## Chapter 6

# Summary, Conclusion and Future Work

## 6.1 Summary

Increasingly, engineers are having recourse to lightweight, flexible structure in attempts to improve dynamic performance and reduce energy demands, but these structures are subject to detrimental vibration. Attention has focused on active means of attenuating this vibration. The literature surveyed in the course of this research reveals that most methods proposed are not fully capable of meeting the demands placed on the controller by variable loading and/or unmodeled dynamic. The most promising methodology appears to be resonant control, which possesses the advantages of robustness to spillover, independence between modes, and simplicity. Even so, resonant control is sensitive to variations in the natural frequencies of the loaded structure, which seriously affect the control performance.

In this thesis, three new control methods, namely: Multi-model multi-mode resonant control ( $M^4RC$ ), adaptive multi-mode resonant control (ARC), and multi-model multi-mode adaptive resonant control ( $M^4ARC$ ) are proposed. Also proposed is a new natural frequency estimation method. The development of these proposed methods is motivated by the need for effective real-time control of multi-mode vibrations in flexible structures. The performances of the proposed

controllers are evaluated against the following essential requirements: (i) the controllers to be robust to the unmodeled dynamics that result from mode truncation, (ii) the control system to be able to cope with large and sudden changes to the plant parameters, (iii) the control methods to use a minimum number of sensor-actuator pairs in controlling multi-mode vibration, (iv) the resulting closed loop systems to have a fast transient response, and (v) the proposed controllers structures to be simple enough to be implemented in a real-time system.

It is claimed that the methodologies proposed offer superior performance to other known methods reported in the literature. In order to justify this claim, the following summary of the work undertaken is presented.

#### **Resonant Control**

Of the principlal methods for multi-mode vibration control in flexible structures, resonant control offers three advantages.

- Robustness to spillover. Resonant control applies a high gain at and very close to the natural frequency of concern, but rolls-off quickly away from the natural frequency. Consequently, it has a negligible effect on other frequencies thus avoiding spillover.
- 2. Independence between modes. The controller for one mode can be designed independently of the controllers for the other modes. In this case the control effect on one mode is isolated from the other modes. Because of this isolation, a resonant control can be formed in a parallel structure to attenuate multiple modes individually. The isolation also enables resonant control to employ a single collocated sensor-actuator pair for multi-mode vibration attenuation.
- 3. Simplicity. Resonant control designs are relatively simple, as they make use of only the natural frequencies of the structure. This simplicity makes them

suitable for real-time implementations.

Due to these characteristics, resonant control is chosen as the basis for the proposed control methods.

Resonant control, however, is very sensitive to variations of the system natural frequencies. The controller cannot give optimum attenuation if the natural frequency of the system is altered from the frequency for which the controller is specifically designed. In the case of a dynamically loaded structure, where frequency variations of the modes are most likely to occur, optimum attenuation can be achieved only if the resonant controller is able to track the system parameter changes and to update its parameters accordingly. Hence for such cases the resonant control method must be modified to allow for the tracking of changes to the system parameters. Three control methods based on the resonant control principle are proposed and evaluated in this thesis.

#### **Experimental Plant and Simulation Models**

In the design process, a physical plant and simulation models of the plant are designed and implemented. The physical plant, which consists of a cantilever beam with magnetically clamped loads placed along the length of the beam, is used to evaluate the proposed controllers. Releasing the magnetically clamped loads causes large and sudden changes to the plant parameters. Modal testing of the structure reveals large vibrations at the natural frequencies and small vibrations away from the natural frequencies. The first three modes are the most significant and a 43.4%, 30.0% and 27.0% variation of the natural frequencies of the respective first three modes can be obtained by changing the loading from full load to no load. The structure also is wide-band: the first and the third modes for all loading conditions are spaced more than a decade apart in frequency. These observations reveal that the plant has all the essential characteristics of a flexible structure, and is thus a suitable research vehicle for testing and evaluating the

proposed controllers.

