ENGR9700-MASTER THESIS

Development of Mathematical Model of a Car Body for Suspension System

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Submitted By

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Declaration

I certify that this thesis does not incorporate without acknowledgment any material previously submitted for a degree or diploma in any university and that to the best of my knowledge and belief it does not contain any material previously published or written by another person except where due reference is made in the text.

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Abstract

Dynamic modelling of a system is essential in the field of engineering and science as it shows the dynamic behaviour of a system when the system is under some external forces. An automobile sector is one of the significant sectors which consists of many subsystems in one system. In order to analyse different systems in the automobile, a dynamic model of each system is necessary. For instance, an automobile consists of many systems such as steering system, braking system, suspension system, electrical system and transmission system where all the systems are developed by engineers to give comfort and safety to their consumers. The vehicle body is a significant part of the vehicle system as all the systems are connected with the body, and it helps to analyse different systems dynamically. The suspension system is one of the essential systems in the vehicle as it gives ride and comfort to passengers and drivers. In order to analyse the dynamic behaviour of the suspension system, a dynamic model of the car body is essential as it gives the overall performance of the suspension system.

There are many lumped parameter models of vehicle used to investigate the dynamic behaviour of different systems. However, when the suspension system is considered for analysis, 2 DOF (quarter car) and 4 DOF (half car) model of vehicle are used. Due to some constraints and assumptions, these models unable to predict real vehicle behaviour. So, in order to predict actual behaviour of the suspension system on car body 6 DOF model of car body required. This research project aims to make 6 DOF model of car body which shows six degrees of freedom motion of the car body in the form of three linear positions of body (longitudinal, vertical, lateral) and three angular positions with respect to the car body axis (pitch, roll, yaw).

Bond graph technique for vehicle dynamics is used to make a 6 DOF dynamic model of the car body. State-space equations are derived for the car body with the standard procedure

of bond graph technique. To investigate the results obtained by the dynamic model of the car body, the most potent tool used in the engineering field for simulation MATLAB-SIMULINK is used. A SIMULINK model of the car body is proposed to simulate the results which show six degrees of freedom motion of the car body for the analysis of the suspension system. Two scenarios are discussed in the result section where in the first scenario, car body only subjected to the gravitational force due to the weight of the car body and in the second scenario sine wave applied to the front left suspension components to analyse the car body behaviour under forces generated by suspension components through road excitation. The obtained results show six degrees of freedom motion of the car body, which includes three translational and three angular motions of the car body with and without external forces acting on the car body through suspension components. This research work is helpful to investigate any suspension system (Active, semi-active, and passive) by investigating car body behaviour under forces generated by the suspension components.

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Nomenclatures

$\dot{x}_m =$ velocity of state variable (m-variable)

$\dot{x} =$ velocity in x-direction

$\ddot{X} =$ acceleration in x-direction for inertial frame

$\ddot{Y} =$ acceleration in y-direction for inertial frame

$\ddot{Z} =$ acceleration in z-direction for inertial frame

$\ddot{x} =$ acceleration in x-direction for non-inertial frame

$\ddot{y} =$ acceleration in y-direction for non-inertial frame

$\ddot{z} =$ acceleration in z-direction for non-inertial frame

$P =$ momentum

$M =$ momentum force

$H =$ angular momentum

$e =$ effort

$f =$ flow

$S_e =$ source of effort

$S_f =$ source of flow

$F_X =$ force in x-direction

$F_Y =$ force in y-direction

F_Z = force in z-direction

M = applied momentum force

I_{xx} = inertia in x-x plane at CG point

I_{yy} = inertia in y-y plane at CG point

I_{zz} = inertia in z-z plane at CG point

a = Distance between the front axel and CG point

b = Distance between rear axel and CG point

c = Distance between suspension reference point to CG point in y-direction

h = Height of CG point from the

ground

d = Wheel track

I = Wheelbase

$p2 = m\dot{x}$ (*linear momentum in x-direction*)

$p23 = m\dot{y}$ (*linear momentum in y-direction*)

$p11 = m\dot{z}$ (*linear momentum in z-direction*)

$p42 = I_{xx}\omega_x$ (*Angular* momentum in *x-axis*)

$p37 = I_{yy}\omega_y$(*Angular* momentum in *y-axis*)

$p32 = I_{zz}\omega_z$(*Angular* momentum in *z-axis*)

e_{45}, e_{26}, e_{16} = effort at the left-front suspension reference point

e_{46}, e_{77}, e_{97} = effort at the right-front suspension reference point

e_{59}, e_{69}, e_{83} = effort at the left-rear suspension reference point

e_{62}, e_{75}, e_{101} = effort at the right-rear suspension reference point

1 Introduction

1.1 Background

Since the wheels have used as a medium to transport cargos and peoples, the evolvement of automobile never stop. Vehicles of all shape and size built to pursue individual requirements. Generally, automobile classified by many factors but one of the main factor is the number of wheels. The number of wheels in automobile never choose according to the advantages of one on another, but it chose based on design and performance needs to accomplish the required potential work. The evolution of the automobile begins with one wheel vehicle to four-wheel vehicle, and evolve as per the requirement of transportation. As the popularity of ground vehicles is increased, engineers and researchers have built a common interest to build a safer and efficient public transport which used for both purposes for humans as well as cargos. Researchers found that vehicle components, such as suspensions, steering, braking, are essential as it gives the power to handle the vehicle according to the situation and gives efficient and safer drive to drivers and passengers. All the systems incorporate with handling, and driving characteristics have come under vehicle dynamics as they connected with the ride and quality of the vehicle. Vehicle dynamics is the primary subject of the automobile industry, where the dynamic behaviour of automobile studied for different driving conditions. The suspension system is a crucial part of the vehicle for the study of vehicle dynamics as it controls the vertical movement of the car body and also helps to control steering geometry such as camber, caster, toe-in and toe-out which helps to improve the drivability of the vehicle . There are mainly three types of suspension systems currently used in the automobile sector passive suspension, semi-active suspension and active suspension system. In the automobile sector, the passive suspension system is widely used because of its simple geometry and availability, which only consist of passive elements such as spring and damper assembly.

In contrast, the semi-active suspension system has a shock absorber which controls the damping force as per the input coming from the road surface. The active suspension system is not widely used in the market, but many researchers have been working on active suspension system nowadays. The active suspension has consist of active elements which are continuously measured shocks coming from the road surface and position of the car body and control vertical movement of the car body according to the position of the car body. For the analysis of any suspension system in vehicle dynamics, one dimensional two degrees of freedom lumped parameter model is used where all the elements of the suspension system are model as spring and damper assembly, and vertical movement of the sprung and unsprung mass has been investigated. Two dimensional four degrees of freedom lumped parameter model used to investigate pitch and roll movement of the car body. However, there is a lake of three dimensional six degrees of freedom model which shows three linear(longitudinal, lateral, vertical) and three angular (roll, pitch, yaw) movements of the car body, so it is essential to have six degrees of freedom model which can show six motion of car body to analyse suspension system.

1.2 Objectives

The objectives of this research project are shown below.

1. To investigate the existing models used for the analysis of the suspension system.
2. To study types of modelling techniques used in the Automobile sector to investigate the behaviour of the system.
3. Choose appropriate modelling technique to accomplish the research objective
4. To make a mathematical model of the car body which shows six degrees of freedom motion.
5. Investigate the results obtained by the mathematical model.

1.3 Thesis Outline

Chapter 1

In this chapter, background information related project title has discussed and objective of the project defined.

Chapter 2

In this chapter, necessary literature required for this project is included.

Chapter 3

It contains equations which are helpful to make a mathematical model of the car body. Newton Euler equations are briefly discussed in this chapter.

Chapter 4

In this chapter methodology used to make a mathematical model of the car body is discussed. The different variable used in the bond graph is discussed briefly and, the procedure of derivation of equations are discussed.

Chapter 5

In this chapter bond graph model of full car body proposed and state-equations are derived by bond graph methodology.

Chapter 6

In this chapter Simulink model of the car body proposed and simulated results are discussed.

