., et al. (2010) used a forward selection method and cross validation to find the best input attributes and number of hidden layers in in ANN. The authors proposed a rigorous model development process for building a multilayer perceptron (MLP) neural network for streamflow forecasting. The process includes constructing calibration and validation datasets based on Korhonen network clustering, employing Levenberg-Marquardt with Bayesian regularization as the calibration procedure, and using the stacking multimodal approach. The MLP architecture, a feedforward-type neural network was, is selected in this comparison because it usually has good model performance and is the most frequently used configuration in hydrology.

2.5 HYBRID SARIMA-ANN MODEL

In SARIMA forecasting of complex non-linear trends can produce unreliable results whilst the e-ANN perform poorly when there are data outliers, multicollinearity of input attributes, limited model training data or noisy (erroneous) measurements. A hybrid model that combines the benefits of the SARIMA with the ANN has been proposed (Azad et al. 2022) for water resources monitoring.

The hybrid SARIMA-ANN model consists of first modelling the linear component of the data via a SARIMA followed by an input of the lagged values of the SARIMA residuals into the ANN.

The hybrid SARIMA-ANN model is steps are represented mathematically as in Equation 11.

$$y_t = L_t + N_t$$

Eq 11

Where is in Equation 11 y_t is the current time value which is additive of linear L_t and non-linear N_t component at time *t*.

The SARIMA model is then fit to y_t , to estimate the linear trend component $\hat{L}_t$ with the residual error component described as in Equation 12.

$$e_t = y_t + \widehat{L_t}$$

Eq 12

In the Equation 12 the relationship's prediction value for time t is denoted as $\widehat{L_t}$. It may possible that the residual of errors has still non-linear component therefore the residuals of SARIMA is passed through ANN as shown below.

$$e_t = f(e_{t-1}, e_{t-2}, \dots, e_{t-n}) + \epsilon_t$$

Eq 13

Where the non-linear function that is model by the ANN is denoted as *f* and ε_t is the random error component at time *t*. The prediction from ANN is denoted as $\widehat{N_t}$ while the combined forecast of the hybrid SARIMA, ANN is in Equation 14:

$$\widehat{y_t} = \widehat{L_t} + \widehat{N_t}$$

Eq 14

The optimum parameters of SARIMA are found through use of the Auto-correlation Function (ACF) and Partial-correlation Function (PCF) plots to specify a model with the lowest Root

Mean Square Error (RMSE) fit, penalised for degrees of model freedom. Whilst the optimal ANN layer is selected through cross-validation and prior knowledge of data patterns.

Azad, et al. (2022) used hybrid SARIMA-ANN to forecast Reservoir water level (RWL) of Red Hills Reservoir (RHR) located at Tamil Nadu, India. The author develops SAMRIMA (0, 0, 1) (0, 3, 2)$_{12}$ and used their residual as an input to ANN. They converted the daily RWL data to average monthly from January 2004 to November 2020. This data set was further partitioned into 80% training and 20% testing. The data was first pre-processed removing outliers and detrending and standardization attributes magnitudes for input to the ANN. The hybrid model has lower RMSE of 430.728 as compared to separate SARIMA and ANN and provide high R^2 value of 0.84. Conclude that to capture the complex linear and non-linear trends present in the RWL data a hybrid model such as the SARIMA-ANN was more suitable than purely SARIMA or ANN models.

2.6 RADIAL BASIS FUNCTION NEURAL NETWORKS (RBF-NN)

RBF-NN are feedforward neural networks that can approximate different types of functions with high accuracy, given enough computational units and data. The RBF-NN has three layers the first of which is the input layer, which receives sensory input; the second is the hidden layer, which performs a non-linear transformation of the input data; and finally, is the output layer, which generates the network's output by combining its inputs linearly. The neurons in the hidden layer are provide nonlinear transformations of their vector inputs and are responsible for the network's ability to approximate complex functions utilising the radial basis function activation function.

$$\Phi_i(x_i, c_i, \sigma_i) = e^{\frac{-\|x-x_i\|^2}{2\sigma_i^2}}, \Phi_0 = 1$$

Eq 15

Where is in Equation 15 x_i are the input attributes, c_i and σ_i is the location (mean) and spread (standard deviation) of the radial basis function in input space. The output of the RBF-NN is given by

$$\hat{y}(x, \alpha, C, \sigma) = \sum_{i=1}^{n} \alpha_i \Phi_i(x, c_i, \sigma_i) = \Phi(x, C, \sigma)\alpha,$$

Eq 16

where is in Equation 16 αi are weights of the network output layer activation functions and n is the number of neurons.

This vector of mode parameters $w = (\alpha, C, \sigma)$ is found through a training data set and the error minimisation of a training criterion.

$$\Omega(\mathrm{X}, \mathrm{w}) = \frac{1}{2}\left||\mathrm{y} - \hat{y}(\mathrm{X}, \mathrm{w})|\right|^2$$

Eq 17

.

Lineros, et al. (2021) used a RBF-NN to model River Carrión flow data located in the northwest of the Iberian Peninsula. The dataset contains 602,928 measurements from 2 February 1999 till 20 July 2010. The data is almost stationary however a seasonality is observed.

2.7 EXTREME LEARNING MACHINE (ELM) MODEL

The author Yaseen, et al. (2019) discussed ELM's performance and concluded that ELM is increasingly popular. It has several advantages over traditional machine learning algorithms such as it is very fast to train, solving a variety of problems, and being easy to understand and use. Besides their application in other fields, the ELM is also used for hydrology, meteorology, and climatic studies. In hydrology, ELMs have been used to model river flow systems.

Yaseen, et al. (2016) investigated the ELM to forecast stream flow discharge rates in the semi-arid region of Tigris River.

The authors first used a partial auto-correlation function (PACF)to choose the most suitable lagged steam-flow as a predictor. A 12-month stream flow was then forecast using an ELM and the results compared with other machine learning algorithms such as support vector regression, and generalized regression neural networks (GRNN). The results reported by Yaseen et al. (2016) show that the ELMs outperform the other machine learning algorithms. The ELMs can accurately forecast stream-flow discharge rates, even in a semi-arid region where the stream-flow patterns are highly variable.

Yaseen et al. (2016) conclude that ELMs are a promising approach for streamflow forecasting in semi-arid regions. They suggest that ELMs could be used to improve the design of water management systems in these regions.

2.8 OPTIMALLY PRUNED EXTREME LEARNING MACHINES (OP-ELMS)

The OP-ELM model is a type of ELM that can improve the generalization performance of ELMs by pruning the hidden nodes that are not contributing to the prediction accuracy. Adnan, et al. (2019) used OP-ELMs for daily streamflow prediction. the Fujiang River, China. partial autocorrelation function (PACF) was used to identify the most important lagged streamflow values to use as predictors. OP-ELMs were then fit to the data and used to forecast streamflow for the next day. The OP-ELM using randomly initialized using a set of weights and biases. The hidden nodes are ranked according to their importance to the prediction accuracy. The least important hidden nodes are pruned (removed) from the ELM. The ELM is then retrained using the remaining hidden nodes. The importance of the nodes is computed through the following equation.

$$Imp = \sum |W_i|^2$$

Eq 18

where is in Equation 18 W_i is the weight of the i^{th} hidden nodes.

In equation 19 used to calculate the output of the OP-ELM:

$$y = f(w^T x + b)$$

Eq 19

where is in Equation 19:

y is the output of the OP-ELM

w is the weight matrix

b is the bias vector

x is the input vector

f is the activation function

The performance of the OP-ELM was compared to other machine learning algorithms, such as support vector regression (SVR) and generalized regression neural networks (GRNN). concludes that the OP-ELMs outperform the other machine learning algorithms and can accurately forecast daily streamflow, even in a river with a highly nonlinear streamflow pattern.

2.9 ADAPTIVE NEURO-FUZZY INFERENCE SYSTEM (ANFIS)

The Adaptive Neuro-Fuzzy Inference System (ANFIS) is a hybrid computational model that combines the strengths of both neural networks and fuzzy logic. It is used for learning of complex patterns in data from the past data and to forecast the future observations.

ANFIS is a combination of fuzzy logic and neural networks to create a fuzzy inference system. Fuzzy logic is a mathematical framework that deals with uncertainty and imprecision in data. It uses linguistic variables and rules to model human-like reasoning. Fuzzy logic allows for the representation of vague concepts and the handling of uncertain information.

On the other hand, neural networks are powerful computational models inspired by human brain functioning. They can learn patterns and relationships from data through a process called training. Neural networks consist of interconnected artificial neurons that process and transmit information.

Galavi, et al. (2013) used the ANFIS model to forecast the flow of water of the Klang River in Malaysia. The water flow level of the river is collected from Sulaiman station from January 2002 to March 2010. The autoregressive parameter of the ANIF model was selected through ACF plot and showed that last three observations has strongly influence the current day water flow level. The general structure of the model can be expressed as

$$X_t = f(X_{t-1}, X_{t-2}, X_{t-3})$$

Eq 20

Here in Equation 20 X_{t-1} ... X_{t-3} is the water flow level at time t-1, t-2, t-3. Galavi et al. (2013) divide data into four equal parts and select the optimal parameters of the model through the k-fold cross-validation technique. The MAPE and RMSE are used as a selection criterion of the ANIF model parameters. A total of 40 Fuzzy Inference Systems (FISs) were constructed using different combinations of squash factor values (0.75, 1.0, 1.25, 1.5) and range-of-influence factors (0.25, 0.3, 0.35, 0.4, ... 0.7). The accept and reject ratios were set to 0.5 and 0.15, respectively. The models utilized first-order Takagi-Sugeno FIS with linear output. Secondly, the selected dataset from the previous step was applied to these 40 FISs. The input data was represented by a Gaussian Membership Function (MF), while a hybrid algorithm was employed for the parameter optimization. the fuzzy inference engine with a range of influence of 0.35 and a squash factor of 0.75 (FIS35075) produced the lowest RMSE on training data.

A subtractive cluster method is used to estimate the initial parameters of the Fuzzy Inference system (FIS) which identify the four clusters in a data set. As a result, each cluster was assigned four Membership Functions (MFs) and four rules. The Gaussian is found to be the most suitable MF for data fuzzification. The data were then divided into four fuzzy sets which is very low, low, average, and high-water levels, which remained consistent across all MFs. Consequently, each input variable was categorized by four fuzzy sets.

The defined rules (Ri) for the water level at lag one (WL1) and the other two lags (WL2, WL3) wearer expressed as:

$$R_i: if\left(W_{L_1}, W_{L_2}, W_{L_3}\right) \text{ belong to cluster } C_i \text{ then } WL_{today} = p_i WL_1 + q_i WL_2 + r_i WL_3 + s_i$$

Eq 21

In the Equation 26, p_i,q_i, r_i, and s_i represent the consequent FIS parameters. The hybrid algorithm was utilized to estimate the FIS parameters, including both premise and consequent parameters. The calibrated consequent parameters are used in the Equation 26.

Galavi et al. (2013) demonstrates that the FIS performs better then ARIMA based on MAPE value. The FIS model produces MAPE value of 0.404, lower than ARIMA in term of MAPE.

2.10 SUPPORT VECTOR MACHINE (SVM).

The SVM is a machine learning algorithm that works on a structure risk minimization function (SRM) principal.

The SRM principle choose the functions fx from a subset of functions based on a given set of observations $(x_1, y_1), \dots, (x_n, y_n)$. The SRM principle aims to minimize the guaranteed risk bound, as shown in following Equation 27, where the actual risk is controlled by two terms.

$$R(\alpha) \geq R_{emp}(\alpha) + \Omega(n/h)$$

Eq 22

The first term in the Equation 22 is an estimate of the risk, while the second term represents the confidence interval for this estimate. The parameter h corresponds to the VC (Vapnik-Chervonenkis) dimension of a function set. The VC dimension measures the capacity of the learning machine's function set to best approximate the problem.

The final approximating function used in SVM regression is shown below.

$$f(x) = \sum_{i=1}^{l} \left(\alpha_i, \widehat{\alpha}_j\right)\left(x_i, x_j\right) + b$$

Eq 23

where is in Equation 23 x_i and x_j are transformed using a kernel function, denoted as $\Phi(x_i)\Phi(x_j)$ which performs an inner product in the feature space. α_i and α_j are Lagrange multipliers.

Moharrampour, et al. (2013) used SVM regression to forecast the flow level of Ghara-soo river located in Golestan province of Iran. The author used 18 years of basin discharge data from 1989 to 2007 collected from Gharasoo station along with data from three different locations and (Ziarat, Shastkalateh, and Kordkooy). Four types of SVM kernels were evaluated with Gaussian kernel (RBF) providing the best performance with an the RMSE of 0.034.

2.11 LARGE SHORT-TERM MEMORY (LSTM) NETWORK.

The LSTM network is a type of neural network. That is designed to process sequential data (such as time series) by repeating the same operation for each element in the sequence, using information from the previous steps. Standard, NNs face challenges when dealing with long sequences and predicting time series data, due to gradient issues encountered when estimating network weights.

To address these challenges, the LSTM was developed. It incorporates memory cells with input, self-recurrent connection, forget, and output gates. These gates help the LSTM network capture and retain relevant information over long periods of time. Specifically, the input gate (i_t) controls the flow of new information, the output gate (o_t) determines the information to be output, and the forget gate (f_t) manages the retention or forgetting of previous information at a given time (t).

The functioning of the LSTM can be described as follows. Let x_t and h_t are the input and state respectively at time t where h and x at t-1, t+1, etc. The long-term and short-term memory in this cell is denoted by C_t and h_t. The following equation shows the calculation C_t and h_t at time-step t step in this process.

$$f_t = \sigma\left(U_f x_t + W_f h_{t-1} + b_f\right)$$

Eq 24

$$i_t = \sigma(U_i x_t + W_i h_{t-1} + b_i)$$

Eq 25

$$o_t = \sigma\left(U_f x_t + W_f h_{t-1} + b_f\right)$$

Eq 26

$$\widehat{C}_t = \tanh(U_0 x_t + W_o h_{t-1} + b_o)$$

Eq 27

$$C_t = f_t c_{t-1} + i_t C_t$$

Eq 28

$$h_t = o_t \tanh(C_t)$$

Eq 29

.

where, b_i is the bias, σ is the sigmoid activation function and Wi and Ui is the weight matrix. The cell-state value candidate is denoted by Equation 28.