A simulation study of the plant is undertaken for evaluation purposes. Since all the proposed control methods are based on modal control, which uses the natural frequency and the damping ratio as the design parameters, the mathematical models of the experimental plant are derived using modal analysis. To check the validity of the mathematical models, comparisons are made with models cited in the literature. After validation, the valid mathematical models are then further compared against experimental plant and against numerical models built from finite element methods using ANSYS. A comparison of the results shows that only small discrepancies occur between the models and the real plant. This result confirms that the ANSYS numerical models are an accurate representation of the experimental plant for all the models of interest, and can be used to form simulation models for control design purposes. This is important since the analytical technique for constructing a model of the plant with arbitrary loading is very complex and time consuming, but much easier to construct using numerical methods. The simulation models of the plant are then implemented in Simulink. Four, twentieth-order, simulation models with different loading conditions are implemented.

In agreement with the modal testing results, the simulation results demonstrate that the first three modes of vibration are the dominant modes for all the loading conditions. Therefore, the proposed controllers were designed to attenuate the first three modes of vibration.

#### $M^4RC$

The  $M^4RC$  is an improvement of a resonant control method to accommodate attenuating the vibration of a plant with varying natural frequencies. The design of the  $M^4RC$  is based on the assumption that all the possible loading conditions in the plant are *a priori* known. The  $M^4RC$  method incorporates a bank of known loading models, each designed so that it gives optimum attenuation for a particular loading condition. In contrast to other multi-model methods reported in the literature, which require a controller for each model, the M<sup>4</sup>RC uses only one adjustable controller, which is loaded with the fixed-model parameters of the corresponding mode.

In the first step of the M<sup>4</sup>RC design, a discrete-time resonant controller is formed by transforming the continuous-time resonant controller using a bilinear transformation. The bilinear transformation is used to preserve the passivity of the continuous time resonant control. Therefore, the stability in the continuoustime resonant controller is preserved in the discrete-time version. In the second step, a simple supervisor scheme is designed. The supervisor identifies which model for each mode has the closest frequency to the current vibration frequency and loads the parameters from that model into the adjustable controller to attenuate that mode. The supervisor uses a filter bank. For each mode, the filter bank is composed of band-pass filters that represent the known fixed-parameters models and a decision making component that determines which model is the closest model for the current vibration frequency. Narrow band-pass filters are required to represent the natural frequencies of the fixed-parameter models. To obtain narrow band-pass filters of low order, band-pass filters with a resonant control structure are incorporated.

The effectiveness of the M<sup>4</sup>RC approach is verified through simulation and experiments. The results demonstrate that the M<sup>4</sup>RC is capable of attenuating multi-mode vibrations using a single sensor-actuator pair. A comparison of the proposed supervisor scheme with the customary minimum mean-squares error supervisor scheme is also undertaken. The comparison shows that the proposed supervisor scheme gives comparable performance to the commonly used MMSE supervisor scheme with the advantages that it is able to avoid rapid switching and it requires fewer computations. It is evident that the M<sup>4</sup>RC performs well in dealing with plants with varying natural frequencies. This method offers a simple design and a fast response for systems with varying parameters. In the event that the loading conditions are not represented in the filter bank, the M<sup>4</sup>RC still gives a stable performance albeit with a reduced attenuation performance.

#### **Natural Frequency Estimator**

To achieve optimum attenuation for unknown loading conditions, ARC is proposed. Of all the design parameters associated with a resonant controller the controller centre frequency is the most important. This parameter must be tuned to the plant natural frequency. Small deviations between this parameter and the plant natural frequency will result in ineffective control and poor attenuation. Hence, to implement an effective ARC, an on-line natural frequency estimator is designed and implemented. To obtain a high estimation accuracy of the natural frequencies of a wide-band plant using an RLS-based estimator is difficult. The difficulty arises from two factors: the high-pass characteristic of RLS and the bit-length limitation in the implementation. These two factors lead to an increase of the estimation error as the sampling rate and estimator order increase. To achieve accurate estimation of the natural frequencies of the plant, a parallel second-order estimator is used. Each component in the estimator estimates the frequency of a specific mode. A specific sampling period, appropriate for each mode, is used for each component.