Chapter 7

In this chapter summary of the thesis project and conclusion of the project discussed. Limitations and future work are also included.

At the last references used in this project are included with appendices on which additional information about the project included,

2 Literature review

2.1 Introduction

This chapter consists of the necessary literature required to make a dynamic model of a car body and modelling techniques which are mainly used for modelling. In the field of engineering and science modelling and simulation of engineering systems play a vital role. Modelling techniques are used to analyse different physical systems for a better understanding. Moreover, These modelling techniques are also used in the early stage of development of new engineering systems where they predict the overall behaviour of the engineering system with the help of simulation.

2.2 Types of models

A model is a simple way to describe a system on which experiments are being done to understand how the system works, and modelling is the process of organising data for a given system (Cellier, 1991). There are various models used in the field of Engineering to investigate the behaviour of any system. A model is nothing but the abstraction of some system which represents the behaviour of that system or predicts the outcome of that system. Models are usually used to study a system or to investigate the system behaviour under some specific inputs. In general, models are classified into four types.

1. Iconic models
2. Graphical models
3. Analog models
4. Mathematical models

2.2.1 Iconic models

Iconic models are defined as real physical models which exactly look like a real system, but the size of those models are different in scale. These types of models are used in the first phase of designing any system. Mockup and prototype models are the best examples of an Iconic model. They are usually used in the designing process of the new systems. Mockup models have the same size as a real system, but the materials used in this model is different. For instance, to design a car body in the automobile sector, a clay model or plastic model of the car body first developed to get a glimpse of the real system. However, a prototype model is a model used to investigate the system behaviour at the designing stage, but the size used for that model is smaller than the real system. The best example of the prototype model is the model of aircraft and submarines used in the wind tunnel to investigate the real behaviour of the system under specific input parameters. To build a real physical system Iconic model is play a vital role as it is cheap to build and gives the same behaviour as a real system gives. In designing stage of any system prototype model of that system first developed and then with the help of dimension analysis, the real system has developed.

2.2.2 Graphical models

Graphical models simply consist of graphs which represent the number of nodes connected with edges and edges represents the direction. Graphical models are developed manually as per the requirement of the designer. Graphical models are widely used in computer programming software nowadays to develop any system based on a graphical method and efficiently simulate the results obtained by the graphical model. MATLAB -SIMULINK software is one of the best examples of graphical modelling.

2.2.3 Analog Models

Analog models are used for analysis before the invention of digital computers. Analog models are developed in the form of graphical representation of system variables, and analog computers are used to simulate the model behaviour.

2.2.4 Mathematical models

In the mathematical model, the system is described by mathematical equations in the form of mathematical language by considering various logic, relations, operations, expressions and symbols. In the engineering field, mathematical models are used widely. There are various equations that are used to develop mathematical models such as linear equations, nonlinear equations, differential equations and many more. According to equations used in the mathematical models, models are classified as linear models, nonlinear models, static models, dynamic models, parametric and non-parametric models.

2.2.5 Static model

Static models described by the relationship of input and output, and they are independent of order and rate of the input signal. In general, the output of the static model only depends on the input value of the model at the instant it does not depends on the past values of the input signal. It is also called a stateless model or zero memory model.

2.2.6 Dynamic model

The dynamic model is a model which has a memory in some form. The output of the dynamic model does not depend on inputs at instant same as a static model but the output of the dynamic model change as input signal changes because dynamic model remembers the previous memory of input variable. Dynamic models are formed as state-space equations in the engineering field. In state-space formalism, dynamic systems are described as a set of inputs, states, and output variables that are described as first-order differential equations. In the first-order differential equations, the first derivative of the state variables are placed at the left-hand side, whereas state variables and input variables are placed at the right side of the equation. Whereas State variables and input variables algebraically define the output variables. For instance, consider a model equation of a system which has n-input variables and m-state variables The state equation can be written as,

$$\dot{x}_1 = f_1(u_1, u_2, \ldots\ldots u_n, x_1, x_2 \ldots\ldots x_m) \quad (1)$$

$$\dot{x}_m = f_m(u_1, u_2, \ldots\ldots u_n, x_1, x_2 \ldots\ldots x_m) \quad (2)$$

Whereas, the below equations define output variables.

$$y_1 = g_1(u_1, u_2, \ldots\ldots u_n, x_1, x_2 \ldots\ldots x_m) \quad (3)$$

$$y_p = g_p(u_1, u_2, \ldots\ldots u_n, x_1, x_2 \ldots\ldots x_m) \quad (4)$$

2.2.7 Parametric model

When the model has a small number of parameters at a different point over a structure, the model is called a parametric model. The parametric model has a small number of parameters and shorter to describe it, and all the parameters used in the model have distinct significance in the system behaviour. Parametric models have either linear or nonlinear according to parameters used. Consider input vector as u, and output vector as y where the parameter of the model can be identified as β1, β2, β3……, βn in the following equation. The following equation can be considered as a nonlinear function in terms of inputs but not in terms of parameter because parameters show a linear relationship. The solution of this equation is also linear in terms of parameters.

$$y = \beta_1 u_1 + \beta_2 u_2^2 + \beta_3 sin(u_3) \tag{5}$$

2.2.8 Non-parametric model

The non-parametric model can be defined by a large number of parameters used in the identification of the system. All the non-parametric model have a similar significance of parameter over the structure of the model. The non-parametric model is also considered as linear or nonlinear in terms of a parameter.

2.3 Importance of Vehicle dynamics

Vehicle dynamics deals with moving vehicles, and resultant forces acted on cars while the car is in moving condition. Vehicle dynamics is defined further in two parts (1) Vehicle Handling dynamics (2) Vehicle driving dynamics. Lateral dynamics and transverse dynamics both are covered in vehicle handling dynamics, which mainly depend on vehicle sideslip caused by tire lateral force, roll, and yaw motion. Whereas, vehicle driving dynamics divided into longitudinal dynamics and verticle dynamics which include braking, driving and ride comfort(Yang et al., 2013).

Modelling of the vehicle dynamics consists of the following system components, analytical and mathematical description, and influence of these components on the system, to evaluate the overall behaviour of the system.

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Figure 1 Vehicle dynamics Environment and related components(Dieter Schramm et al., 2018).

2.4 Methods of modelling used in vehicle dynamics

In the field of vehicle dynamics, various modelling methods have used to evaluate the performance of the different systems depends on a type of research held out by a researcher. Some of the modelling techniques used in modelling for the vehicle dynamics are lumped mass parameter modelling, finite element modelling, multibody system dynamic modelling, kinematics and dynamics modelling of system and bond graph technique. Every modelling techniques have their advantages and disadvantages.

2.4.1 Lumped parameter modelling

This technique usually deals with vehicle handling and driving dynamics. In this technique, the vehicle system is divided into three component elements, such as mass, spring, and damper. Many researchers used this technique to develop vehicle models of two, three, four, seven and eighteen degrees of freedom car models to evaluate the different characteristics of the system. (Huh et al., 2000) have modelled 18 degrees of freedom model using lumped parameter technique and simulate it in Matlab/ Simulink to investigate the handling parameters such as yaw rate, lateral acceleration in frequency and time domain. There are mainly three types of models used to investigate vehicle handling and ride dynamics 1. Quarter car model 2. Half car model and 3. Full car model of a lumped parameter as per the investigating properties.

2.4.1.1 Quarter car model

Quarter car model mainly used to investigate vertical dynamics of suspension in the frequency domain when the range of frequency in vertical vehicle dynamics in between (0-25Hz). This model primarily deals with the dynamic behaviour of the sprung and unsprung mass of the vehicle connected to a suspension system where sprung mass is considered as a mass attached to the suspension system(mainly considered as a quarter of chassis mass, payload including passenger weight) and unsprung mass is the mass of wheel carrier, tire, brake, and the wheel which are not supported by the suspension. Indirectly it also measures the performance of the vertical dynamic of the suspension system(Sun et al., 2020).

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Figure 2 Lumped parameter model of a quarter car for an active suspension system(Sun et al., 2020).