In summary, LSTM networks are a specialized type of neural network that excel in handling non-linear time-series problems, such as hydrological time series. They overcome the limitations of ANNs by utilizing memory cells and gate mechanisms to effectively capture and retain information over extended sequences.

Atashi, et al. (2022) forecast the stream flow level of Red River of the North (USA) using an LSTM model. Flow level data from three stations namely Pembina satiation, Drayton station, and Grand Fork station was collected from 1 January 2007 to 5 August 2007. Forecasts were produced for different time intervals such as 6 hours, 12 hours, 1 day, 3 days, and a week ahead. The LSTM model results were compared to SARIMA and Random Forest (RF) models. The results shows that the LSTM forecasting for one week ahead forecasting provide more accurate results as compared to SARIMA and RF. The one-week ahead prediction of the LSTM model has an RMSE of 0.107, 0.151, and 0.190 for Grand Forks, Drayton, and Pembina station respectively. Compared to RSME of (2.027, 1.491, 2.268) for the SARIMA and (2.673, 1.819, 2.287) for the RF model.

Chapter 3: Methodology

This chapter introduces the methodology and the specific methods and procedures used to conduct the research, providing a clear roadmap for how to conduct the study. This chapter includes details about the research design, data collection methods, sample selection, and data analysis techniques used. It explains the rationale behind the chosen methodology and justifies why it is appropriate to address the research questions or objectives.

3.1 DATA COLLECTION AND PRE-PROCESSING

An accurate dataset is indispensable to analyse the river water levels and understand the underlying hydrological trends. Given the vast geography of Australia, the Murray River holds significance, acting as a significant river system. This study primarily utilizes the MDBA River Data website dataset(https://riverdata.mdba.gov.au/murray-river-albury-union-bridge), focusing on the Murray River at Albury (Union Bridge) (MDBA, 2023).

Sources of River Water Level Data in Australia

The data is extracted from the MDBA River Data website. This platform offers a holistic view of the River Murray System's flow and storage details.

3.2 ADAPTIVE ARIMA

3.2.1 Introduction and Theoretical Background

The ARIMA (Autoregressive Integrated Moving Average) model has long been a cornerstone in time series forecasting. Essentially, it amalgamates the Autoregressive (AR) and Moving Average (MA) models, with the addition of differencing the series to make it stationary (Integrated). As introduced in Section 2.1, the ARIMA model has form,

$$\sum_{i=1}^{p} \varphi_i \nabla X_{\{t-1\}} + \sum_{\{i=1\}}^{q} \theta_i \varepsilon_{\{t-1\}}$$

Eq 30

The Adaptive ARIMA model is a variant proposed to extend the ARIMA through the use of incremental learning (van de Ven et al. 2022). As new river level water arrives the ARIMA model is updated via the fitting of new parameters, in this manner the ARIMA model adapts to the changing river water conditions. By adapting to the underlying dynamics of the data, the adaptive ARIMA thus making the model provides a more flexible model with potential and improvements in modelling forecasting accuracy.

3.2.2 Implementation Details

The adaptive ARIMA model generally involves two main steps:

1. **Parameter Estimation:** The initial parameters (p, d, q) are estimated from previous observed data set (for instance a time series of 1000 previous water level observations) based on model selection criteria like AIC (Akaike Information Criterion). These criteria balance the model's goodness-of-fit and the complexity (the number of parameters).

2. **Adaptive Learning:** As new observations become available over time, the model parameters are re-estimated. This could be done after a fixed number of new observations or when the prediction error exceeds a certain threshold. In this thesis, a 'running window' approach was utilised, for each new instance that arrived the ARIMA model was updated using this new instance and the previous 7 days river water level observations.

3.3 SHORT-TERM MEMORY PROCESSES AND ADAPTIVE LEARNING

One of the fundamental strengths of the ARIMA model lies in its ability to model short-term memory processes using the AR and MA components. The AR component captures the momentum and drift of the series, while the MA component captures the shocks or abrupt changes.

However, these short-term memory processes might change in a rapidly changing environment. The adaptive component of the Adaptive ARIMA allows the model to adjust its short-term memory to these changes, improving its accuracy in such volatile scenarios (De Livera et al., 2011).

3.4 EXTREME LEARNING MACHINE (ELM) MODEL

3.4.1 Introduction to ELM

The Extreme Learning Machine (ELM) is a single layer feed-forward neural network for classification and regression tasks. Introduced by Huang et al. (2006), ELM stands out due to its rapid learning speed and generalization performance. Rather than the traditional backpropagation training method that is typically used for feed-forward neural networks, the ELM randomly initializes the weights of the hidden layer, eliminating the need for iterative tuning. The output weights are then determined analytically, via a Moore-Penrose Inverse resulting in a single step learning process.

Mathematically, for N arbitrary distinct samples (x_i, t_i), the ELM aims to approximate the target function as:

$$\sum_{i=1}^{N} \beta_i h(w_i, x_i) = t_i$$

Eq 31

where is in Equation 31:

x_i is the input sample (e.g. river water observation number).

t_i is the corresponding target (e.g. river water height).

h represents the neural network activation function.

w_i and β_i denote the input weight and output weights, respectively.

3.4.2 Network Architecture and Training

The ELM network comprises three layers: an input layer, a hidden layer, and an output layer.

1. **Input Layer to Hidden Layer:** The weights and biases between the input and hidden layers are initialized with random values. For a given input vector x, the output h_j of the j-th hidden neuron can be expressed as:

$$h_j(x) = g(w_j \cdot x + b_j)$$

Eq 32

where is in Equation 32:

w_j is the weight vector for the j-th hidden neuron.

b_j is the bias for the j-th hidden neuron.

g is the activation function, which could be a sigmoid function, a sine function, or others.

2. Hidden Layer to Output Layer:

   The output weights between the hidden and output layers are determined using a matrix representation and Moore-Penrose generalised inverse:

$$\beta = H^{\dagger}T$$

Eq 33

where is in Equation 33:

β is the output weight matrix.

H is the hidden layer output matrix.

T is the target matrix.

$H^{\dagger}$ denotes the Moore-Penrose generalized inverse of H (Huang et al., 2006).

3.4.3 Long-term Data Considerations

Long-term data provides essential insights into understanding and predicting seasonal and yearly fluctuations for river water level forecasting. When employing ELM, there are unique considerations for such long-term datasets:

1. **Data Volume:** Long-term datasets typically have a large volume of data, making the hidden layer output matrix H extremely large. Efficient computational methods or parallel processing might be needed.

2. **Sequential Learning:** ELM has been extended for sequential prediction to handle the continually incoming river water level data without retraining the entire model.

3. **Temporal Dependencies:** River water levels exhibit temporal dependencies. Incorporating memory mechanisms, such as sliding windows or recurrent structures, can assist the ELM in capturing such dependencies.

3.5 7-DAY SLIDING WINDOW APPROACH

3.5.1 Theory and Rationale

The sliding window approach, commonly utilized in time series forecasting, involves taking a subset or "window" of consecutive data points and using this window to predict the subsequent point. As the forecast progresses, the window "slides" forward by one-time unit, dropping the oldest observation and including the most recent one. This ensures that the model makes predictions based on the most recent data.

A 7-day sliding window refers to using the past seven days of data to predict the next day. The choice of 7 days is particularly apt for river water forecasting, given that hydrological processes may exhibit weekly patterns due to factors such as rainfall, industrial discharges, and human activities (Montanari et al., 2009).

Mathematically, for a time series $x(t)$, the windowed data at time t can be represented as:

$w(t) = [\, x(t-6), x(t-5), \ldots, x(t-1), x(t)]$

This window $w(t)$ is then used to predict $x(t+1)$. via input to the Adaptive ARIMA or ELM models.

3.6 APPLICATION TO BOTH ARIMA AND ELM

For ARIMA: ARIMA models, inherently consider past values and errors to predict future values. Combined with the 7-day sliding window, the ARIMA model parameters (p, d, q) are specifically fine-tuned based on the recent week's data.

For ELM: In the context of ELM, the 7-day sliding window serves as the input features to the network. Given the feed-forward nature of ELM, it does not maintain a state or memory from the previous inputs. The ELM was trained on a large dataset of 3 years of previous river water levels at the Albury location. This ELM then utilised the past 7 days of observations to forecast the observation h days ahead. Multiple ELMS were trained, each providing a separate forecast window, e.g. h = 1, h = 3, h = 5 and h = 7 days ahead. By feeding the past 7 days of data as features, the network inherently considers this temporal information for its prediction. This sliding window ELM model does not implement an incremental learning, as the ELM is not updated as the current data arrives. Nonetheless due to access of the ELM to a much larger dataset, it is possible for the sliding window ELM to observe similar previous 7-day patterns and utilise this knowledge when forecasting based on the current 7-day window. Note that this sliding-window ELM model is unlike the ARIMA model, which utilised short-term memory of only the past p and q observations. Both the Adaptive ARIMA and Sliding-Window ELM have advantages and disadvantages. The Adaptive ARIMA has far less data and computational requirements, however it has a 'short-term' memory of the past 7-days. In contrast, the Sliding-Window ELM requires more training data and greater computational requirements, along with H separate models (one for each forecast). The Sliding-Window ELM does however allow for the development of models with 'long-term memory' utilising data observed over several years or more. Mathematically, for the input matrix X, each row representing a day's data would have seven columns representing the past week's values, helping in training the ELM network to recognize patterns over the week.

3.7 EXPERIMENTAL ANALYSIS

3.7.1 Model Performance Metrics

When the forecasting is applied to time series, the model's performance must be evaluated through serval metrics. The most commonly used model performance metrices are as follows:

- *Root Mean Square Error (RMSE)*

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}$$

Eq 34

where is in Equation 34:

y_i *is* the actual value

$\hat{y}_i$ *is* is the predicted value

(Hodson, T. O. (2022).

The RMSE take the difference between the forecast value and observed value, then square it, sum up all the squared differences, then divide the sum by the number of data point which is n, and the last step is take the square root of the result to obtain the RMSE value.

- Mean Absolute Error (MAE)

$$MAD = \frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i|$$

Eq 35

(Chai & Draxler 2014)

MAE is defined as measure of the difference between forecast and actual values. It is calculated as the mean of the absolute differences between the predicted and actual values.

- Mean Square Error (MSE)

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \hat{Y_i})^2$$

Eq 36

(Hodson, T.(2021))

The mean squared error (MSE) is the average squared difference between the estimated values and the actual value.

3.7.2 Methods:

Model	Data	Forecast Horizon (Days)	Model Performance Metrics
Adaptive ARIMA	Water level (m)	*h = 1, 3, 5, 7*	*MSE* *RMSE* *MAD*
Sliding Window ELM			

Table 1 Methods. Data and Model Performance Metrics.

Models	Functions	Packages	Resources
Adaptive ARIMA	auto.arima	Forecast	(Hyndman & Khandakar 2008)
Sliding Window ELM	elm	nnfor	(Kourentzes 2022)

Table 2 The Implementation of the Functions and Package used in R programming.

Table 1 describes the model forecasting evaluations performed in this thesis. The Adaptive ARIMA and Sliding-Window ELM are both evaluated at the h =1, 3, 5, and 7-day forecasting windows. These forecasts are performed incrementally, as each new observation arrives the Adaptive ARIMA and Sliding ELM models provide h =1, 3, 5, and 7-day forecasts and the forecast errors (difference between forecast river water level and actual water level is recorded). From these datasets of forecast errors, the MSE, RSME and MAD are calculated.

The entire project was implemented using the R statistical software package in Table 2. Table 2 describes the software packages utilised when developing and implementing the forecasting models described in this thesis.

Chapter 4: Results

This chapter presents the results in a logical and systematic way, using tables, charts or graphs to enhance clarity and facilitate understanding. The separation of results is crucial in conveying the empirical evidence and insights gained from the study, contributing to the overall knowledge in the field and potentially informing future research or practical applications.

4.1 ADAPTIVE ARIMA FORECASTING ERROR

Days ahead	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
h = 1	-0.36714	-0.06075	-0.0005	0.00113	0.053	0.492
h = 3	-1.227	-0.07704	0.01329	0.01653	0.11375	0.769
h = 5	-2.56	-0.08064	0.01186	0.03024	0.16211	1.447
h = 7	-4.463	-0.08682	0.01429	0.04236	0.17939	2.125

Table 3 Summary for Adaptive ARIMA Forecasting Error (units in metres)

- The forecast errors tend to widen as the forecast horizon increases.
    - **Minimum Error:**

      The minimum error grows more negative, starting from -0.36714 (m) at h=1 and dropping to -4.463 (m) at h=7. This suggests that the model can sometimes underestimate the actual river level values by a more significant margin as we forecast further into the future.
    - **Maximum Error:**

      Similarly, the maximum error, indicating an overestimation, rises from 0.492 (m) at h=1 to 2.125 (m) at h=7.
    - **Mean Error:**

The mean error, on the other hand, shows a slight positive bias, suggesting that, on average, the model may slightly overestimate across all horizons. The tendency grows somewhat as the forecast horizon increases.

- **Median Error:**

  The median forecast error remains close to zero across all horizons, indicating that at least 50% of the forecasts have errors clustered around zero. This is a positive sign, suggesting that at least half the time, the model is very close to the actual values.

- **Quartiles:**

  The range between the first and third quartiles also widens as the forecast horizon increases, which is consistent with the idea that forecast uncertainty grows with longer horizons.

In summary, the Adaptive ARIMA model's performance degrades as the forecast horizon increases, a common characteristic of time series forecasting models. The model performs quite well for short-term forecasts (e.g., h=1), but as we attempt to predict further into the future, the errors increase, and the model's accuracy decreases. This trend is evident from the metrics and the statistical summary of forecast errors.

It's also worth noting that while there's an increase in underestimation and overestimation as the forecast horizon grows, the median error remains close to zero, suggesting that the model is often close to the mark.

4.1.1 Adaptive ARIMA Model Performance Metrics

Days ahead	RMSE	MAD	MSE
h = 1	0.113867	0.0872616	0.012966
h = 3	0.201862	0.1418001	0.040748
h = 5	0.342922	0.1628742	0.117595
h = 7	0.506595	0.1721934	0.256638

Table 4 Model Performance Metrics for water level of Adaptive ARIMA

- As the forecast horizon increases (from *h*=1 to *h*=7), all metrics (RMSE, MAD, MSE) also increase. This suggests that the model's accuracy decreases as we try to forecast further into the future.