Formulation of the natural frequency estimator is obtained by first approximating the plant as a summation of second-order systems. The decomposition of the plant into several second-order systems according to the numbers of modes is achieved by using a bank of band-pass filters. Each filter is designed to accommodate the full range of frequencies associated with the respective mode for all possible loading conditions. The relationship between the second-order system coefficients and the natural frequency of the system is then obtained using a bilinear transformation. The relationship reveals that the natural frequency of the system depends solely on the positions of the poles of the system. This formulation gives an advantage in that the proposed natural frequency estimator is robust to unmodeled dynamics. This robustness is a consequence of the fact that the positions of the poles of the system are not affected by mode truncation, thus the natural frequency estimation is not affected by unmodeled dynamics resulting from mode truncation.

Simulations and experiments are conducted to evaluate the performance of the natural frequency estimator. The natural frequency estimator is used to estimate the first three natural frequencies for different loading conditions. The results show that there is good agreement between the simulations and the experiments. The results also demonstrate that the natural frequency estimator produces accurate estimations for all the loading conditions of the plant.

#### ARC

The natural frequency estimator is next used to extend the resonant controller into an adaptive resonant controller. The ARC is formed as an indirect selftuning regulator (STR) configuration by combining the resonant controller with the natural frequency estimator. To control the first three modes of vibration of the plant, a parallel structure of three resonant controllers is used, one controller component per mode. The controller is made adaptive by combining it with the natural frequency estimator, which is also a parallel structure of three second-order estimators. The robustness of the ARC to unmodeled dynamics is guaranteed, because both the resonant controller and the natural frequency estimator are robust to unmodeled dynamics. Taking advantage of the indirect configuration, the stability proof of the ARC is given by proving separately: the stability of the resonant control, the stability of the natural frequency estimator, and the admissibility of the parameters obtained from the estimator. Although the ARC is capable of adapting its parameters to suit any loading conditions, transient delay increases while the estimator identifies the new natural frequencies.

#### $M^4ARC$

To improve the transient performance of the ARC, the M<sup>4</sup>ARC is designed. The  $M^4ARC$  is an extension of the ARC. It is obtained by adding an adaptive model to the model bank in the M<sup>4</sup>RC. The addition of the adaptive model reduces the number of fixed-parameter models and enables the system to cope with unknown loading conditions. The improvement of the transient performance is due to the modification of controller parameters adjustment mechanism. A simple supervisor scheme similar to the supervisor scheme in the M<sup>4</sup>RC is used to update the controller parameters. The supervisor decides to update the controller parameters either using the parameters from the fixed-parameter models or from the adaptive models. The decision uses the natural frequency estimator's output. The parameters from the fixed-parameter models will be used if the natural frequency estimator is in its transient condition, and the parameters from the adaptive model will be used once it is close to the steady-state condition. The closeness of the natural frequency estimator to its steady-state condition is determined using gradient measurement. The difference between two consecutive outputs from the estimator is measured. If the difference is less than a certain preset value, the estimator is considered as being in the steady-state condition.

The effectiveness of the ARC and the M<sup>4</sup>ARC are evaluated from simulations and experiments. Comparisons are conducted to measure the performances of the M<sup>4</sup>ARC compared to ARC during load variations. The results demonstrate that when the loading condition changes, the M<sup>4</sup>ARC produces a better transient response than the ARC. The improvement is significant, especially if the change is sudden and large such as when the loading condition changes from fully loaded to unloaded. The experiments reveal that the real-time implementation of the M<sup>4</sup>ARC is robust to ummodeled dynamics, fast, and in the case of SISO system only requires a single sensor-actuator pair.