2.4.1.2 Half car model

The half-car model is a 4 DOF model which usually used to investigate vertical and pitch motion of the sprung mass and vertical motion of the unsprung mass. In order to investigate pitch and vertical movement of the car, the car has considered symmetric from the left and right sides (Sun et al., 2020).

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Figure 3 Lumped parameter model of half car for an active suspension system (Sun et al., 2020)

2.4.1.3 Full car model

The entire car model consists of 7 DOF due to four-wheel attached to the vehicle body. Due to the symmetric body of the car from left to right, full car model investigates various motions of the vehicle from vertical, pitch and roll movement of the sprung mass to vertical motion of all the four-wheel unsprung masses which helps to examine the performance of different suspension systems such as Active, semiactive, and passive(Sun et al., 2020).

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Figure 4 Lumped parameter model of Full car for an active suspension system (Sun et al., 2020).

With the help of Lumped parameter models of 2 DOF, 4 DOF, 7 DOF(Manoj k. Mahala et al.) studied and analysed different responses for different road conditions such as pitch and roll motion of the body to given input parameters, and body acceleration and body displacement for the passive suspension system with the help of simulation in Simulink. Lumped parameter models are simple to understand as it gives simplified actual vehicle structure and measure vehicle vibration characteristics with the effect of the structural parameters from the vehicle performance. Though lumped parameter technique used to analyse the overall system, this method does not include nonlinearity comes from the suspension structure.

2.4.2 Finite Element Modeling

Finite element method is virtual prototype method which is generally used for dynamic design of complicated parts such as vehicle components like vehicle body, engine components, and suspension components. This modelling technique used to analyse nonlinear models of the system as it is challenging to interpret a nonlinear system, with the help of the lumped parameter model. The final element method subdivides the mathematical model of a complex nonlinear system into disjoint components called as finite elements. These finite elements consist of pure geometry and also it comprises simple algebraic equations. The response of all finite elements is recorded as a finite number of degree of freedom at nodal points. The reaction of all finite elements is assembled to get a response of a full mathematical model (Friswell M. I and E, 1995).

Many researchers used this technique for the analysis of different parameters during the design stage. Dynamic stress analysis of vehicle frame using VPG (Virtual Proving Ground) approach with Finite Element code has investigated by Gyu Ha Kim et al. (2003) and predicted results were compared with experimental data. (Gyu Ha Kim et al., 2003) have found that the VPG method is useful to analyse dynamic stress applied on the vehicle frame and also state that this method could be helpful for suspension kinematic analysis and vehicle dynamic analysis.

Finite element modelling is also useful because the FE model can be used in any multibody system simulation program to analyse the system precisely. Multibody system simulation is a computer program which simulates the behaviour of the system model efficiently. Many researchers used the FE model with a multibody systems simulation model to evaluate the performance of vehicle dynamics. (Chang-Ro Lee et al., 1997) established an FE model of a tire and used this model to analyse vehicle dynamics in VPG environment. (C. W. Mousseau et al., 1999) have used Multibody system simulation program to evaluate the vehicle dynamic performance with multibody MSS vehicle model and FE tire model. (C. W. Mousseau et al., 1999) found that the simulation of the combined model of multibody and FE tire model is beneficial to predict vehicle handling performance, but this approach is not sufficient for the vehicle ride performance.

2.4.3 Multibody system dynamic Modeling

Multibody systems are considered as multi bodies connected with appropriate joints. Usually, multibody systems consist of multi motions relative to each other, and this model can show more than six degrees of freedom motion of the system. Generally, mechanical systems are described by a multibody system which contains mostly rigid bodies connected with bearings and the joint of another body called nodes. In multibody system, rigid bodies are defined by the mass of body and moment of inertia of a body. All the forces and moments are acting at a node point where other bodies are connected. Moreover, elasticity and damping are modelled as massless force elements. Some typical modelling symbols are shown below (Dieter Schramm et al., 2018).

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Figure 5 Elements of multibody systems (Dieter Schramm et al., 2018)

For a complex multibody system such as in the vehicle model, it is possible to make a model of the different subsystem of the vehicle and analyse the subsystem individually also add different models of subsystems afterwards if required.

Multibody systems are incorporate with many computer-based software to analyse multibody mechanical systems. (S Hegazy et al.) has presented a multidegree of freedom nonlinear model of vehicle for vehicle handling analysis in which all subsystems like front and rear suspension with rack and pinion steering system incorporated. The vehicle model made on ADAMS and various vehicle handling parameters such as lateral acceleration, roll angle, and vertical tire forces were investigated. Though the multibody system gives an accurate analysis with multibody dynamic software like ADAMS, and Simpack it is complicated to study the six degrees of freedom motion of the car body as it contains several components as a multibody with appropriate constraints and joints. Because of multi degrees of freedom motion of this multi bodies, this technique usually used for the analysis of the subsystems by subsystem synthesis method on which every subsystem modelled as a separate multibody system and evaluate the performance of that particular subsystem such as suspension system, steering system. So mostly this method is used for analysing vehicle handling characteristics but not for ride control as it is a very complicated method to investigate ride dynamics of the vehicle.

2.4.4 Kinematics and dynamic modelling

The role of kinematics and dynamics modelling is crucial in vehicle dynamics as kinematics describe the orientation of the system model with position and appropriate motion of joints whereas dynamic model deals with the analysis of the system model which gives approximate system behaviour. In the field of robotics, this modelling technique is popular as robotics contains many rigid bodies attached with appropriate joints and constraints (Chenguang Yang et al., 2016). The vehicle is also a combination of rigid components where subsystems like steering, rigid body and suspensions are connected with multibody components with suitable joints. Therefore, To predict particular behaviour of such a subsystem, this technique of modelling is used in vehicle dynamics. (Jorge Hurel et al., 2013) proposed a Mcpherson suspension planer quarter car model with including kinematics of the suspension and derive the mathematical equations based on the kinematic model and make a dynamic model. Dynamic behaviour of suspension has investigated in terms of vertical motion of the wheel, sprung mass acceleration, sprung mass displacement and compared with an Adams model.

(Jorge Hurel et al., 2013) believes that Though multibody dynamics software used for investigation of vehicle suspension, there is still need one accurate analytical model to use in hardware in the loop application. Moreover (M. S. Fallah et al., 2008) have proposed a new nonlinear model of Mcpherson suspension system for ride control and investigate the vertical acceleration of sprung mass, rotation motion of unsprung mass due to control arm and also evaluate the kinematic parameters such as camber, caster and kingpin inclination on the ride vibrations. This technique is useful to derive an analytical model of the system, but due to kinematics modelling, it is challenging to make a model of six degree of freedom for car rigid body.

2.4.5 Bond graph technique for vehicle dynamics

In the field of designing a dynamic system, it is common for engineers and designers to express their model graphically to communicate with each other and to exchange their model with each other. Bond graphs are the definition of formalism that is best suited for the modelling of multidisciplinary dynamic systems, including components from different energy domain such as mechanical, electrical, thermal and hydraulic domain. In the engineering field, the different graphical model used for different fields. For instance, free body schematic in mechanical, circuits in electrical, and block diagram, linear graphs, signal flow diagram for the hydraulic systems. Block diagrams present the structure of the mathematical model, but they do not replicate the physical structure. Same as signal blocks can not be connected with system components. Whereas in bond graph modelling dynamic system includes system components and elements that are interacting by exchanging energy. In short, bond graph reflect the physical structure of the system (Borutzky, 2010). The bond graph represents energy transfer between subsystems associate with physical quantity like mass, momentum, electric charge and entropy as per the subsystems that's why it is called as physical-based system modelling

(Breedveld, 1993). The bond graph uses multiport modulated transformers and the modulated gyrators to express transformation between forces and velocities between inertial to rotating frames and handle the motion of a rigid body. It is convenient to derive the equation of motion from a riders point of view in vehicle dynamics. The bond graph gives the same kind of physical insight as an electric circuit diagram when the equation of motion derives from standard bond graph technique (Karnopp, 1976).