The key points to note are that:

- **RMSE:** It starts at 0.1138665 for *h*=1 and goes up to 0.5065946 for *h*=7.
- **MAD:** The increase is more gradual, from 0.0872616 at *h*=1 to 0.1721934 at *h*=7.
- **MSE:** It starts at 0.01296557 for *h*=1 and increases to 0.2566381 for *h*=7.

4.1.2 Adaptive ARIMA 1st Day:

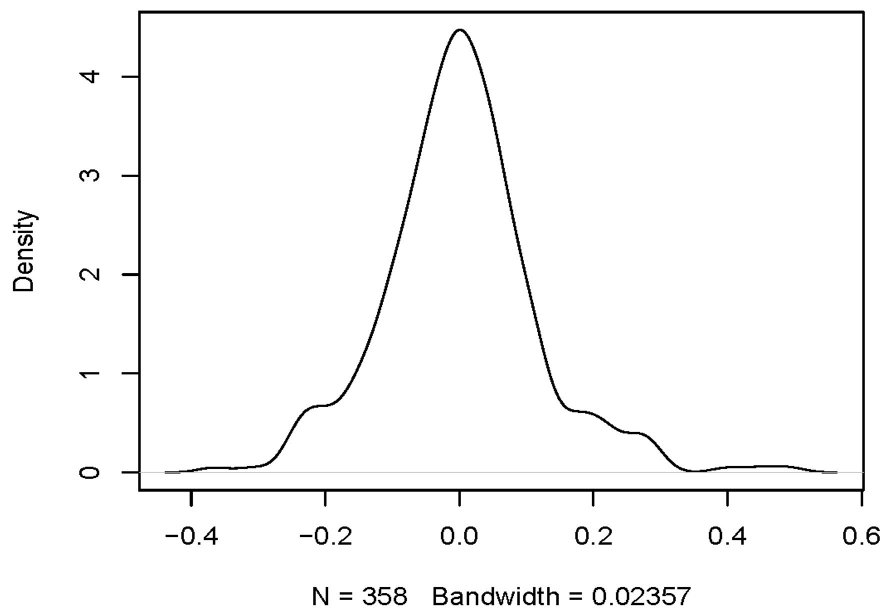

Figure 1 Kernel Density Estimate of Adaptive ARIMA Forecast Error 1st Day.

Density plots are used to visualize the distribution of a continuous variable. The y-axis often represents the probability density function (from which probability can be calculated), and the x-axis represents the data's values. In this case, the x-axis is the magnitude of the forecast error and the y-axis is the corresponding kernel density estimate. Peaks in a density plot show where values are concentrated over the interval.

Observations:

1. The plot potentially depicts two overlapping distributions. One is narrower with a peak around the centre, and the other is broader. This intepretation can change however depending on the bandwidth used in the kernel density estimate, which is an automated process in this case.

2. The narrower distribution indicates a higher concentration of data around its peak. The broader distribution suggests that some of the errors are more extreme and spread over a more extensive range.

3. Time series plots of the forecasted water levels are also another very useful to visualise these errors.

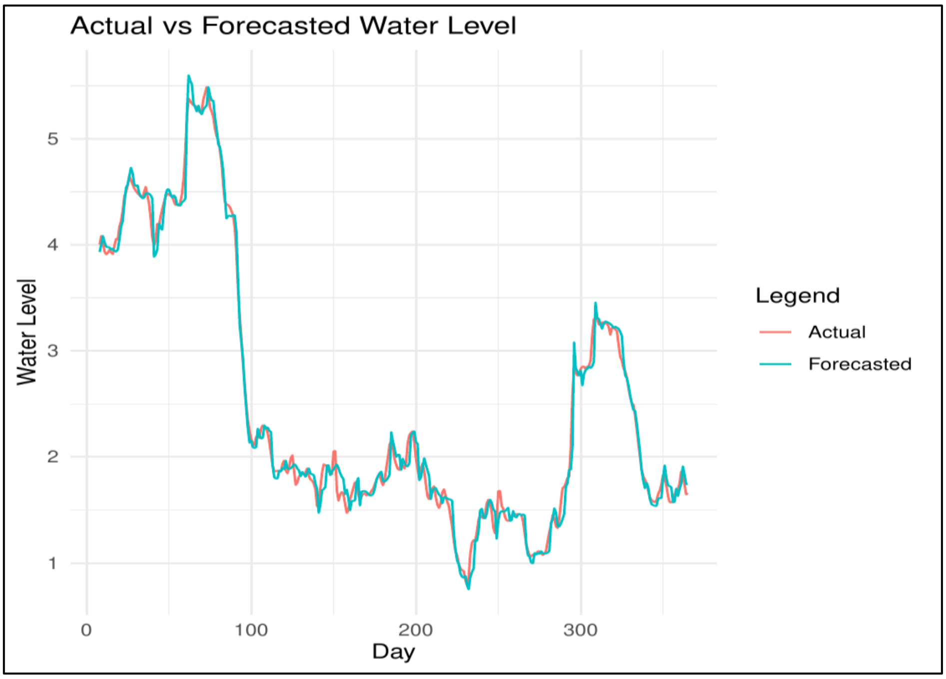

Figure 2 "Actual vs. Forecasted Water Level" for the 1st day

Observations:

1. **Time Series Plot:** The **Figure 3** presents a time series with two lines. One line is the represents the actual water level, while the other represents the h =1 forecasted values using the Adaptive ARIMA model.

2. **Close Match:** The actual and forecasted lines follow a similar trajectory, suggesting that the Adaptive ARIMA model's forecasts are relatively close to the actual values for the 1st day.

3. **Deviations:** At some points in time, the forecasted line deviates from the actual bar. These deviations represent forecast errors. These errors tend to occur during phases of rapid variation of daily water levels.

4. **Consistent Scale:** Both actual and forecasted values follow a similar scale, indicating the model's reasonable accuracy for the 1st day.

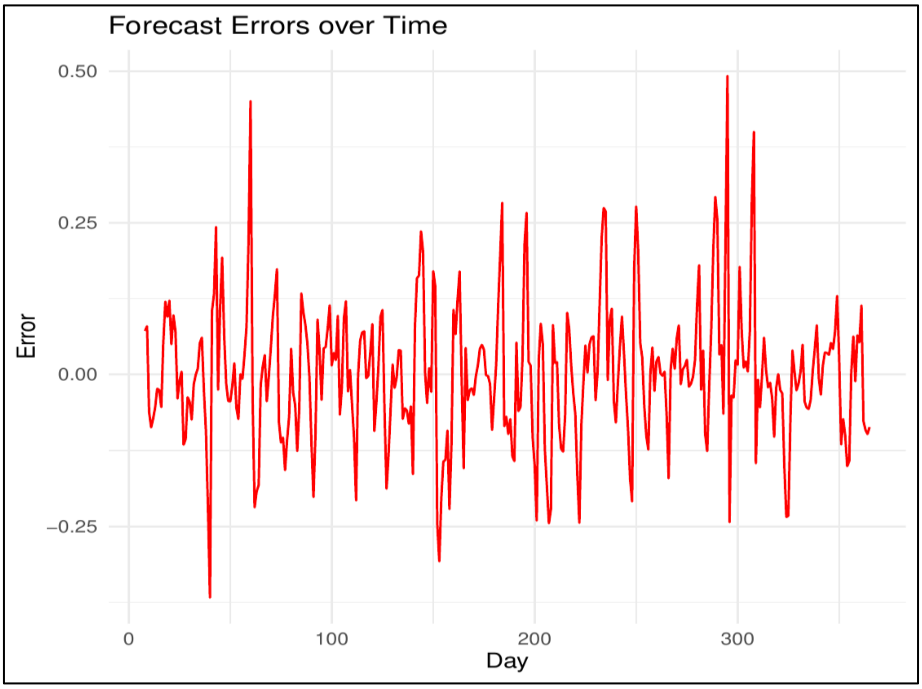

Figure 4 "Forecast Errors over Time" for the 1st day

Examining the forecast errors as a detrended (mean value removed) time series is also very informative, as per Figure 3

Observations:

1. **Nature of Errors:** The Figure 3 displays the Adaptive ARIMA $h = 1$ forecast errors as a detrended time series. Positive values indicate overestimations by the model, while negative values suggest underestimations.

2. **Zero Line:** The horizontal line at zero represents a perfect forecast. Deviations from this line (either above or below) reflect the magnitude of the error at that specific point in time.

3. **Variability:** While there are periods where the errors are clustered around zero, suggesting good forecast accuracy, there are also instances of spikes, both positive and negative. These spikes indicate moments when the model's forecasts deviated

significantly from the actual values. There are several large magnitude errors ('spikes') which contribute substantially to the wide error range observed

4. **No Systematic Bias:** There isn't a consistent positive or negative bias in the errors, as they oscillate around the zero line. This suggests that the model isn't consistently overestimating or underestimating the values, once the mean trend has been accounted for in the forecast erors.

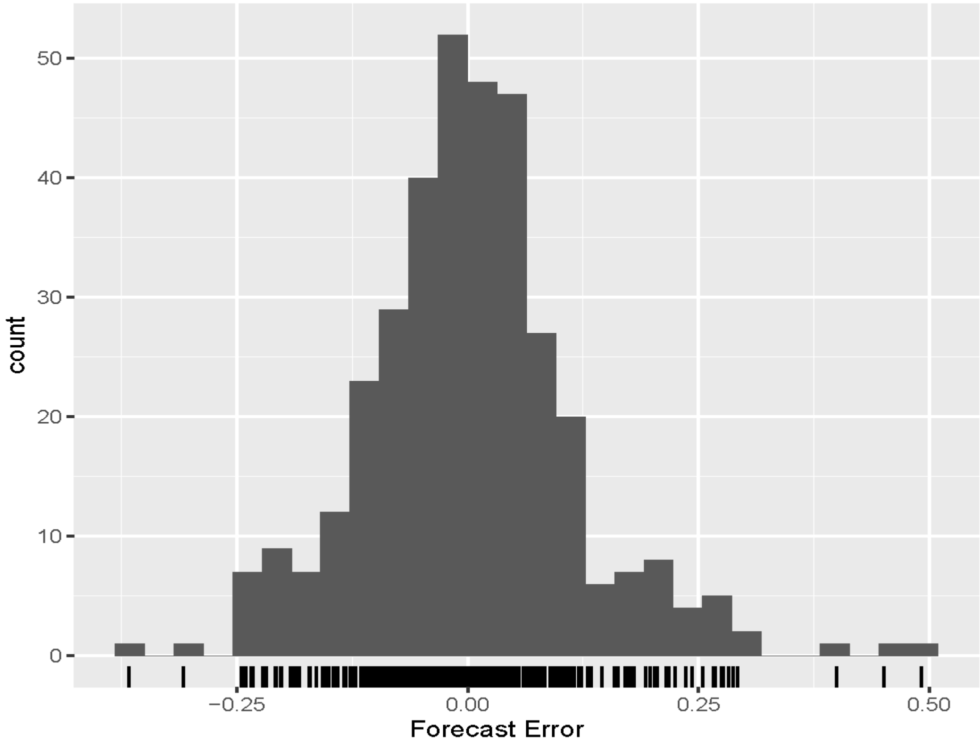

Figure 5 Histogram depicting the distribution of Adaptive ARIMA forecast errors for the 1st day.

Figure 4 displays a histogram of the Adaptive ARIMA forecast errors for the 1-day ahead forecast (h = 1). This histogram provides an alternative approach to kernel density estimates in the exploration of forecast error.

1. **Histogram Structure:** Histograms are used to show the distribution of a dataset. Each bar represents an interval range of forecast error values, and the height of the bar indicates the counts (or frequency) of errors within that range.

2. **Centred Around Zero:** The distribution is roughly symmetrical and centred around zero. This is a good sign as it indicates that there's no systematic bias in the forecast errors. In other words, the Adaptive ARIMA model tends not to consistently overestimate or underestimate the values.

3. **Spread of Errors:** While a significant portion of errors is close to zero (as indicated by the tall bars around the centre), there are errors spread across a broader range, suggesting that there were instances of both significant overestimations and underestimations.
4. **Tail Behavior:** The tails of the histogram show that extreme errors (both positive and negative) are less frequent, which is expected in a well-performing model.

Overall Discussion:

1. The density plot suggests two overlapping distributions, likely representing actual and forecasted data, indicating that predicted values are generally close to the actual ones.
2. The time series plot of actual vs. forecasted water levels showed a close match, with some noticeable deviations.
3. The forecast errors over time plot provide insights into the moments when the model's predictions deviated from the actual values, with errors oscillating around zero, indicating no systematic bias.
4. The histogram of forecast errors confirms the lack of bias and shows that while many predictions were close to the mark, there were instances of both overestimation and underestimation.

Given these visualizations and the previously discussed metrics, the Adaptive ARIMA model performs reasonably well for the 1st-day forecast, though it's essential to be aware of moments of significant deviations.

4.1.3 Adaptive ARIMA 3rd Day:

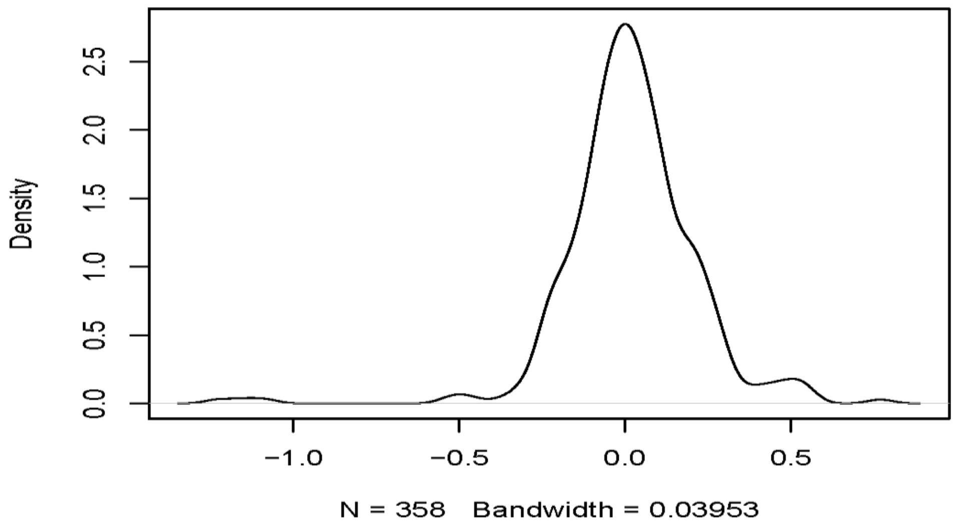

Figure 6 Adaptive ARIMA density plot for the 3rd-day results

Observations:

1. **Two Distributions:** Similar to the 1st-day results, we observe two overlapping distributions. These likely represent the presence of several very large errors in the forecast error.