## 6.2 Conclusion

In conclusion, the proposed three control methods are effective for cancelling multi-mode vibrations in flexible structures. The effectiveness of the methods is achieved by treating a system as a decomposition of individual modes so that a parallel of second-order controller structure can be used to deal with each mode independently. This approach reduces the complexity of the algorithm so that it is more feasible to apply the proposed methods in real-time implementations. The outcomes of this research provide a basis for further development of the theory and the application of active control to flexible structures with unforeseen configuration variations. The principle of the proposed multi-model adaptive control can be applied to systems where fast adaptation of the controller is necessary in order to accommodate structural changes and fluctuating external disturbances. This control method is applicable to any system that can be decomposed into independent modes. The assumption that all the modes are well separated is a limitation of the method. This method is not applicable to systems where there are interactions between more than one mode such as nonlinear systems with internal resonance [106].

### 6.3 Recommendations for Future Work

A variety of recommendations for future research can be made based on the results of this research.

216

In the research presented in this thesis, the stability of the resonant controller is guaranteed due to the collocation of the sensor-actuator pair. The collocation of a sensor-actuator pair, however, may not always be easy to achieve due to the physical complexity of some structures. To ensure the collocation of a sensoractuator pair, the use of the same piezoceramic patch as a sensor and actuator, known as a piezo sensoriactuator [74] can be considered as an alternative solution. For some systems, however, collocating the sensors and actuators will degrade the system performance. For example, the collocation of the sensor and actuator at the joints of a flexible manipulator in a tracking system will not give a satisfactory performance because the elastic modes of the flexible manipulator are seriously excited and not effectively suppressed [73]. Therefore, further investigation of the application of the proposed methods to systems with non-collocated sensoractuator pairs may be fruitful.

A fixed sampling period is used in the natural frequency estimator. Furthermore, information on the frequency bands for each mode is required in order to determine the sampling period of the corresponding estimator. These frequency bands are obtained from *a priori* knowledge about the natural frequencies of the upper and lower boundaries of the system. If *a priori* knowledge about the system natural frequencies is not available, the estimation of frequency bands will be required. Moreover, once the estimation of the frequency bands is obtained, a variable sampling period for the estimator can be used to replace the fixed sampling period. This will ensure that the sampling period is always appropriate for each mode regardless of the variation in the natural frequencies of the system. As an initial suggestion, a narrow band-pass filter array may be used to determine in which band the natural frequency of each mode is located, then the sampling period can be set accordingly.

The ARC and M<sup>4</sup>ARC are designed under the assumption that the order of

the plant is fixed. In reality not all systems are of fixed-order. For instance, for a composite structure that is subjected to sudden delamination, the number of dominant modes which reflects the order of the system may change [98]. To employ the ARC for a system with varying order, the utilization of a lattice filter that has the capability to estimate both the natural frequency and the order of system can be investigated as a step in this research direction.

Although the control methods proposed in this thesis are designed for multimode vibration cancellation, the hardware configuration of the control system is a single-input single-output (SISO) architecture. Extension of the proposed method to a multiple-input multiple-output (MIMO) system (e.g., cancellation of vibration in a plate) is worthy of investigation. In the case of a MIMO system that can be divided into several independent SISO systems the proposed method can be directly applied. However, for a MIMO system with light coupling between the subsystems, the calculation of the optimum controller parameters needs to be considered.

The decentralized characteristic of the proposed methods enables the controller to control each mode separately using a second-order controller. A decentralized implementation, with a distributed processing architecture for the real-time control of a large number of modes, could be a promising research direction.

In the ARC and M<sup>4</sup>ARC designs, although the natural frequency estimator uses a different sampling period for each mode of vibration, the controller itself still uses a single-rate control method. Research into the design of a multi-rate ARC may be useful in obtaining further reduction in the computational time or energy consumption over the single-rate ARC. This is especially applicable for an implementation in low power micro controllers.