According to (Karnopp) bond graphs and circuit diagrams are more potent than equations as each bond graph of subsystem generates various equations depending on the connection of the subsystems. (Karnopp) developed a six-port bond graph model of a rigid body which shows the nonlinear motion of the rigid body for analysing the stability and control properties of automobiles, aircraft and for a ship. (Filippini et al., 2005) constructed a dynamic bond graph model of the nonlinear vehicle using 20sim modelling and simulation. Nonlinearity coming from vertical, lateral, and longitudinal dynamics, as well as geometric nonlinearity coming from the suspension unit, are taken into account. Bond graph technique of vehicle dynamics is the best technique to make a dynamic model of the system as it incorporates the physical structure of the system, and also it is an object-oriented technique of modelling.

2.5 Gap statement

The review of this literature shows that there are many ways to model any specific subsystems in vehicle dynamics. However, in terms of a vehicle body, there is very less work done on a vehicle body modelling to define the dynamic behaviour of the car body. The car body is the common subsystem connected with many other subsystems through multibody mechanics. Therefore, the dynamic response of the car body used to analyse the performance of other subsystems connected with it.

2.6 Contribution

Gap statement shows that the vehicle body is an essential subsystem of the vehicle as it helps to analyse the performance of the other subsystems in order to design and development. So it is necessary to have a dynamic model of the car body which shows the six degrees of freedom motion under some external forces.

3 Newton Euler Equations of Motions for Rigid Body

3.1 Introduction

A rigid body has six independent motion in the space which represent six degrees of freedom motion of the rigid body. To define the equation of motions of any rigid body, it is easier to define velocities of the body with respect to the body-fixed frame, These velocities than transferred to equivalent inertial frame velocities. The body-fixed frame is considered as a noninertial frame. Due to constant geometric distance of the rigid body, The velocities of the rigid body efficiently computed in a body-fixed frame. Modelling of the complex multibody system is always be considered in the noninertial frame.

3.2 Inertial and Noninertial reference frame

An inertial frame is a reference frame through which velocity, acceleration vectors of the rigid body are measured for the noninertial frame. Consider O-X-Y-Z as an inertial frame of the rigid body, and o\`-x-y-z is the noninertial frame of the rigid body with origin O\`. The noninertial frame is passing through the centre of mass of body which is translated and rotated when the body is translating and rotating about its principal axis, the noninertial frame is momentarily aligned with the principal axis of the system body. Where V_{XYZ} represents the velocity vector in the inertial frame, and Vxyz represents the velocity vector in the noninertial frame. If the angular velocity vector is defined by $\omega_{xyx,}$ then the acceleration vector in the inertial frame can be represent by the following equation.

$$a_{XYZ=}\, a_{xyz} + \omega_{xyz} \times V_{xyz} \tag{6}$$

Where a_{xyz} is the acceleration observed in the noninertial frame. If ($\ddot{X}, \ddot{Y}, \ddot{Z}$) are the acceleration components of the inertial frame and($\ddot{x}, \ddot{y}, \ddot{z}$) are the acceleration component of the body-fixed frame than acceleration in the inertial frame can be defined by following equations.

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Figure 6 Inertial and non-inertial frame of body

$$\ddot{X} = \ddot{x} + \left(\dot{z}\omega_y - \dot{y}\omega_z\right), \quad (7)$$

$$\ddot{Y} = \ddot{y} + (\dot{x}\omega_z + \dot{z}\omega_x), \quad (8)$$

$$\ddot{Z} = \ddot{z} + \left(\dot{y}\omega_x + \dot{x}\omega_y\right). \quad (9)$$

If F_x, F_y and F_z are the component of resultant force towards the centre of mass of the rigid body in the noninertial frame, then newtons equation of motions for a three-dimensional rigid body can be derived as

$$m\ddot{X} = m\ddot{x} + m\left(\dot{z}\omega_y - \dot{y}\omega_z\right) \quad (10)$$

$$m\ddot{Y} = m\ddot{y} + m(\dot{x}\omega_z - \dot{z}\omega_x) \quad (11)$$

$$m\ddot{Z} = m\ddot{z} + m\left(\dot{y}\omega_x - \dot{x}\omega_y\right) \quad (12)$$

Where $m\left(\dot{z}\omega_y - \dot{y}\omega_z\right)$, $m(\dot{x}\omega_z - \dot{z}\omega_x)$, $m\left(\dot{y}\omega_x - \dot{x}\omega_y\right)$ are the pseudo forces acting on the noninertial frame and derived equations are called as newtons equation of motion in the noninertial frame.

3.2.1 Eulers equations of motions

Euler's equation of motions are the extended version of the newtons equation of motions for point particle to rigid body motions. Euler's first law states that the linear momentum of any rigid body is equal to the product of the mass of the body and the velocity of the centre of mass of that body.

$$P = mv_{cm}$$

Euler's second law states that if the angular momentum of any rigid body is H about a fixed point on the inertial reference frame, then the rate of change of angular momentum is equal to the moment force(Torque) acting on that body about the fixed point of reference

$$M = \frac{dH}{dt}$$

Euler also states that for rotational dynamics angular momentum H of the rigid body is defined by the product of the moment of inertia (I) of the rigid body towards the centre mass and angular velocity (ω) of the rigid body with respect to the principal axis of the body.

$$H = I\omega$$

Let be consider a rigid body which has three angular velocities respective to its principal axis. If ω_x, ω_y, ω_z are the angular velocities of the rigid body with respect to its principal axis. According to Euler's law, angular momentum can be defined by the following equations.

$$H_x = I_{xx}\omega_x - I_{xy}\omega_y - I_{xz}\omega_z \tag{13}$$

$$H_y = I_{yy}\omega_y - I_{yx}\omega_x - I_{yz}\omega_z \tag{14}$$

$$H_z = I_{zz}\omega_z - I_{zx}\omega_x - I_{zy}\omega_y \tag{15}$$

Where I_{xx}, I_{yy}, I_{zz} represents the second mass moment of inertia whereas I_{xy}, I_{xz}, I_{yx}, I_{yz}, I_{zx}, I_{zy} are the product of inertia of the rigid body If the rigid body is symmetry with respect to the principal axis product of inertia will have vanished.

$$I_{xy}=I_{xz}= I_{yx}= I_{yz}= I_{zx}= I_{zy}= 0$$

$$H_x = I_{xx}\omega_x$$

$$H_y = I_{yy}\omega_y$$

$$H_z = I_{zz}\omega_z$$

Now let's consider three external moments acting on a rigid body towards its centre of mass are M_x, M_y, and M_z Then according to Eulers second law

$$M_x = \frac{dH_x}{dt}$$

$$M_y = \frac{dH_y}{dt}$$

$$M_z = \frac{dH_z}{dt}$$

$$\frac{dH}{dt}_{XYZ} = \frac{dH}{dt}_{xyz} + \omega \times H \tag{16}$$

$$\dot{H} = \hat{\imath}I_{xx}\dot{\omega}_x + \hat{\jmath}I_{yy}\dot{\omega}_y + \hat{k}I_{zz}\dot{\omega}_z + (\hat{\imath}\omega_x + \hat{\jmath}\omega_y + \hat{k}\omega_z) \times (\hat{\imath}H_x + \hat{\jmath}H_y + \hat{k}H_z)$$

$$M_x = I_{xx}\dot{\omega}_x + I_{zz}\omega_y\omega_z - I_{yy}\omega_z\omega_y$$

$$M_y = I_{yy}\dot{\omega}_y + I_{xx}\omega_x\omega_z - I_{zz}\omega_z\omega_x$$

$$M_z = I_{zz}\dot{\omega}_z + I_{yy}\omega_x\omega_y - I_{xx}\omega_y\omega_x$$

So the general form of the equations are written as follows

$$M_x = I_{xx}\dot{\omega}_x - (I_{yy-}I_{zz})\omega_y\omega_z \tag{17}$$

$$M_y = I_{yy}\dot{\omega}_y - (I_{zz-}I_{xx})\omega_z\omega_x \tag{18}$$

$$M_z = I_{zz}\dot{\omega}_z - (I_{xx-}I_{yy})\omega_x\omega_y \tag{19}$$

The derived equations are called Euler's equations of motion for the rigid body, which shows due to external moments applied on the rigid body, the rigid body moves angularly with its principal axis.