2. **Spread & Peaks:** Both distributions have a noticeable peak, suggesting areas where data points are concentrated. The broader spread of one of the distributions indicates more variability in those forecast error values.

3. **Similarity to 1st Day:** The distributions appear somewhat similar to the 1st-day results.

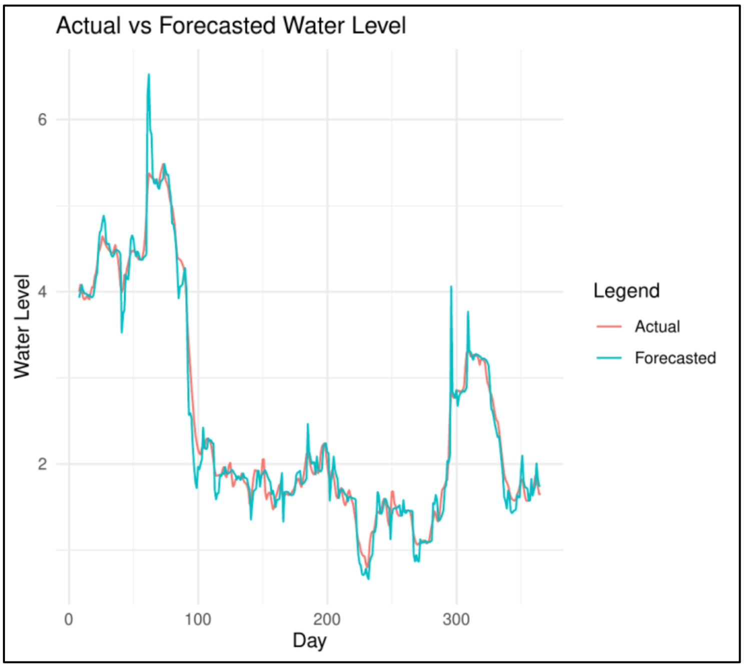

Figure 7 "Actual vs. Forecasted Water Level Adaptive ARIMA 3rd day forecast

Observations:

1. **Time Series Plot:** The Figure 6 represents a time series with two lines, one representing the actual water level and the other the forecasted values.
2. **Trajectory & Deviations:** While the actual and forecasted lines seem to follow a similar path, there are evident deviations between the two, especially in some plot areas (days 50 and 300).
3. **Scale Consistency:** Both the actual and forecasted water levels appear to operate within a similar scale, there are however significant over-estimates of forecast water level.

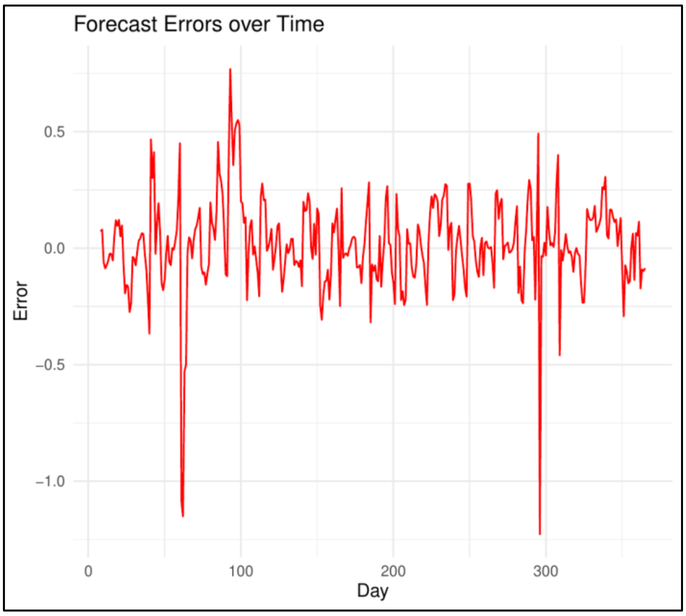

Figure 8 "Forecast Errors over Time" for the 3rd day

Figure 7 displays the 'detrended' (mean removed) forecast errors in the form of a time series.

Observations:

1. **Error Trend:** The Figure 7 displays the time series of forecast errors. As before, positive values represent overestimations by the model, while negative values indicate underestimations. The errors indicates that the forecast tends to be within the [-0.5, 0.5] metre range but there are occassional significant over-estimations by more than 1 metre.

2. **Zero Line:** The zero line signifies an ideal forecast. Deviations from this line reflect the magnitude of the error.

3. **Fluctuations:** There are evident fluctuations in forecast errors, with positive and negative deviations from the zero line. Some spikes indicate instances where the model's predictions significantly differed from the actual values.

4. **No Systematic Bias:** Similar to the 1st-day results, the errors oscillate around the zero line, indicating no consistent bias in either direction.

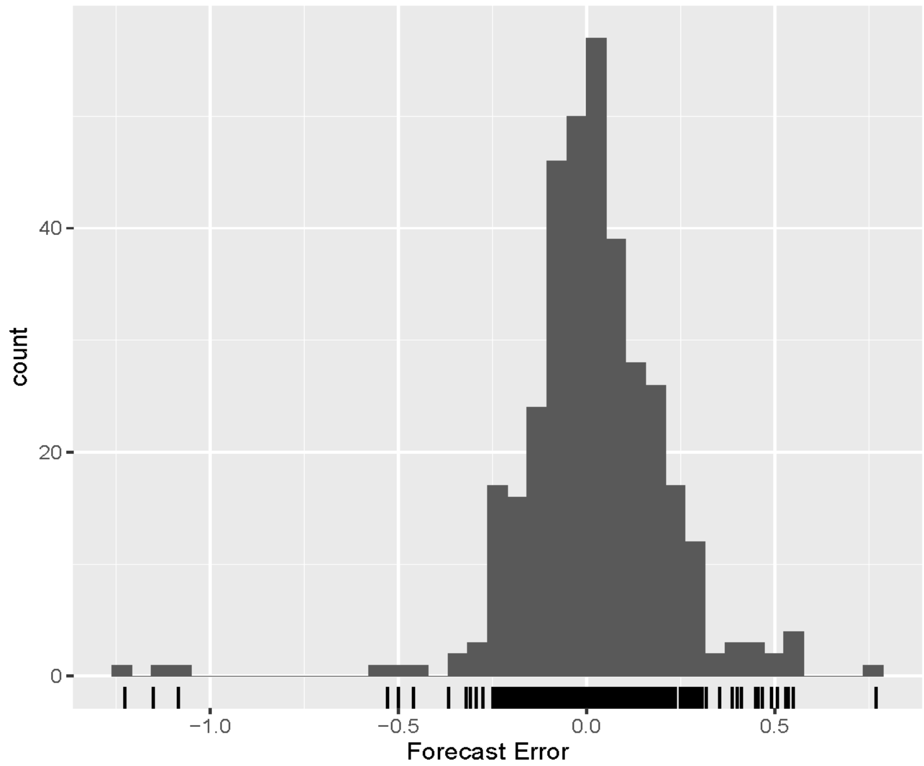

Figure 9 Histogram of Adaptive ARIMA forecast errors for the 3rd day

Observations:

1. **Histogram Distribution:** Each bar represents a range of forecast error values, with the height reflecting the counts of errors within that range.
2. **Centred Distribution:** Similar to the Adaptive ARIMA 1st-day results, the error distribution is roughly symmetrical around zero, indicating in general no systematic overestimation or underestimation by the ARIMA model.
3. **Error Spread:** Most forecast errors are clustered near zero, suggesting that many of the model's forecat were accurate and within the [-0.5, 0.5] metre range. However, errors are spread across a broader range with a few forecasts out by more than ±1.0 metres, signifying both overestimation and underestimation.
4. **Tail Behavior:** The tails of the histogram show that extreme errors are less common, reinforcing that significant deviations are infrequent.

Summary of Adaptive ARIMA 3rd-day Forecast results:

1. **Density Plot:** The density plot of the 3rd day forecast errors indicates a central cluster around zero describing most of the errors along with a more dispersed distribution describing errors of more extreme magnitude.

2. **Actual vs. Forecasted:** While there's a general alignment between the real and forecasted water levels, there are noticeable deviations at specific points in time.
3. **Forecast Errors over Time:** Errors fluctuate around zero, with both overestimations and underestimations, but overall there is no evidence of systematic bias.
4. **Histogram:** Similar to the density plot, the histogram of Adaptive ARIMA 3-day ahead forecast errors is centred around zero, with most predictions being accurate with a tolerance band, but a small number of significant deviations occur.

The Adaptive ARIMA 3rd-day forecast results show a similar trend to the Adaptive ARIMA 1st-day forecasts but differ in the magnitude and frequency of deviations. The Adaptive ARIMA model still performs reasonably well, but given the nature of time series forecasting, it's not uncommon to see increasing errors as the forecast horizon extends.

4.1.4 Adaptive ARIMA 5th Day:

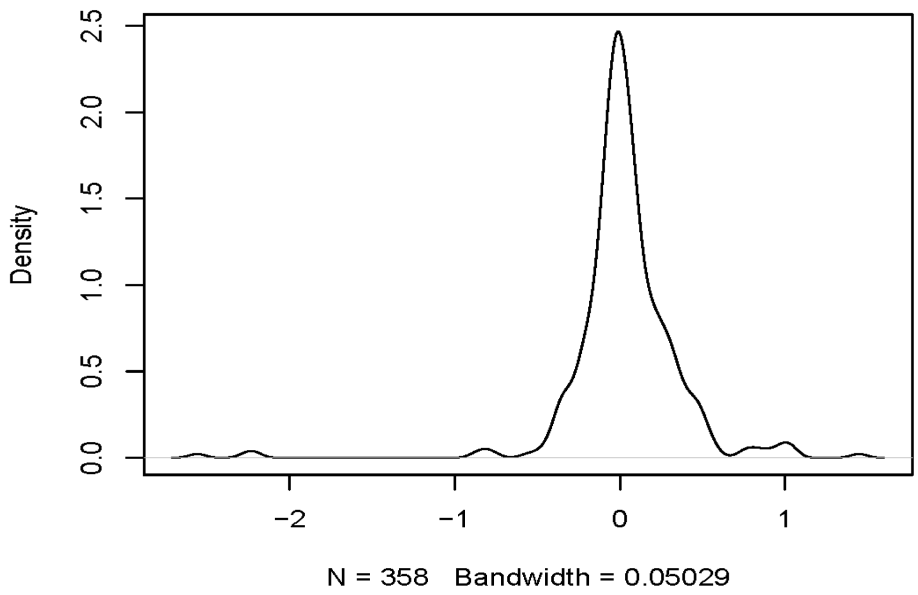

Figure 10 Adaptive ARIMA density plot for the 5th-day results

Figure 9 displays the density plot for the Adaptive ARIMA 5-day ahead forecast errors.

Observations:

1. **Distributions:** There is one main distribution describing the errors, again most errors are within a tolerance band but there are some very large forecast errors up to the order of 2 metres.

2. **Comparison with Previous Days:** While the distributions share similarities with the 1st and 3rd-day results, the forecast errors are more extreme indicating greater uncertainity in the forecasts.

3.

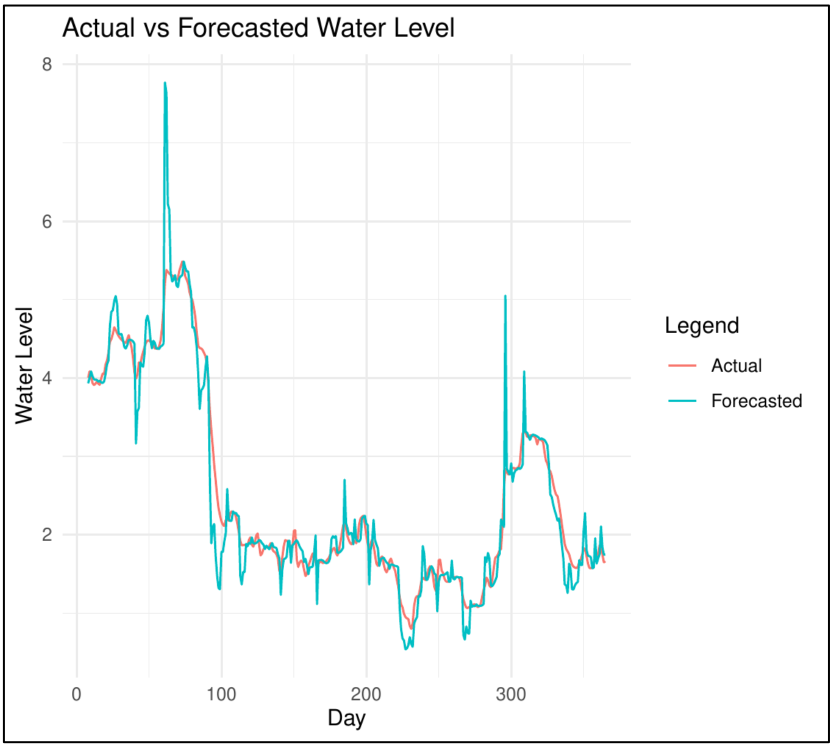

Figure 11 "Actual vs. Forecasted Water Level" for the Adaptive ARIMA 5th day forecast

Figure 10 displays the time series of the actual water levels and the 5-day Adaptive ARIMA forecasts.

Observations:

1. **Time Series Trend:** The Figure 10 displays a time series with two distinct lines, one representing the actual water level and the other the forecasted values.

2. **Similarity & Deviations:** The actual and forecasted lines follow a similar path, but there are evident deviations between them with the rapid-short term water level variations missed. These deviations represent the forecast errors for those particular times.

3. **Scale Consistency:** Both actual and forecasted values continue operating within a similar range, suggesting the model maintains its scale consistency.