Generally, Newton's equation of motions derived for three-dimensional rigid body shows three translation motion of the rigid body, whereas Euler's equation of motion shows three angular motion of the rigid body. If Newton and Euler's equation of motion combined together, it shows six independent motion of the body which is able to show six degrees of motion of the rigid body in space.

3.3 Eulers angles

A rigid body has six different motions in the space. The orientation of any rigid body can be defined by a given frame of reference by which the angular position of any rigid body can be defined. Euler's rotational theorem states that two consecutive rotations about two different fixed axis of the body is equal to the single rotation about another fixed axis. Therefor three or more than three rotations about fixed axes give a single rotation of the body. A rotation vector is used to represent the orientation of any rigid body, and it is a product of elementary rotational matrices which represents the rotation of the body about a fixed or rotating axis these rotational matrices are described as Eulers angles in the cartesian coordinate system.

If the X-Y-Z is the reference frame of body and x-y-z is the body-fixed frame than the intersection of homologous planes are taken as a nodal line of Eulers angles. In-vehicle dynamics intersection of the non-homologous plane is always preferable to find the nodal line of the body. However, there are some possibilities to define improper Eulers angles. There are various names used for improper Eulers angles such as Cardan angles, nautical angles, and Roll-Pitch-Yaw angles, together these angles are known as Tait-Bryan angles. In the field of vehicle dynamics, Cardan angles are used to illustrate yaw, pitch, and roll motion of the car body. Let be consider two non-homologous planes X-Y, and x-z to define Cardan angles for

vehicle body through which nodal line can be defined. To evaluate Cardan angles, lets rotate X-Y-Z axes by an angle Ψ about Z-axis so that a new coordinate system is defined as x_1-y_1-z_1.

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Figure 7 Eulers angle notation

If we rotate x_1,y_1,z_1 axis by angle θ about y1 axis, so that x_2-y_2-z_2 defines a new coordinate system of the body. Where x_2 is aligned with the x-axis. Finally, further rotation of coordinate system about x-axis by the angle ϕ gives y2 and z2 axis in terms of y, z-axis due to alignment of the axis. These successive rotations about Z-Y-X axis give Cardan angles, which represents body yaw, pitch, and roll respectively. For vehicle dynamics, the z-axis is taken as a vertical axis to ground, and x-axis should be aligned with the length of the vehicle.

4 Methodology

4.1 Introduction

This chapter consists of the methodology used to make a mathematical model of the car body which shows six degrees of freedom motion relative to the axis of the car body. A bond graph technique of vehicle dynamics has used to make a model of the car body. Different elements used in bond graph modelling are discussed briefly in this chapter.

4.2 Bond graph technique

A system is a combination of different subsystems connected with each other. Bond graph technique is the unified technique to connect system components graphically. A graphical representation is vital in the engineering field as it simplifies the system. There are several graphical methods available for different engineering field such as for Mechanical engineering free body diagrams, for electrical engineering circuit diagrams, flow charts and many more. However, in the mechanical system, it is tough to represent the behaviour of system components whose connected and exchanging energy with the help of free body diagram as it does not show the physical properties of components such as how much power exchanged between two connected components. The bond graph is the unified technique to connect two physical components of the system, which shows the physical behaviour or power exchanged between them by the connection of half arrow connected called as a bond. In short, The bond graph is a collection of multiport elements. In the bond graph power is transferred by two generalised variable effort(e) and flow(f). In mechanical engineering, efforts are considered as intensive parameters such as pressure, force, or torque applied on a model whereas, flows are the extensive parameters such as volume flow, entropy flow, velocity and many more.

4.2.1 Port

Port is the connection point or node where all the bonds are connected with elements of the bond graph.

4.2.2 Power port

The power port is the connection point where bonds are connected. It unable the energy transfer between other nodes connected with power bonds. In short, power ports are considered as a place where energy can either enter or leave the system. Powersports are not only used for energy transfer between elements, but it is also used as a medium to transfer information such as sensors used in a system to transfer information to operate mechanisms.

4.2.3 Multiport

A bond graph node is called multiport if it connects with more than one bond. Consider a system which consists of the number of subsystems in it. In below system, S is the primary system in which S1, S2, S3, S4, S5, and S6 are the subsystems connected with each other here S3 and S4 has one port whereas S2 has a multiport which is connected with more than one system.

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Figure 8-Figure 8 General bond graph structure of systems (Borutzky, 2010)

4.2.4 Power Variables used in bond graph

In general domain, power is the multiplication of two variables which shows the power transfer in between two subsystems. In bond graph power variables are taken as effort (e) and flow (f) because these variables are used to find what amount of power transfer in between two subsystems. Consider an electric system in which power is the function of voltage(V) and current(I) where, V is effort, and I is the flow of the system.

$$Power = Effort * Flow$$

$$P = V * I$$

4.2.5 Energy Variables used in bond graph

Energy variables are the most important variables used in the bond graph as it represents the energy transferred between two elements of the bond graph. P and q are the standard variables used in the bond graph for the mechanical domain, which shows the momentum and position of the system, respectively. The table represents the various energy variables used in different domains. Generalised momentum and displacement can be defined by the integration of given effort and flow respectively.

Table 1- Bond graph variables for different energy domains (Borutzky, 2010)

Energy domain	**Effort(e)**	**Flow(f)**	**Generalised Energy variables**	**Generalised displacement**
Translational mechanics	Force(F) (N)	Velocity(v) (m/s)	Momentum(p) (N/s)	Displacement(x) (m)
Rotational Mechanics	Angular moment(M) (Nm)	Angular velocity(ω) (rad/s)	Angular momentum(p_ω) (Nms)	Angle(θ) (rad)
Hydraulic	Total pressure(P) (N/m^2)	Volume flow(Q) ($m^{3/}s$)	Pressure momentum(p) (N/m^2s)	Volume(Vc) (m^3)

$$q = \int_0^t f\,dt \tag{20}$$

$$P = \int_0^t e\,dt \tag{21}$$

4.3 Power direction

When two or more than two submodels of the system are connected through a bond graph, power ports of submodels exchange energy through non-directed edges of the bond graph. But, when submodels are represented in the form of equations sign convention is more important to derive a set of equations from the bond graph where all the variables used in the equations are consistent with their sign. In the bond graph technique, this type of sign convention is achieved by adding a half arrow to the front edge of the bond, which represents a positive reference direction of energy flow. In short, half arrow represent the positive direction of energy flow, but edges do not show the actual direction of energy flow. When the bond is connected to any power port of model, and its orientation is towards the power port, the direction of power taken as positive. However, when a bond is connected to the power port of energy source, the positive direction of the power flow considered as bond oriented away from the port.

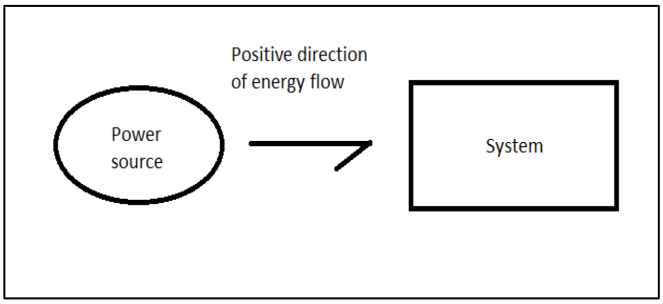

Figure 9- Energy flow direction away from the source

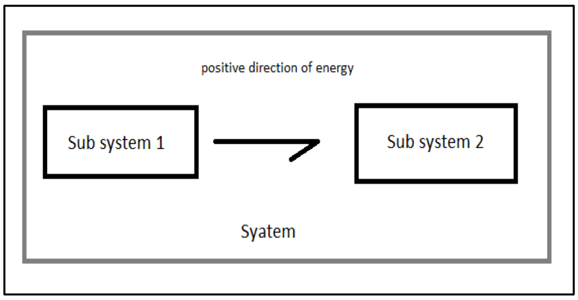

Figure 10- Energy flow direction towards subsystem

4.4 Basic bond graph elements

Bond graph technique has various elements to describe a system graphically. According to the physical process involved in the system model bond graph described various elements which are stated below.

- Source element

- Dissipators
- Storage elements
- Power couplers and transducers
- Power nodes (Junctions)

4.4.1 Source element

Source elements used in the bond graph are the primary element which gives energy to the system in terms of the generalised variable of energy such as effort (e) and flow (f). According to (Cellier, 1992) source elements are not a part of the system, but it represents the boundary conditions of the system. In general, source elements are used to generate the impact of surrounding on the system. In bond graph methodology, two types of source variables are used by which one is denoted by Source of effort (Se) and other is denoted by the source of flow (Sf). Source of effort only mandates effort according to the function of time and ignores flow. Whereas the source of flow only mandates flow and neglect the effort.

The best example of source elements is the gravitational force acting on the system and zero-valued flow source grounded to the earth.

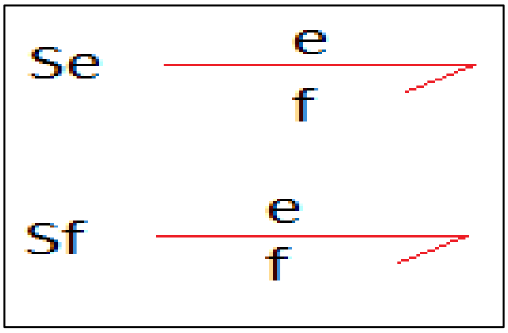

Figure 11- Bond graph symbol of Source elements

4.4.2 Dissipators

Dissipators are generally designed to dissipate energy coming from the system. In the engineering field, there is one type of element used to describe dissipator, and that is resistor R. Resistors are absorb an infinite amount of energy coming from the system. Same as other engineering system bond graph also have R element which is used to absorb the energy. As per the law of energy conservation energy neither be created nor be destroyed but it can change its form. R elements generally convert energy coming from the system to heat energy. The graphical representation of R-element is denoted below for the bond graph

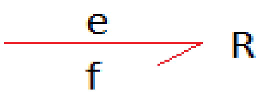

Figure 12 Graphical representation of R element

The following equation defines effort and flows at the corresponding bond connected with R element.

$$e = Rf \tag{22}$$

$$f = \frac{1}{R}e \tag{23}$$

4.4.3 Storage elements

In bond graphs, there are two types of energy storage elements used to describe energy stored in different domains. Symbol C and I denote these elements. These energy storage elements are multiport elements which are allowed to connect more than one element of a bond graph. Symbols C and I denote capacitor and Inertia property of system respectively.

4.4.3.1 C- element

In the mechanical energy domain, C-element is used to store energy in the form of generalised displacement(q), which is proportional to the applied effort(e). for different energy domains, C elements are different. For instance, if C-element is used in the mechanical energy domain, it represents mechanical translation or rotation in the form of spring. In the hydraulic energy domain, C-element is taken as an accumulator. For electric circuits, it is used as a charged storage device. The generalised power variables effort (e) and flow (f) attached to 1 port C element can be defined by the following equation, which is called as the constitutive equation of the C element.

$$q = Ce \tag{24}$$

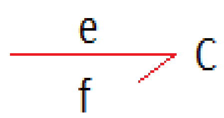

Figure 13 Graphical representation of C element

4.4.3.2 I-element

I-element in the bond graph represents the inertia of the system component. I-element simply stores kinetic energy in the form of generalised momentum (P), which is proportional to the applied flow(f). In the mechanical energy domain, I-element represents translation or rotation of the mass. In hydraulic domain, it represents mass flow rate, and in the electric circuit, it considered as a coil or other magnetic field storage device. The following equation can represent the constitutive equation for I-element.

$$P = If \tag{25}$$

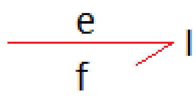

Figure 14 Graphical representation of I element

4.4.4 Power coupler and transducers

Power couplers and transducers are the elements of the bond graph which are not stored any energy neither convert the energy into heat energy. In short, it complies with the power conservation principal. In the bond graph, two elements are considered as power couplers, and transducers and those elements are transformers and gyrators. The transformer and gyrator elements are two-port elements which are connected with two ports of the bond graph. The symbol of the transformer is denoted by (TF) whereas Gyrator is defined by the symbol(GY).

4.4.4.1 Transformer and modulated transformer

Transformers are used to transfer power variables from one node to another node of the element. It complies with the power conservation law. Power conservation in two-port element means shown by the following equation.

$$e1f1 = e2f2 \tag{26}$$

Now for the transformer, the following equations give constraint between two efforts.

$$e1(t) = m \times e2(t) \tag{27}$$

Same as for flow variable, equation of constraint can be written as follows

$$f2(t) = m \times f1(t) \tag{28}$$

In short, Two-port transformer is an element of the bond graph which follows above two constraint equations, where m is the non-negative integer number. Transformers are denoted as TF in the bond graph. Modulus m is annotated with colon connected with transformer element if the (m) is a constant value. If the modulus connected with transformer element is not constant, then it is denoted by just adding M later prefix to TF symbol and transformer called as a modulated transformer. Modulated transformers have three ports connected on it. Where Two ports are transferred power variables, and the third bond is connected to transfer information signal, so the modulated transformer is a three-port element. Graphical representation of the transformer and modulated transformer are given below

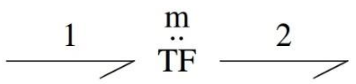

Figure 15 Graphical representation of transformer element

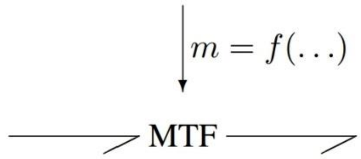

Figure 16 Graphical representation of Modulated transformer

4.4.4.2 Gyrator and modulated gyrators

The gyrator is also a two-port element which is connected to a bond graph elements. Gyrators are used to convert the power variables from an effort to flow or flow to an effort. In short if at one port of gyrator receive effort, it converts that effort to flow on the second port. Basically, gyrators are working on faraday's law. The constitutive equations for gyrators are shown below.

$$e1 = r \times f2 \tag{29}$$

$$e2 = r \times f1 \tag{30}$$

Where r is the gyrator ratio, and it is any constant real integer value. In bond graph gyrators are denoted by symbol GY. If the gyrator ratio is not a constant value then modulus M is connected at the prefix position of gyarator called modulated gyrator. It is denoted by symbol MGY. Gyrator ratio r always has a physical dimension because the consecutive equations are symmetric. It is an element which converts energy from one form to another same as transducers. The best example of gyrators used in the engineering field is DC motor which converts mechanical energy into electrical energy. The graphical representation of gyrator and modulated gyrator is shown below.

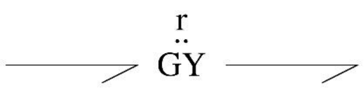

Figure 18 Graphical representation of Gayrator

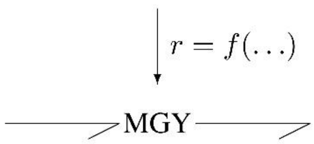

Figure 17 Graphical representation of Modulated gayrator

4.4.5 Power nodes (Junctions)

Power nodes are used to connect the multiport of the system to conserve energy from one port to another port. Power nodes are also known as junctions which are used to transfer power. Basically, there are two types of junctions used in bond graph to conserve energy. 1-junction and 0- junction are two junctions which are used to connect the bond. 0 and 1-junctions are multiport elements which permit multi bond connections to input and output.

4.4.5.1 0- junction

Zero junction is the element where all the flows are added, so it is known as flow sum junction. At zero junction effort comes from the corresponding bond is common same as parallel connectin of electric circuit and hydraulic circuits. The graphical representation of 0-junction in the bond graph is shown below.