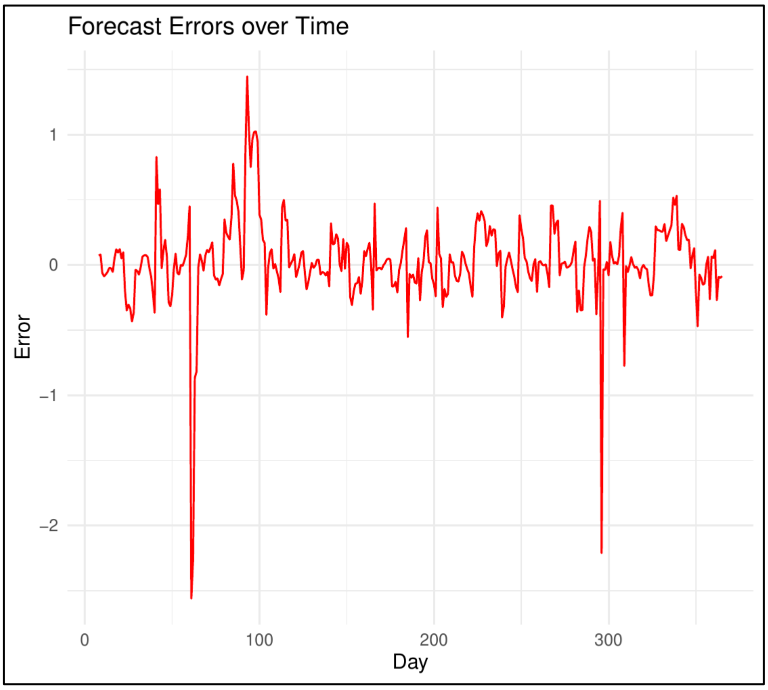

Figure 12 " Forecast Errors over Time" for the Adaptive ARIMA 5th day forecasts

Figure 11 displays the detrended ('mean' removed) Adaptive ARIMA 5-day ahead forecast errors.

Observations:

1. **Error Oscillation:** The Figure 11 displays the time series of forecast errors. As with previous days, positive values suggest overestimations by the model, while negative values indicate underestimations.

2. **Zero Line:** The line at zero represents an ideal forecast. Any deviations from this line, either above or below, describe the magnitude and direction of the error.

3. **Error Fluctuations:** The forecast errors show fluctuations around the zero line. Some pronounced spikes, both positive and negative, indicate moments when the model's predictions significantly deviated from the actual values. The Adaptive ARIMA 5-day ahead forecast tends to have under-estimate water levels for the largest magnitude errors.

4. **Bias Indication:** The errors oscillate around the zero line, suggesting no consistent bias in the Adaptive ARIMA models forecast.

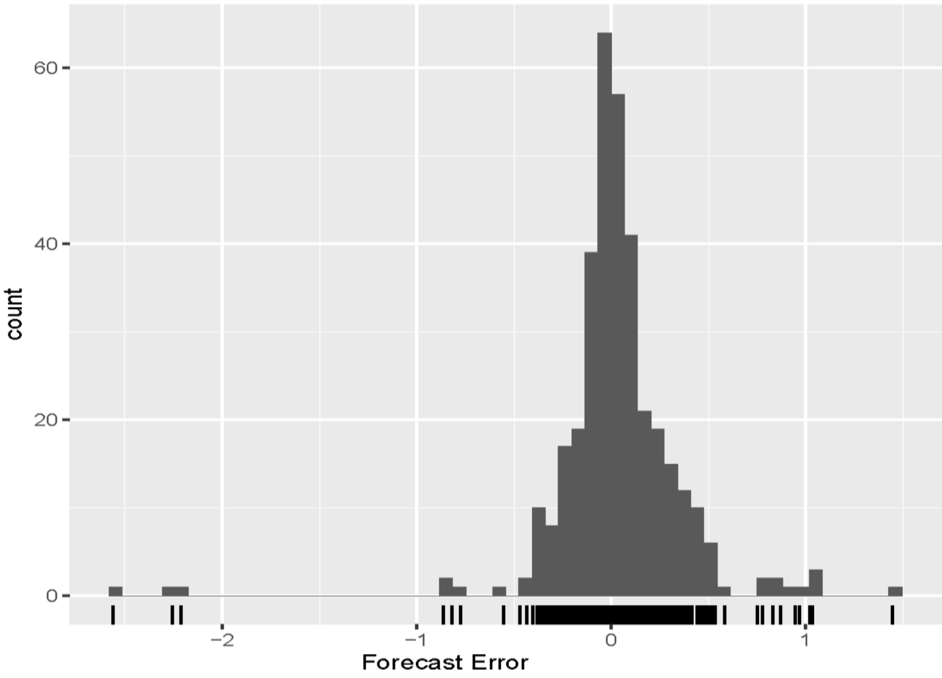

Figure 13 Histogram of the Adaptive ARIMA forecast errors for the 5th day

Figure 12 displays the histogram of 5-day ahead forecast errors for the Adaptive ARIMA model.

Observations:

1. **Histogram Distribution:** The bars in Figure 12 depict ranges of forecast error values, with the height of each bar representing the counts of errors within that range.

2. **Symmetry:** The error distribution is roughly symmetrical around zero, indicating that there's no consistent overestimation or underestimation by the Adaptive ARIMA model.

3. **Error Clustering:** A significant portion of the forecast errors is clustered close to zero, suggesting many accurate predictions by the model within a [-0.5, 0.5] metre band.

4. **Spread of Errors:** While many errors are close to zero, errors are spread across a broader range up to 2 metres, indicating moments of both signficant overestimation and underestimation.

5. **Tail Behavior:** The tails of the histogram, similar to previous days, show that extreme errors are rarer.

Summary of Adaptive ARIMA 5th-day forecast results:

1. **Density Plot:** There is one main distribution that is centered around zero, describing the majority of forecasting errors but there is evidence of a small number of large magnitude errors.

2. **Actual vs Forecasted:** There's an alignment between the natural and forecasted water levels but with evident deviations at specific points, especially when there are rapid and short-term flucatuations in water levels.

3. **Forecast Errors over Time:** The detrended errors fluctuate around zero, with several spikes indicating significant deviations at specific times.

4. **Histogram:** The forecast error distribution is centred around zero, with most predictions being accurate within a [-0.5, 0.5] metre tolerance band, but there are moments of both overestimation and underestimation.

Similar to the 1st and 3rd-day results, the 5th-day results indicate that the Adaptive ARIMA model performs reasonably well but with some noticeable deviations. The model's performance is consistent across the days, but as always with time series forecasting, there's an inherent challenge in predicting further into the future, leading to increased errors.

4.1.5 Adaptive ARIMA 7th Day:

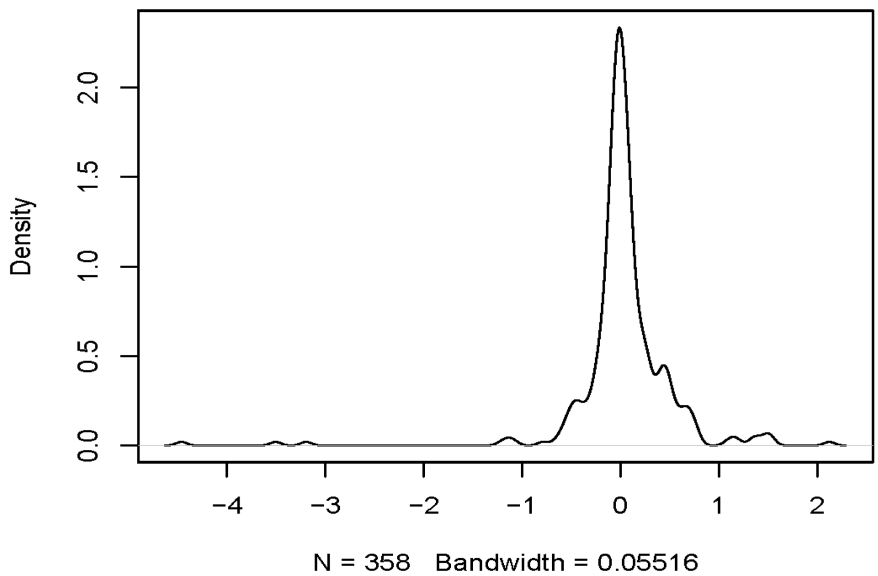

Figure 14 Density plot for the Adaptive ARIMA 7th forecast errors

Figure 13 displays the density plot of the 7-day forecast errors for the Adpative ARIMA model.

Observations:

1. **Distributions:** As with previous days, there is a main distribution that describes the majority of errors (within a 1 metre tolerance band around zero) super-imposed with a broader scale distribution containing larger magnitude errors (up to 4 metres).

2. **Spread & Peaks:** The density has distinct peaks, indicating where the data points are most concentrated. There density has a greater spread, with 'long-tails' indicating the presence of larger-magnitude errors, particularly compared to shorter-term (1, 3, 5-day) Adaptive ARIMA forecasts.

3. **Comparison with Previous Days:** As anticipated the Adaptive ARIMA 7th-day forecast has greater magnitude errors than previous shorter-term forecasts, the majority of forecasts errors are however within a [-0.5, 0.5] metre tolerance band around zero.

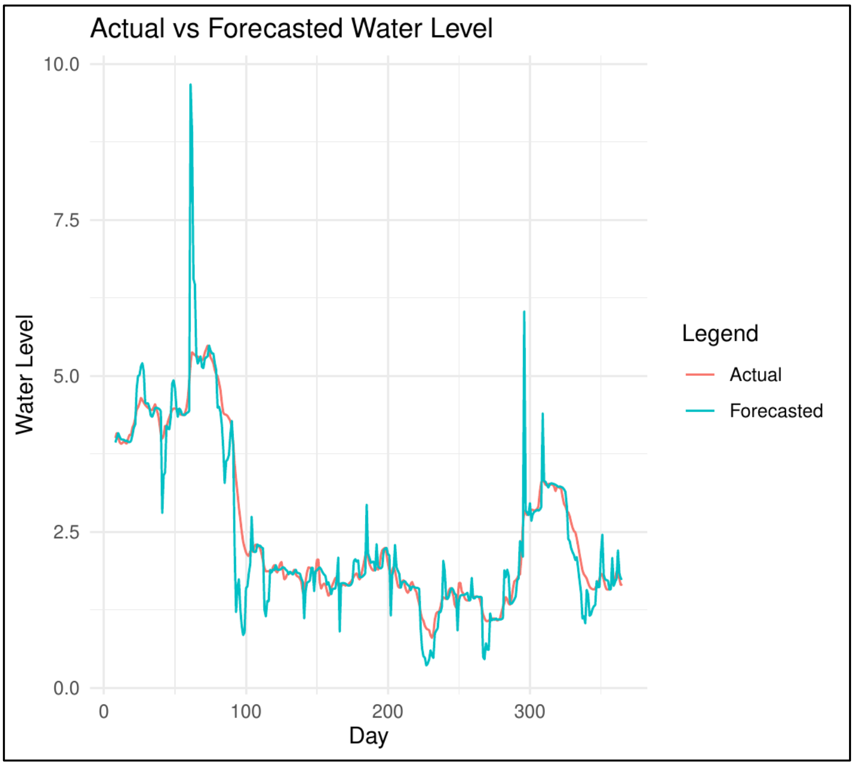

Figure 15 "Actual vs. Forecasted Water Level" for the Adaptive ARIMA 7th day forecasts

Figure 14 displays the time series plots of the actual and forecast water levels for 7 days ahead using the Adpataive ARIMA model.

Observations:

1. **Time Series Trajectory:** The Figure 14 showcases a time series with two distinct lines representing the actual water level and the forecasted values.

2. **Alignment & Deviations:** The actual and forecasted lines generally follow a similar path, but there are apparent substantial deviations at specific intervals, indicating forecast errors at those moments.

3. **Scale Consistency:** Both the actual and forecasted water levels appear to be within a similar range, suggesting the model's consistency in scale even on the 7th day.

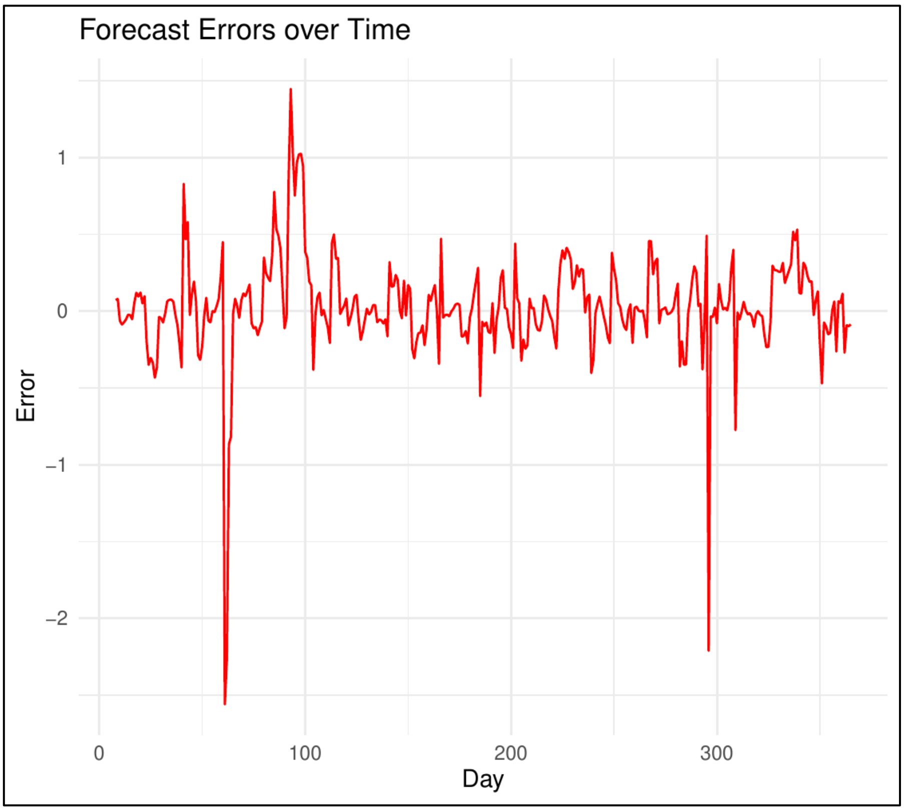

Figure 16 " Forecast Errors over Time" for the Adaptive ARIMA 7th day forecasts.

Figure 15 displays the detrended ('mean removed') forecast erorrs for the Adaptive ARIMA 7-day ahead forecasts.

Observations:

1. **Error Fluctuations:** The graaph in Figure 15 showcases the detrended time series of forecast errors. As in previous days, positive values suggest overestimations by the model, while negative values denote underestimations.

2. **Zero Line:** The horizontal line at zero signifies a perfect forecast. Deviations from this line (above or below) show the magnitude and direction of the error.

3. **Variability in Errors:** There are pronounced fluctuations in the forecast errors, with positive and negative deviations from the zero line. Some spikes highlight moments when the model's predictions significantly deviated from the actual values. There are

two major forecasting errors on the order of 2 metres magnitude and another close to 1.5 metres.

4. **Bias Indication:** The errors oscillate around the zero line, indicating no consistent bias in the predictions.

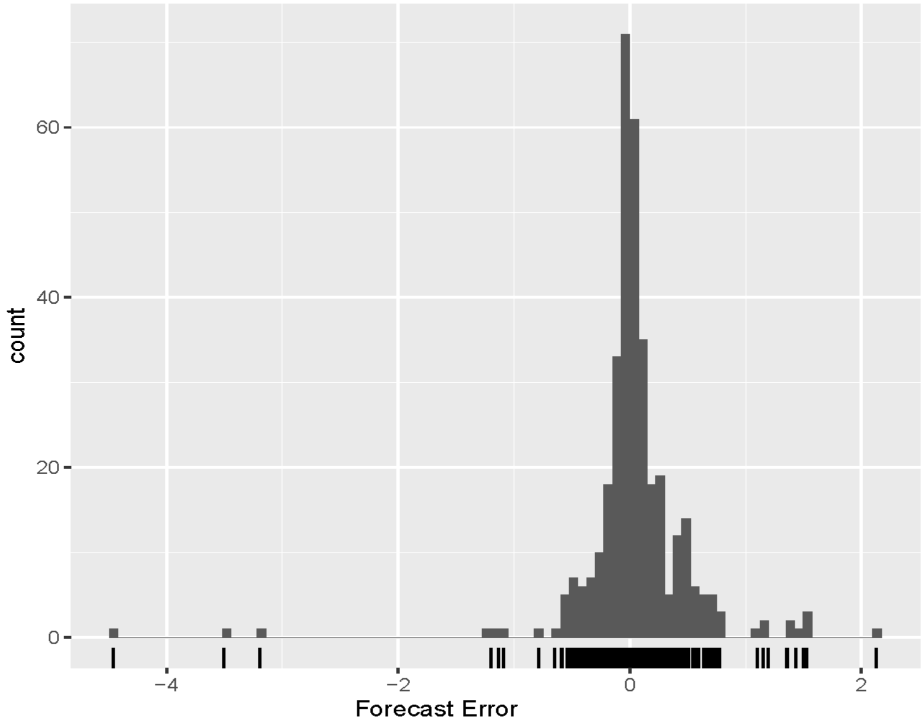

Figure 17 Histogram of forecast errors for the Adaptive ARIMA 7th day forecasts.

Figure 16 displays the historgram of 7 day forecast erorrs for the Adpative ARIMA model.

Observations:

1. **Histogram Distribution:** As with previous days, each bar in the histogram represents a range of forecast error values, with the height of the bar indicating the counts of errors within that range.

2. **Centred Distribution:** The error distribution is centred around zero, suggesting no consistent overestimation or underestimation for the majority of forecasts provided by the Adaptive ARIMA model.

3. **Majority of Errors Near Zero:** A substantial portion of the forecast errors is clustered close to zero, signifying that many of the model's predictions were relatively accurate on the 7th day.

4. **Spread of Errors:** While many forecast errors are near zero, errors are distributed across a broader range, indicating moment of significant overestimation and underestimation.

5. **Tail Behavior:** The tails of the histogram, as observed in previous days, suggest that extreme errors are more frequent.

Summary of Adaptive ARIMA 7-day forecast results:

1. **Density Plot:** The density plot indicates a central distribution containing most of the errors within a band close to zero, however there larger magnitude outliers are present.

2. **Actual vs Forecasted:** There's an overall alignment between the actual and forecasted water levels but with evident deviations at specific points.

3. **Forecast Errors over Time:** Errors fluctuate around zero, with spikes indicating significant deviations at specific times but otherwise no specific trends that indicate model bias.

4. **Histogram:** The histogram of forecast errors is centred around zero, suggesting that the model's predictions, for the most part, were accurate, but there are moments of both overestimation and underestimation. The number of larger magnitude errors has increased compared to shorter term (1, 3, 5) day Adaptive ARIMA model forecasts, this is to be expected as the forecast is relatively far ahead and the Adaptive ARIMA model only utilises a 7-day window of data.

The 7th-day results, as expected, show some similarities with the 1st, 3rd, and 5th-day results. However, as we move further into the forecast horizon, the challenges of time series forecasting can lead to increased error variability. While demonstrating reasonable accuracy, the Adaptive ARIMA model shows moments of significant deviations, primarily as the forecast horizon extends.

4.2 SLIDING WINDOW ELM ERROR

4.2.1 Summary of Sliding Window ELM Error

Table 5 displays the summary of Sliding Window ELM across all forecast windows (h = 1, 3, 4 and 7 days ahead).

Days ahead	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
h = 1	-0.33010	-0.04800	-0.01414	-0.01123	0.02165	0.25454
h = 3	-0.70618	-0.14291	-0.02917	-0.01830	0.08306	0.98408
h = 5	-0.71007	-0.22366	-0.03535	-0.0260	0.13242	1.06827
h = 7	-0.68889	-0.23043	-0.02849	-0.01840	0.17052	1.04258

Table 5 Summary of Sliding Window ELM Error (units in metres)

- **Minimum and Maximum**: The range between the minimum and maximum forecast errors tend to broadens with an increasing forecasting horizon. The more extensive spread indicates that error variability augments as the forecast extends.
- **1st Quartile, Median, and 3rd Quartile**: These metrics shed light on the distribution of forecast errors. The median error is relatively stable, although slightly negative, suggesting a minor tendency for the model to underestimate. As the forecast horizon widens, the interquartile range (difference between 3rd and 1st quartiles) expands, implying heightened error variability.
- **Mean**: The mean error is also relatively stable and leans slightly negative across forecast horizons, affirming the model's slight underestimation propensity.

Overall Analysis:

While displaying reasonable accuracy for short-term forecasts, the Sliding Window ELM model encounters mounting challenges as the forecast horizon stretches. All error metrics (Min, 1st Qu, Mean, Median, 3rd Qu, Max) show a clear trend of increased error with lengthier forecasts, indicating the intrinsic complexities of long-term predictions.

The descriptive statistics underline this observation, with the spread of forecast errors becoming more pronounced for extended horizons. This implies that the model's predictions become more dispersed and less consistent over longer durations.

For better forecasting performance, especially in longer horizons, refinements in model parameters, feature engineering, or incorporating external influencing factors could be considered. The results significantly emphasize the necessity of cautious interpretation when leveraging the model for long-term predictions.

4.2.2 ELM Model Performance Metrics

Days ahead	RMSE	MAD	MSE
h = 1	0.06846	0.048906	0.004687
h = 3	0.206522	0.155525	0.042651
h = 5	0.277334	0.213925	0.076914
h = 7	0.31984	0.246136	0.102297

Table 6 Model Performance Metrics for Sliding Window ELM water level forecasts

- **Root Mean Square Error (RMSE)**: RMSE measures the model's prediction error. The results show that as the forecast horizon (**h**) increases, the RMSE also rises. This suggests that the Sliding Window model's accuracy diminishes with a longer forecasting horizon. The rise from h=1 to h=7 is notable, indicating increased uncertainty in longer-term forecasts.
- **Mean Absolute Deviation (MAD)**: Similar to RMSE, MAD also depicts the forecast error, but without the squaring effect. The consistent growth of MAD from h=1 to h=7 mirrors the RMSE trend, reinforcing the notion that the model's precision lessens with a broader forecasting horizon.
- **Mean Squared Error (MSE)**: MSE amplifies more significant errors by squaring them. The upward trajectory of MSE values from h=1 to h=7 corroborates the findings from RMSE and MAD. The model needs to work on extended forecast horizons.

The provided visualizations give an insightful look into the performance of the Sliding Window ELM (Extreme Learning Machine) model for water level forecasting on the 1st day. Here is a brief discussion and commentary on the results:

4.2.3 Sliding Window ELM model 1st day

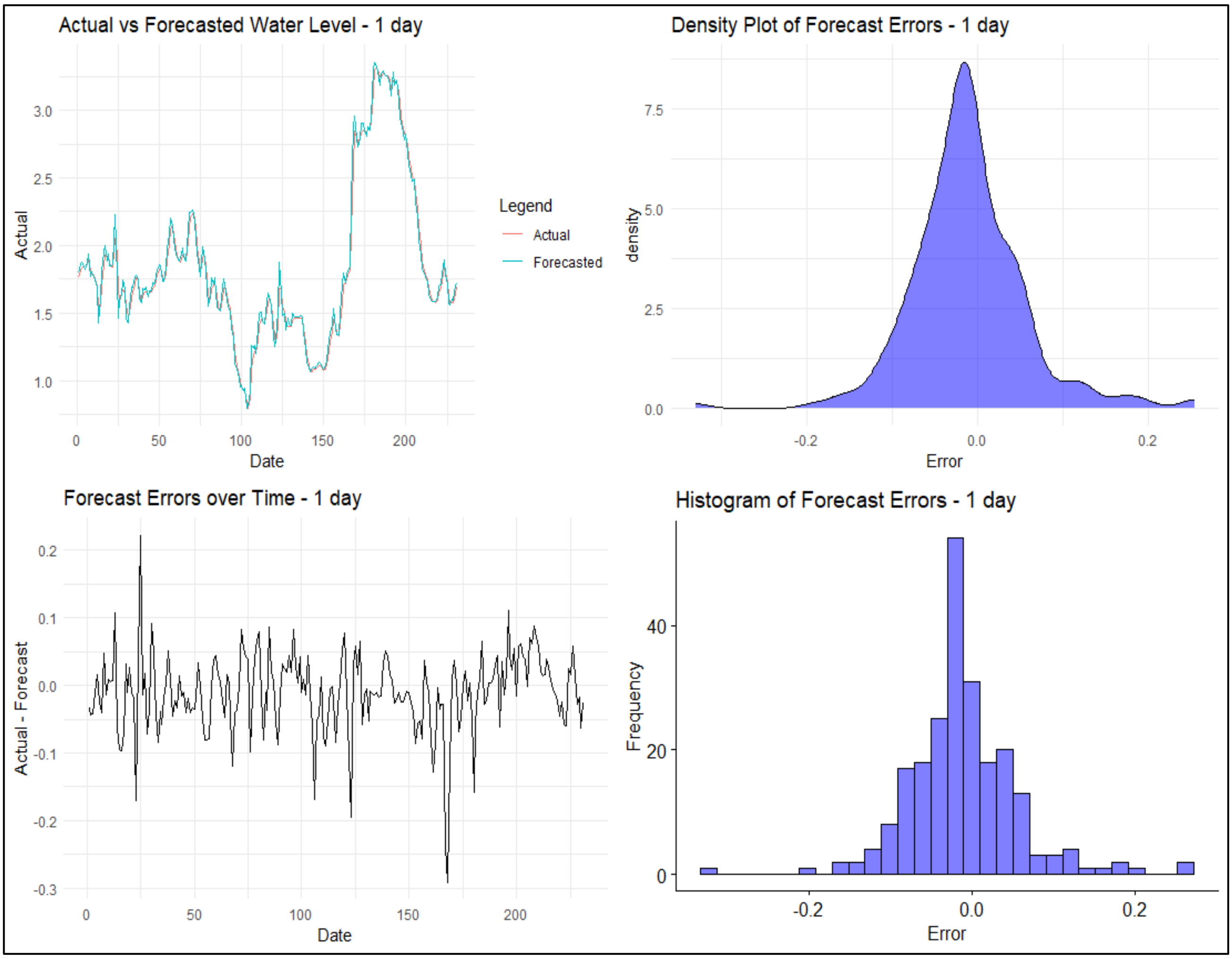

Figure 18 Sliding Window ELM plots for 1 day ahead forecasts.

Actual vs Forecasted Water Level - 1 day:

- The forecasted values (in blue) generally follow the actual values (in red) trend. This indicates that the Sliding Window ELM model has managed to capture the main trends in the water level data.
- There are instances where the forecasted values deviate from the actual ones, especially in regions with sharp peaks. It is essential to consider these areas for further improvement.

Density Plot of ELM Forecast Errors - 1 day ahead:

- The density plot showcases the distribution of the Sliding Window ELM forecast errors. The peak of the curve suggests that most of the forecast errors are centered around zero, indicating that the model's predictions are generally accurate.

- However, the presence of a wide spread in the tails of the density plot implies that there are instances where the model has made significant errors. It is essential to diagnose these instances further to understand their cause and refine the model.

Forecast Errors over Time - 1 day:

- This plot in Figure 17displays how the difference between thedetrended actual and forecasted values (errors) evolves.
- While most errors hover around zero (which is desirable), there are clear spikes representing significant overestimations or underestimations by the model. These could be attributed to sudden changes in the water level that the model failed to anticipate.

Histogram of Forecast Errors - 1 day:

- The histogram provides a count of occurrences for specific error ranges. The tall bar around the zero-error range further confirms that a significant portion of the forecasts was entirely accurate.
- The symmetric shape of the histogram suggests that the model's errors are evenly distributed, with both overestimations and underestimations. This is a good sign as the model has no consistent bias in one direction.

Overall Analysis of the 1st Day: As visualized, the Slding Window ELM model's predictions on the 1st day indicate a commendable level of accuracy in forecasting the water levels.
While the model captures the primary trends and patterns, there are noticeable discrepancies.
For the model's improvement, it would be worthwhile to investigate these discrepancies further, by looking at external factors that might influence water levels or by exploring refinements in the model's architecture and training process.

These visualizations serve as a foundational step in evaluating the model's performance and pave the way for a deeper dive into model diagnostics, hyperparameter tuning, and potential ensemble methods to enhance prediction accuracy further.