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Figure 20 Graphical representation of 0-junction

Figure 19 Graphical representation of 0-junction in mechanical system

Consider a mechanical system used in fig 17. The constitutive equation for 0 junction is written by the following equation

$$f1 + f4 - f2 - f3 = 0 \tag{31}$$

$$e1 = e2 = e3 = e4 = 0$$

Where the power direction plays a vital role in which power directed bonds coming toward 0 junction taken as positive (+) and power directed bonds going away from the 0- junction taken as negative (-).

4.4.5.2 1-junction

1 -junction is the element where all the efforts are summed up, so it is known as effort sum junction. At the 1-junction flows coming from corresponding bonds are common. Graphical representation of 1 junction is shown below.

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Figure 22 Graphical representation of 1-junction

Figure 21 Graphical representation of 1 -junction for mechanical system

Consider the above figure to define the constitutive equation for 1-junction. Same as o -junction power directed bond are very important to derive the equation. At 1-junction flow is common for all the bonds which are connected to the 1-junction. And power directed bonds coming towards 1-junction is taken as positive(+) and bonds going away from the junction taken as negative(-). The constitutive equation for 1-junction is shown below.

$$e1 + e4 - e2 - e3 = 0 \tag{32}$$

$$f1 = f2 = f3 = f4 = 0$$

4.4.6 Concept of causality

Model is used to evaluate the physical behaviour of any system. In the bond graph technique, to evaluate the behaviour of any system, it is important to know at which side variables (efforts and flow) are to be defined. Because when two components are interconnected, they exchange some power. Each component has its own behaviour which is represented by the constitutive equations of the component. Moreover, to define the actual behaviour of the component, the order of power variables have to be defined. Causality is nothing but simple process to define the order of power variables with the help of cause and effect decision. To make a causal bond graph a perpendicular stroke is added at the bond which shows the location at which power variable are to be known. This perpendicular stroke is called a causal stroke. The causal stroke indicates how variables effort(e) and flow(f) are computed in a causal bond. The causal bond gives information about the effort and flows direction. The bond side where causal stroke connected gives information about the effort, and the other side gives information about the flow of the system.

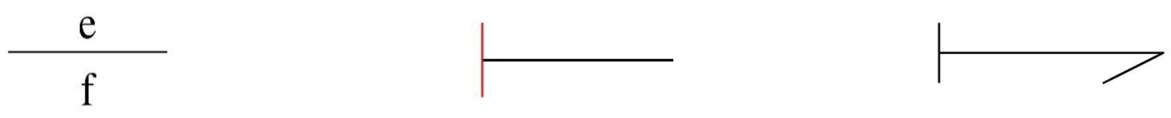

Figure 23- Bond, causal stroke and its power direction

Causal bonds are showing in fig above which shows at which side effort and flow of system are defined. A perpendicular stroke showing the direction of effort or effort is known at the side of causal stroke.

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Figure 24 Description of causal stroke

To understand precisely consider an electric circuit shown below where the bond graph of R element describes with causal stroke. Source of effort gives effort as an electric potential E to R element causal stroke added at the side of R element where effort is applied. Whereas flow in terms of current i is shown at the empty edge of bond in the figure. In this case, effort is known for the R element.

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Figure 25 Example of causality for R element (effort is known)

Consider the same electric circuit, but instead of the source of effort, now the source of flow is applied on R element. So flow is known for the R element in this case, and effort is going at the opposite direction of flow where the causal stroke is added to show the direction of effort.

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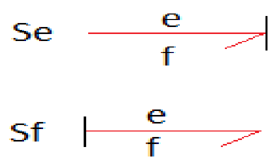
Figure 26 Example of causality for R element (Flow is known)

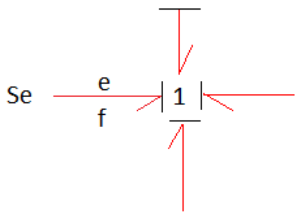
4.4.7 Systematic causality assignment procedure for bond graph

Causality is the very important concept for the bond graph as it helps to understand where power variables can be defined in the model or which power variable known for the system at a specific point or at the junction. In the bond graph for any specific power bond, there is only one side of the bond effort is applied so for a single bond only one causal stroke exist which shows the direction of effort in bond. To understand how to causality assign to any system, systematic procedure is defined below.

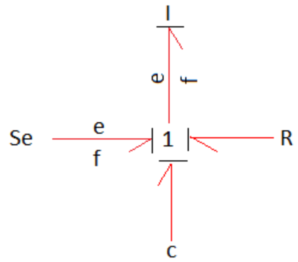
- Causality assignment held through Source elements because source elements are the power source for power variables. For instance, source of effort (Se) always gives effort to the system, whereas the source of flow (Sf) always give flow to the system. According to Source elements connected to the bond, add causal stroke on bond where power variable is known.



- Second step is to assign causality to other elements such as 0-junction, 1-junction, TF, GY connected with the source elements. Note that power source has only one causal stroke which defines causality for other elements. For example, if the power source element (Se) is connected to 1-junction, where other bonds of elements are connected with 1 -junction. Now from definition 1-junction is the effort sum junction where all the efforts coming from other elements are summed up so, at 1-junction, only one bond causes the flow. Same for the 0-junction all the flows are summed up, and only one bond cause the effort in 0-junction. In short 1- junction has (n-1) causal stroke toward the 1-junction if the n number of bonds are connected on it.



- Assign causality to the elements which are connected with 0 and 1-Junctions such as I-elements, C-elements, and R elements. Note that I elements always have causal stroke away from the junction. Whereas for C and R components causality is arbitrary.



- If the two-port elements such as transformers and gyrators are connected with junction structure and causality at one port is known than causality for the second port can be defined easily.

The above procedure is used to assign causality to the system. There are two types of causalities used for elements of the bond graph, integral causality and, derivative causality. However, Integral causality is more preferable for the analytical solution of mechanical systems. Causality assignment of different elements of the bond graph elements are represented below with its causal equation of each bond.

Table 2- Bond Graph, Causal Equations and Block Diagram

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4.4.8 The procedure of equation derivation

The bond graph is the graphical representation of any system components which are interconnected with each other and transfer energy in between. So, bond graph technique is work on energy exchange principle. To derive a set of equations from the bond graph, power direction and causal paths are very important as they define the sign of power variables. Power direction shows the direction of power moves from the one power port to another power port of the bond graph element, and the causal path shows the direction of the power variable in a bond graph.

Consider a simple bond graph model of RLC circuit used in electric equipment to understand the concept of the causal path. In the below circuit, a source of effort is the power source by which system gets power, so power direction always starts from the source elements. The red dotted line in the figure shows the causal path of the bond graph, which shows the path of power variables. In the below model effort is the power variable which is going to the I-element, and I- element always gives flow to the system when the effort is applied to the I-element.

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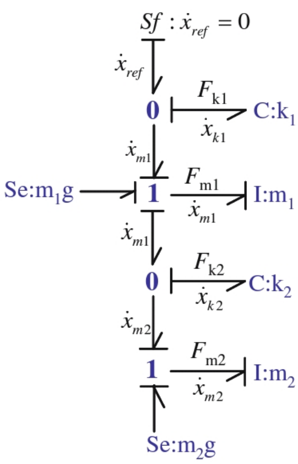
Figure 27 Causal path of simple RLC circuit

4.4.9 Bond graph model for Mechanical systems

Bond graphs model of electrical and hydraulic systems are easy to construct by the method of gradual uncover and method of point potential (Mukherjee and Karmakar, 2000) but, when the mechanical system is considered a method of flow map and method of effort map are used. Generally, the method of the flow map is preferable for mechanical systems because, in mechanical systems, bond graphs are construted from the kinematic relations of the components.

In the method of the flow of map for mechanical systems, 1-junctions are considered as a distinct velocity point which represents the velocity of the component and, 0-junctions are used to compute relative velocities between two system components. 0-junction always come in between two 1 -junctions.