4.2.4 Slding Window ELM model 3rd day

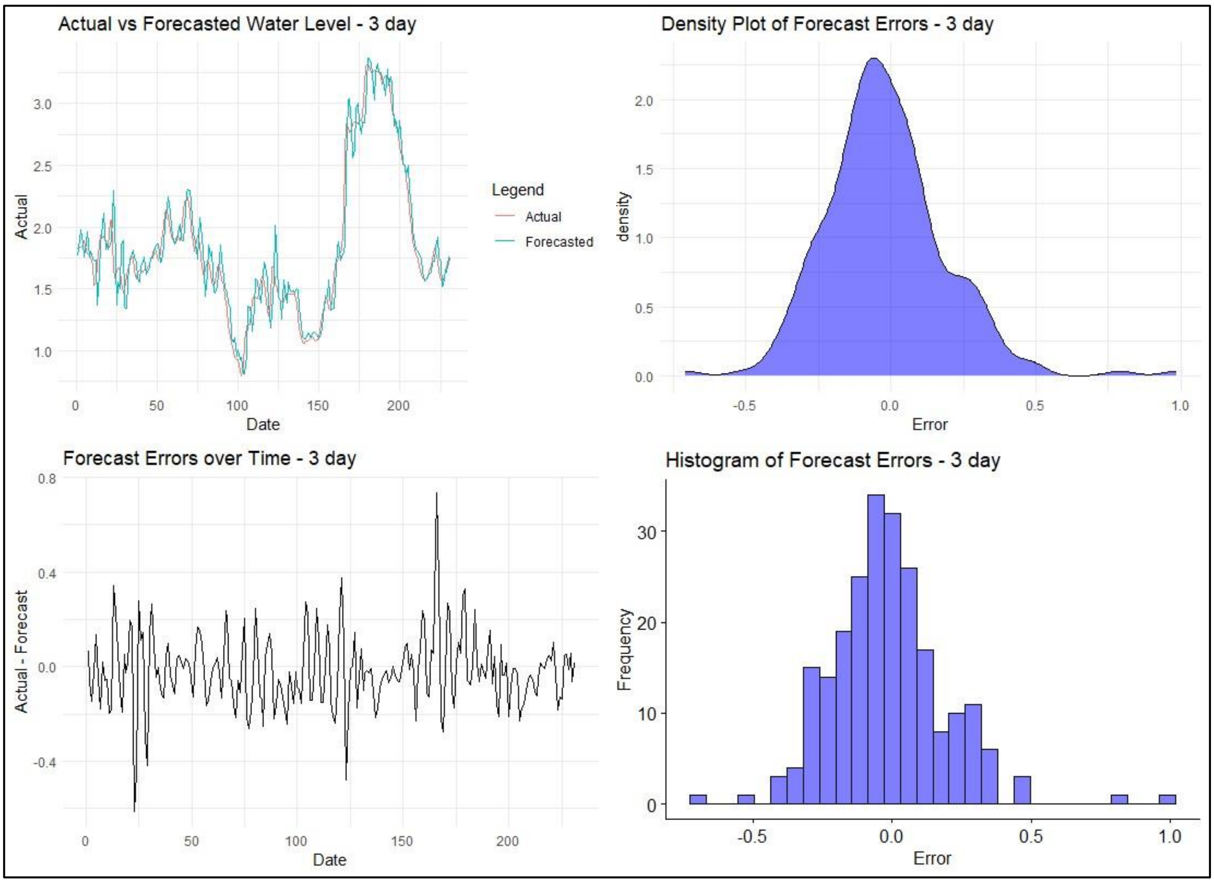

Figure 19 Sliding Window ELM plots 3 day ahead forecasts.

Actual vs Forecasted Water Level - 3 day:

- The forecasted values (in blue) generally follow the actual water level trend (in red), suggesting that the Sliding Window ELM plots 3 day ahead forecasts ELM model has a decent grip on the data's underlying patterns.
- There are notable deviations between the forecasted and actual values in areas of sudden water level changes, indicating potential challenges in capturing abrupt fluctuations for 3-day forecasts.

Density Plot of Forecast Errors - 3 day:

- Most forecast errors are centered around zero, evident from the density plot's peak. This is a positive sign, indicating that the model's predictions are often close to the actual values.

- The wider tails of the density plot highlight instances of more significant errors. This suggests that while the model performs well in general, there are instances of noticeable deviations that need to be addressed.

Forecast Errors over Time - 3 day:

- The error over time plot displays varying magnitudes of discrepancies between the forecasted and actual values throughout the dataset.
- Spikes in this plot suggest moments of significant overestimations or underestimations. These pronounced errors may be tied to unforeseen factors or indicate the model's limitations in predicting more distant future values.

Histogram of Forecast Errors - 3 day:

- The central peak in the histogram reinforces that most of the forecasts had minimal error.
- The gradual spread of errors towards the sides, especially on the positive side, indicates that the model sometimes overestimates the water levels. Examining what leads to these overestimations is crucial and considering refining the model accordingly.

Overall Analysis of the 3rd Day: The Sliding Window ELM model demonstrates a respectable forecasting ability for the 3rd day, capturing the central trends of water levels. However, compared to the 1-day forecast, there are more significant discrepancies in the 3-day predictions. This is expected as forecasting further into the future inherently comes with more uncertainties.

While the model performs adequately for many data points, it is essential to address the pronounced errors. Investigating the causes behind these discrepancies, whether from the model's architecture, training process, or external influential factors, will be vital for improving its predictive capabilities.

For a more refined model, additional input features that influence water levels should be considered or ensemble methods could be considered that combine the strengths of various models to enhance prediction accuracy.

4.2.5 Sliding Window ELM model 5th day

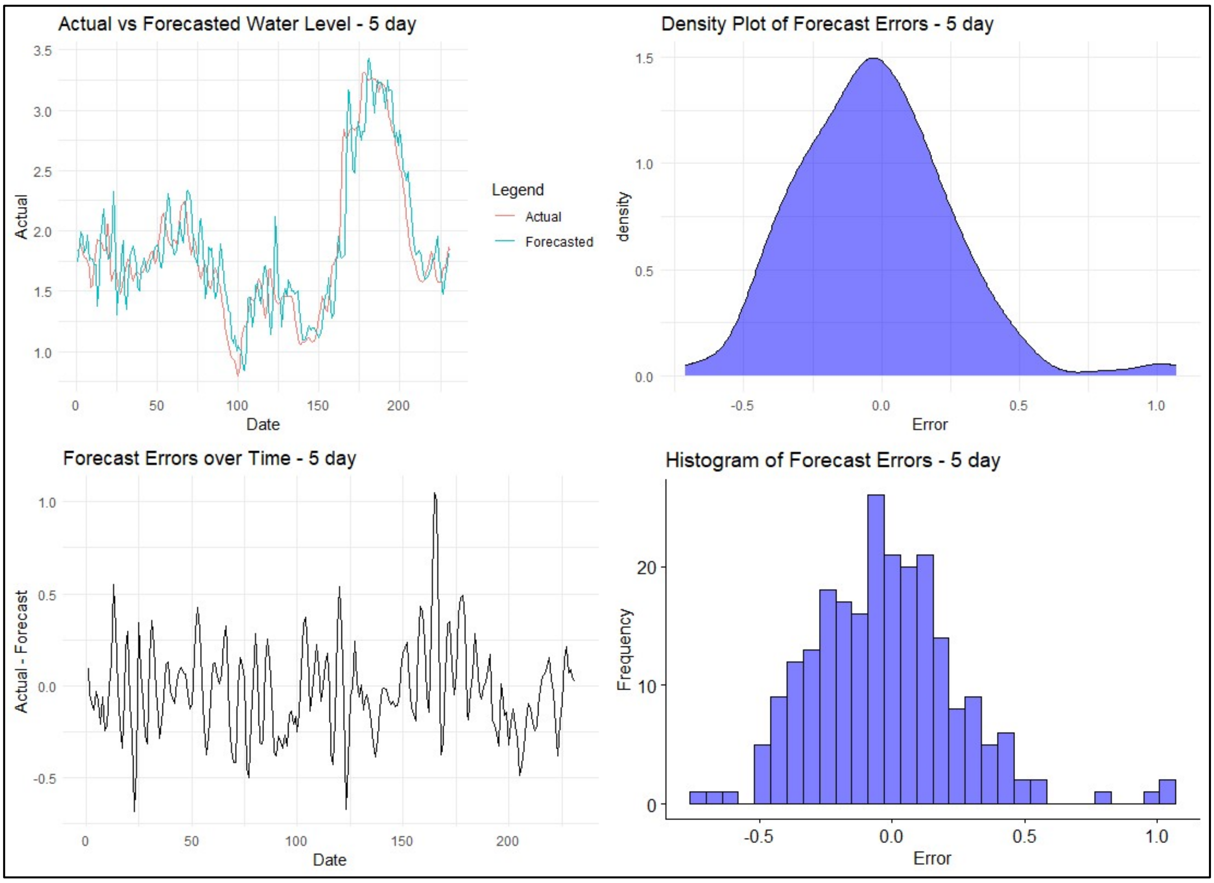

Figure 20 Sliding Window ELM plots 5 day ahead forecasts

Actual vs Forecasted Water Level - 5 day:

- The forecasted curve (in blue) roughly follows the trajectory of the actual water levels (in red), indicating a basic grasp of the underlying trends. However, deviations are more pronounced compared to shorter-term forecasts, which is to be expected as predicting further into the future is inherently more challenging.
- The areas of notable discrepancy, especially in zones with sharp changes, emphasize the model's difficulty in accurately predicting sudden fluctuations five days in advance.

Density Plot of Forecast Errors - 5 day:

- Most forecast errors cluster around zero, suggesting that many of the model's predictions are close to the actual values.
- However, the density plot's tails are spread out, highlighting instances of more significant forecasting errors. The broader spread compared to the 3-day forecast indicates increased uncertainty for 5-day predictions.

Forecast Errors over Time - 5 day:

- The temporal view of detrended errors showcases the variability in the model's performance over time. There are evident instances of overestimation and underestimation, with some errors being considerably significant.
- Compared to shorter-term forecasts, the 5-day forecast has more pronounced spikes in errors, suggesting increased challenges in long-term predictions.

Histogram of Forecast Errors - 5 day:

- The histogram presents a central concentration of errors near zero, which is encouraging. However, there is also a visible spread towards both positive and negative error values, indicating that the model has moments of both underestimating and overestimating the water levels.
- The histogram of errors appears slightly right-skewed, hinting that the model may be more prone to overestimation in this 5-day forecast.

Overall Analysis of the 5th Day: The Sliding Window ELM model exhibits a commendable capacity to grasp the general trends of water levels in the 5-day forecast, but with evident challenges in accuracy as the forecast horizon increases. As we move further from the observation date, uncertainties become more prominent, leading to more significant prediction errors.

For future iterations, additional features or external factors that can influence water levels in the long term couid be considered. Alternatively, exploring hybrid models or ensemble methods could also help improve the 5-day forecasting performance. Addressing the discrepancies will be crucial for a more reliable and robust predictive model.

4.2.6 Sliding Window ELM model 7th day

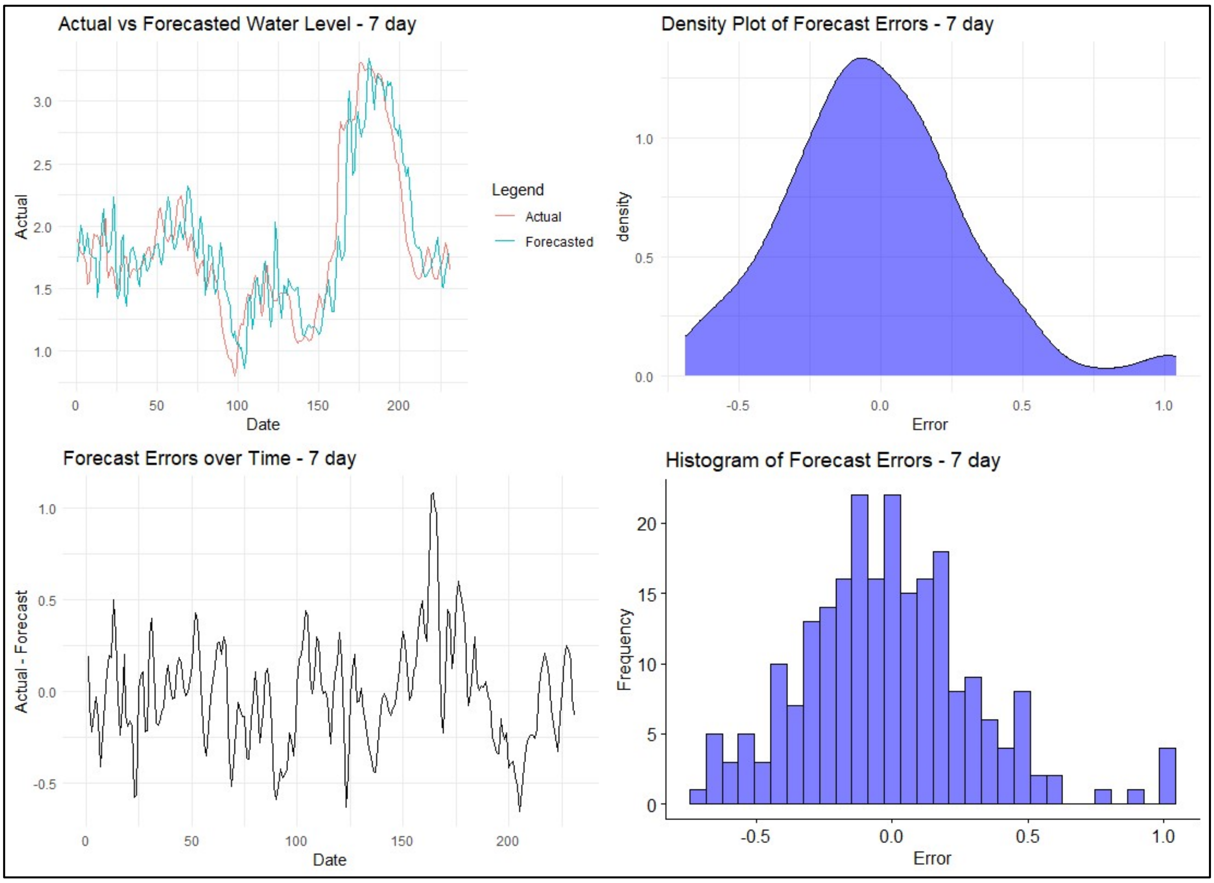

Figure 21 Sliding Window ELM plots of 7 day ahead forecasts

Actual vs Forecasted Water Level - 7 day:

- The forecasted trajectory (in blue) predominantly trails the actual water levels (in red). While the general pattern is captured, the model struggles to capture sharp turns and abrupt changes. This indicates the inherent challenges in predicting water levels a week in advance.

- The deviation zones, particularly around points with significant spikes or drops, accentuate the model's challenges in anticipating sudden changes.

Density Plot of Forecast Errors - 7 day:

- Most forecast errors center around zero, suggesting that most predictions are closely aligned with the actual values. However, the density plot's tail, especially on the right, suggests that overestimations can occasionally be quite pronounced.

Forecast Errors over Time - 7 day:

- This temporal depiction of the detrnded errors provides insight into the model's performance consistency. Overestimations and underestimations are present throughout the forecasted period, with some pronounced spikes suggesting significant discrepancies at certain times.

The error variability appears higher than shorter-term forecasts, showcasing the intricacies of 7-day predictions.

Histogram of Forecast Errors - 7 day:

- A considerable concentration of errors close to zero reiterates the model's general accuracy. Nevertheless, there is a spread in both directions, indicating occasions where the model underestimates and overestimates.
- The histogram of errors is slightly right-skewed, hinting at a slight tendency of the model to overestimate water levels in this forecast horizon.

Overall Analysis of the 7th Day:

While capturing the broad trends of water levels in the 7-day forecast, the Sliding Window ELM model exhibits increased difficulty in maintaining high accuracy as the forecast horizon extends. This aligns with the challenges associated with long-term predictions, where factors influencing water levels can become increasingly unpredictable.

Incorporating additional external factors, seasonal trends, or leveraging ensemble forecasting techniques could be beneficial for enhancement. The discrepancies noted emphasize the importance of continuous refinement to bolster the model's long-term forecasting capabilities.

4.3 COMPARING THE ADAPTIVE ARIMA AND SLIDING WINDOW ELM

	1 Day Ahead				
Model	**Min.**	**Max.**	**RMSE**	**MAD**	**MSE**
Adaptive ARIMA	-0.36714	0.492	0.113867	0.0872616	0.012966
Sliding Window ELM	-0.3301	0.25454	0.06846	0.048906	0.004687

Table 7 Adaptive ARIMA and Sliding Window ELM For 1 Day Ahead Forecasts Comparisons

Table 7 compares the Adaptive ARIMA and Sliding Window ELM model metrics for the 1-day ahead forecasts. The Sliding Window ELM demonstrates a smaller range of error magnitudes, and correspondingly lower model performance metrics. The MAD, for instance, of the Sliding Window ELM is almost half that of the Adaptive ARIMA.

	3 Day Ahead				
Model	Min.	Max.	RMSE	MAD	MSE
Adaptive ARIMA	-1.227	0.769	0.201862	0.1418001	0.040748
Sliding Window ELM	-0.70618	0.98408	0.206522	0.155525	0.042651

Table 8 Adaptive ARIMA and Sliding Window ELM For 3 Day Ahead Forecasts Comparisons

Table 8 compares the Adaptive ARIMA and Sliding Window ELM 3-day forecast error metrics. Similar to the 1-day ahead forecasts, the Sliding-Window ELM has a substantially lower range of errors. The RSME, MSE and MAD are however of similar magnitudes indicating that for the majority of 3-day ahead forecasts both models perform similarly.

	5 Day Ahead				
Model	Min.	Max.	RMSE	MAD	MSE
Adaptive ARIMA	-2.56	1.447	0.342922	0.1628742	0.117595
Sliding Window ELM	-0.71	1.06827	0.277334	0.213925	0.076914

Table 9 Adaptive ARIMA and Sliding Window ELM For 5 Day Ahead Forecasts Comparisons

Table 9 compares the Adaptive ARIMA and Sliding Window ELM 5-day forecast error metrics. The Adaptive ARIMA model has a substantially greater range of error magnitudes. Both the MSE and RSME of the Sliding Window ELM are substantially lower than that of the Adaptive ARIMA model. Oddly however, the MAD of the Sliding Window is larger than that of the Adaptive ARIMA ELM. This could indicate a tendency of the Sliding Window ELM to over-predict at longer forecast horizons.

	7 Day Ahead				
Model	**Min.**	**Max.**	**RMSE**	**MAD**	**MSE**
Adaptive ARIMA	-4.463	2.125	0.506595	0.1721934	0.256638
Sliding Window ELM	-0.68889	1.04258	0.31984	0.246136	0.102297

Table 10 Adaptive ARIMA and Sliding Window ELM For 7 Day Ahead Forecasts Comparisons

Table 10 compares the Adaptive ARIMA and Sliding Window ELM 3-day forecast error metrics. There is quite a substantial difference in the range of the forecast error magnitudes. The Adaptive ARIMA has substantially larger errors than the Sliding Window ELM. This pattern is also evident in the RSME, MAD and MSE metrics, the Sliding Window ELM tends to perform better for 7-day ahead water level forecasts, some prediction errors whilst large are not as extreme as the errors that can occur when using the Adaptive ARIMA model.

Chapter 5: Discussion

The discussion chapter provides an opportunity to interpret and analyse the results of the study in light of the research questions or objectives. It goes beyond simply presenting the findings and delves into their significance, implications and wider context. This chapter presents the limitations or weaknesses of the study and they suggest avenues for further research. The discussion chapter aims to provide a comprehensive understanding of the study results, contribute new insights into the field, and provide recommendations for future research or practical applications based on the results and analysis.

5.1 ADVANTAGES AND LIMITATIONS OF ADAPTIVE ARIMA

The ARIMA (Autoregressive Integrated Moving Average) model, a mainstay in time series forecasting, possesses certain inherent advantages and limitations. These advantages and limitations can be magnified when specifically applied to Australian river systems, providing insights and context crucial for effective river water level forecasting.

5.1.1 Contextualizing within Australian river systems

Advantages:

1. **Seasonality Handling:** ARIMA models are adept at managing seasonal data, and Australian river systems often exhibit seasonal variability. In regions like the Murray-Darling Basin, marked wet and dry seasons can be effectively forecasted using ARIMA's seasonal differentiation. The Adaptive ARIMA model utilised did not include a seasonal component, instead consisting of a ‘short-term’ 7-prior day history with a updatable model. A future model improvement might be to include a seasonal component to the Adaptive ARIMA model.

2. **Flexibility:** Adaptive ARIMA's parameters (p, d, q) can be fine-tuned for specific rivers, catering to their unique hydrological characteristics. For instance, certain rivers might require a higher autoregressive component due to consistent patterns from the past. In contrast, recent shocks might influence others, warranting a more extensive moving average term. A major benefit of the Adaptive ARIMA is that the model parameters are

incrementally updated, thereby allowing dynamic response to changing water conditions.

3. **Interpretable Results:** The coefficients of the Adaptive ARIMA model provide insights into the nature and lag of dependencies, making it easier for hydrologists and policymakers to interpret and strategize accordingly.

Limitations:

1. **Non-linearity:** While Adaptive ARIMA is effective for linear data, Australian river systems can sometimes display non-linear patterns due to complex ecological interactions, severe weather events, or anthropogenic interventions. In such scenarios, Adaptive ARIMA's performance can be compromised. This issue might have been present in the Albury water level data set that was evaluated, there were occasional abrupt changes in the water levels and associated inaccuracies in water level forecasts.

2. **Stationarity Requirement:** Adaptive ARIMA models require data to be stationary. While the model can handle seasonality (and related trends) through differentiation, sudden and prolonged changes in river water levels, as seen during droughts or significant infrastructure projects, can disrupt this stationarity, necessitating additional data transformations.

3. **Short-Term Forecasts:** The ARIMA is traditionally more accurate with short-term than long-term forecasts. This effect is even more pronounced in the Adaptive ARIMA model, it only utilised 7-days of previous observations, resulting a model that only utilises short-term patterns in the water level data. This limitation can be a significant drawback in the context of river systems, where long-term forecasts are vital for water resource management and planning. The Adaptive ARIMA is designed and best utilised for short-term forecasting and water level management.

In sum, while the Adaptive ARIMA offers a robust method for forecasting time series data, its application to Australian river systems necessitates consideration of the unique characteristics and challenges these rivers present. With adequate data pre-processing and parameter tuning, Adaptive ARIMA can be a valuable tool for river water forecasting in the Australian context.

5.2 ADVANTAGES AND LIMITATIONS OF THE SLIDING WINDOW ELM

The Extreme Learning Machine (ELM) is a rapid learning algorithm primarily designed for single-hidden-layer feedforward neural networks. Over the past years, its usage has expanded beyond its initial scope due to its fast-training speed and generalization capability. In this research a Modified version of the ELM was utilised called the Sliding Window ELM. The Sliding Window ELM, trained on a larger set of prior data but only utilised the previous 7-days of river level data when forecasting the future water level. However, like any machine learning model, the ELM (and Sliding Window ELM) has its own advantages and drawbacks, mainly when applied to intricate hydrological datasets.

Advantages:

1. **Fast Learning Speed:** One of the most significant benefits of ELM is its rapid learning capability. Traditional back-propagation neural networks require iterative weight adjustments, whereas ELM randomly assigns weights and biases to the hidden layer, leading to a considerably faster learning process.

2. **Generalization:** in many cases, ELM exhibits better generalization than standard neural networks. This is particularly valuable for hydrological datasets, which often have varied and complex underlying structures that need a model that can generalize well to unseen data.

3. **Scalability:** ELM can manage large-scale datasets effectively. The vast amount of data often associated with river systems makes ELM a suitable choice, particularly for initially training the ELM model on a large amount of prior data.

Limitations:

1. **Overfitting:** Though ELM possesses robust generalization capability, there is still a risk of overfitting, especially when dealing with datasets with a high dimensionality or noise. Careful tuning and regularization techniques are necessary to mitigate this risk.

2. **Interpretability:** Neural networks, including ELM, are called 'black boxes.' This lack of transparency can be a drawback, mainly when model decisions must be explained or justified to stakeholders in hydrological management.

3. **Random Initialization:** The random assignment of weights and biases to the hidden layer can lead to variability in model results. While this random initialization allows ELM its rapid learning speed, it can mean that different runs of the model may yield slightly different outcomes.

Therefore, the ELM (and Sliding Window ELM) provides a compelling alternative to traditional neural networks, especially in scenarios where rapid training is paramount. This property of rapid training is a potential improvement that could permit the development and implementation of an Adaptive ELM in future work. The potential advantages of the ELM in handling vast hydrological datasets make it appealing for river water forecasting. However, careful implementation and understanding its limitations are crucial to harness its full potential effectively.

5.3 IMPLICATIONS FOR WATER RESOURCE MANAGEMENT

River water forecasting is crucial in ensuring that water resource management strategies are practical and sustainable. Incorporating predictive tools like Adaptive ARIMA and Sliding Window ELM can provide precise insights that influence water policy, infrastructure decisions, and community preparedness.

5.3.1 Practical applications and potential impact.

1. **Flood Preparedness:** Predictive models enable better forecasting of extreme river water levels. With precise predictions, communities can prepare better for potential flood events, minimizing loss and damage.

2. **Optimizing Water Usage:** Accurate forecasting allows for better management of water reserves, ensuring sufficient supply during dry spells and avoiding wastage during periods of abundance.

3. **Infrastructure Planning:** Predictive data can guide decisions on building dams, reservoirs, and other infrastructure. An accurate understanding of future water levels can lead to more cost-effective and sustainable designs.

4. **Environmental Conservation:** Predictive models can also help anticipate adverse environmental conditions like droughts. This can guide interventions to protect aquatic life and maintain the ecological balance of river systems.

5.4 RECOMMENDATIONS FOR FURTHER STUDIES AND IMPLEMENTATIONS.

1. **Integration with IoT Devices:** Future studies could explore the integration of Adaptive ARIMA, Sliding Window ELM, and other predictive models with Internet of Things (IoT) devices for real-time data collection and processing. This would enhance the precision and timeliness of forecasts. Both the Adaptive ARIMA and Sliding Window

ELM have relatively small computational requirements and this could be a major advantage of such systems compared to more complicated deep learning (and similar) models which are more suited to cloud deployment.

2. **Hybrid Models:** There is potential in combining the strengths of different predictive models to enhance accuracy. Future research could delve into hybrid models, blending ARIMA's time series analysis with ELM's rapid processing capabilities.

3. **Stakeholder Collaboration:** Implementation of predictive models should be done in close collaboration with local communities, policymakers, and other stakeholders. This ensures that forecasts are aligned with ground realities and can be translated into actionable strategies.

4. **Addressing Model Limitations:** As technology advances, there is an opportunity to address the limitations of existing models, be it in terms of overfitting, computational demands, or data requirements. Continuous refinement will further solidify the role of these models in water resource management.

Chapter 6: Conclusions

SUMMARY OF RESEARCH FINDINGS

The research embarked on an explorative journey to understand the potential and limitations of Adaptive ARIMA and Sliding Window ELM predictive models in river water forecasting, specifically within an Australian river system. Below, we summarize the key findings:

1. **ARIMA's Applicability in Time Series Analysis:** The ARIMA model showcased its well-established capability in handling time series data. Its prowess in determining patterns from historical datasets allowed for nuanced and often accurate predictions about future river water levels.

2. **ELM's Rapid Computational Benefits:** Extreme Learning Machine stood out for its computational efficiency, learning a large amount of prior data followed by a large number of incremental forecasts. Its single-layer feedforward network allowed for rapid learning, making it suitable for scenarios where real-time predictions are crucial.

3. **Data Pre-processing Importance:** Data collection from sources such as the MDBA River Data highlighted the essentiality of data cleaning and normalization. This ensures that the predictive models function optimally and deliver accurate forecasts.

4. **Adaptive Learning in Adaptive ARIMA:** The study illuminated the short-term memory processes inherent in the Adaptive ARIMA model, making it adapt continually to changes in the dataset and refining its predictive capabilities.

5. **Long-term Data Considerations in ELM:** While Sliding Window ELM displayed rapid learning abilities, its dependence on ample training data was evident. In general, the richer the dataset, the better its predictions, emphasizing the importance of long-term data collection.

6. **Implications for Water Resource Management:** Both Adaptive ARIMA and Sliding Window ELM models can significantly affect water resource management in Australia. Their forecasting abilities can guide decisions ranging from flood preparedness and infrastructure planning to environmental conservation.

The confluence of both models, especially with methodologies like the 7-Day Sliding Window Approach, can provide a holistic view of water forecasting. Therefore, this research has laid the groundwork for the broader implementation of these models in river water forecasting across Australia, ensuring that the continent is better prepared for both the challenges and opportunities its river systems present.

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