Consider double spring and mass damper assembly to construct a bond graph from the kinetic relations of the component. The corresponding bond graph model and the simplified version of the bond graph model shown in the figure below.



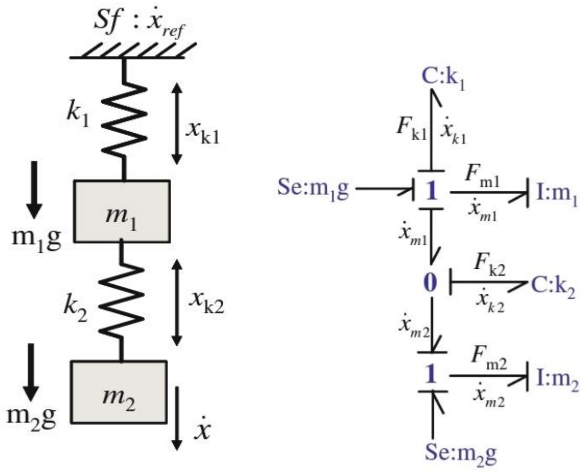


Figure 28 Mechanical double oscillator and corresponding bondgraph

To construct the bond graph of double oscillator 1- junction is assigned as a distinct velocity of mass-1 and mass-2 whereas relative velocities of mass 1 and 2 are assigned at 0-junctions. Connect I-element and C-elements, which represent inertia of masses and spring with corresponding 1-junction and 0-junction. Source of efforts are applied at the 1-junction, which represents the gravitational force of mass weight.

4.4.10 Procedure to derive state-space equation from Bond Graph

State variables are the elements by which the behaviour of the system can be analysed. In the bond graph technique, C-elements and I-elements are considered as state parameters, and P and q are the state variables where P represent momentum of inertia and q represent the position of the spring which shows the state of the system. State equations can be derived by following the general procedure. The general procedure is stated below.

4.4.10.1 General procedure

- Define the system components.

- Construct bond graph model of system components according to constraints and structural parameters
- Define source elements and power direction of energy flow
- Connect the bond graph model of different components with 0, 1, TF, GY elements
- Assign the numbers to the bonds connected with elements
- Write constitutive equations for each element of the bond graph
- Finally, combine all the laws and equations in sequential order to obtain state equations

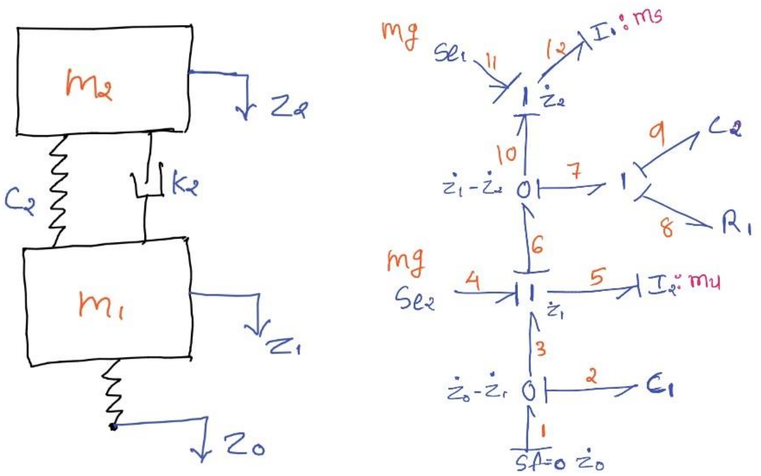


Figure 29 Bond graph model of two degrees of freedom suspension system

In this bond graph model elements such as I and C are define the state of the system. State variable, inputs and output of the system for two degrees of the freedom suspension system are defined as

$$y = [p5\ p12\ q2\ q9]^T\ , \qquad u = [Se1, Se2]$$

Where u is the input of systems and, $p5, p12, q2, q9$ are the generalised energy variables from which state variables can be defined because ($p(momentum) = m\dot{x}$)

$$x = \frac{1}{m}\int p \ dt$$

$$q = c\int f \, dt$$

To derive state equations first of all consecutive equations of all the elements are derived by junction equations.

The consecutive equation for 1-junction of $\dot{z_1}$ is describe below,

$$\dot{p_5} = e_5 = e_4 + e_3 - e_6 \tag{33}$$

Where the numbers such as 5,4,3,6 in front of effort (e) represent the bond number which is connected to the power port.

$$\dot{p_5} = e_5 = m_u g + kq_2 - e_7$$

$$\dot{p}_5 = m_u g + k_1 q_2 - (k_2 q_9 + Rf_7)$$

$$\dot{p}_5 = m_u g + k_1 q_2 - (k_2 q_9 + R(f_5 - f_{12}))$$

$$\dot{p}_5 = m_u g + k_1 q_2 - (k_2 q_9 + R((\frac{p_5}{m_u}) - (\frac{p_{12}}{m_s})) \tag{34}$$

$\dot{p}_5$ represents the momentum rate of unsprung mass by which we can find the state of the system easily

Same as above state equation for sprung mass attached to the 1- junction represent as $\dot{z}_2$ can be find

$$\dot{p}_{12} = e_{10} + e_{11} \tag{35}$$

$$\dot{p}_{12} = (e_7) + m_s g$$

$$\dot{p}_{12} = (e_9 + e_8) + m_s g$$

$$\dot{p}_{12} = (k_2(q_9) + f_8(R)) + m_s g$$

$$\dot{p}_{12} = (k_2(q_9) + (f_6 - f_{10})(R)) + m_s g$$

$$\dot{p}_{12} = (k_2(q_9) + ((\frac{p_5}{m_u} - \frac{p_{12}}{m_s})(R)) + m_s g \tag{36}$$

Consecutive equations for 0-junctions are described below

For the spring c_1

$$\dot{q}_2 = f_2 = f_1 - f_3 \tag{37}$$

$$\dot{q}_2 = s_f - (\frac{p_5}{m_u}) \tag{38}$$

Same as for spring c_2

$$\dot{q}_9 = f_6 - f_{10} \tag{39}$$

$$\dot{q}_9 = \left(\frac{p_5}{m_u}\right) - \left(\frac{p_{12}}{m_s}\right) \tag{40}$$

The above equations are the state equations which will be used for the analysis of the passive suspension system.

5 Bond graph model of six degrees of freedom car body

5.1 Introduction

After the reviewing of the bond graph technique, bond graph model of car body presented, which is capable of showing six degrees of freedom motion of the car body. To develop six degrees of freedom car body model, the car body is taken as a symmetrical rigid body.

The bond graph model of car body developed with the help of Newton-Eulers equation of motion which is already discussed in the above chapter.

Newton Euler equations of the rigid body represented by

$$F_X = m\ddot{x} + m(\dot{z}\omega_y - \dot{y}\omega_z)$$

$$F_Y = m\ddot{y} + m(\dot{x}\omega_z - \dot{z}\omega_x)$$

$$F_Z = m\ddot{z} + m(\dot{y}\omega_x - \dot{x}\omega_y)$$

$$M_x = I_{xx}\dot{\omega}_x - (I_{yy-}I_{zz})\omega_y\omega_z$$

$$M_y = I_{yy}\dot{\omega}_y - (I_{zz-}I_{xx})\omega_z\omega_x$$

$$M_z = I_{zz}\dot{\omega}_z - (I_{xx-}I_{yy})\omega_x\omega_y$$

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Figure 30 Bond graph model of Newton's equations of motion in noninertial frame

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Figure 31 Bond graph model of Euler junction structure

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Figure 32 Bond graph model of Newton Euler equations

Bond graph model of the Newton-Euler equation is derived by a combination of two bond graph model of the newtons equation of motion and Euler junction structure. It is also denoted by the star model of the rigid body. Six port of this star model gives three transitional velocities($\dot{x}, \dot{y}, \dot{z}$) velocities and three angular velocities($\omega_x \omega_z \omega_y$). Angular velocities are converted into Euler's angle(θ,ϕ,ψ) by using coordinate transform to get angular positions of the car body with respect to the inertial frame of the body.

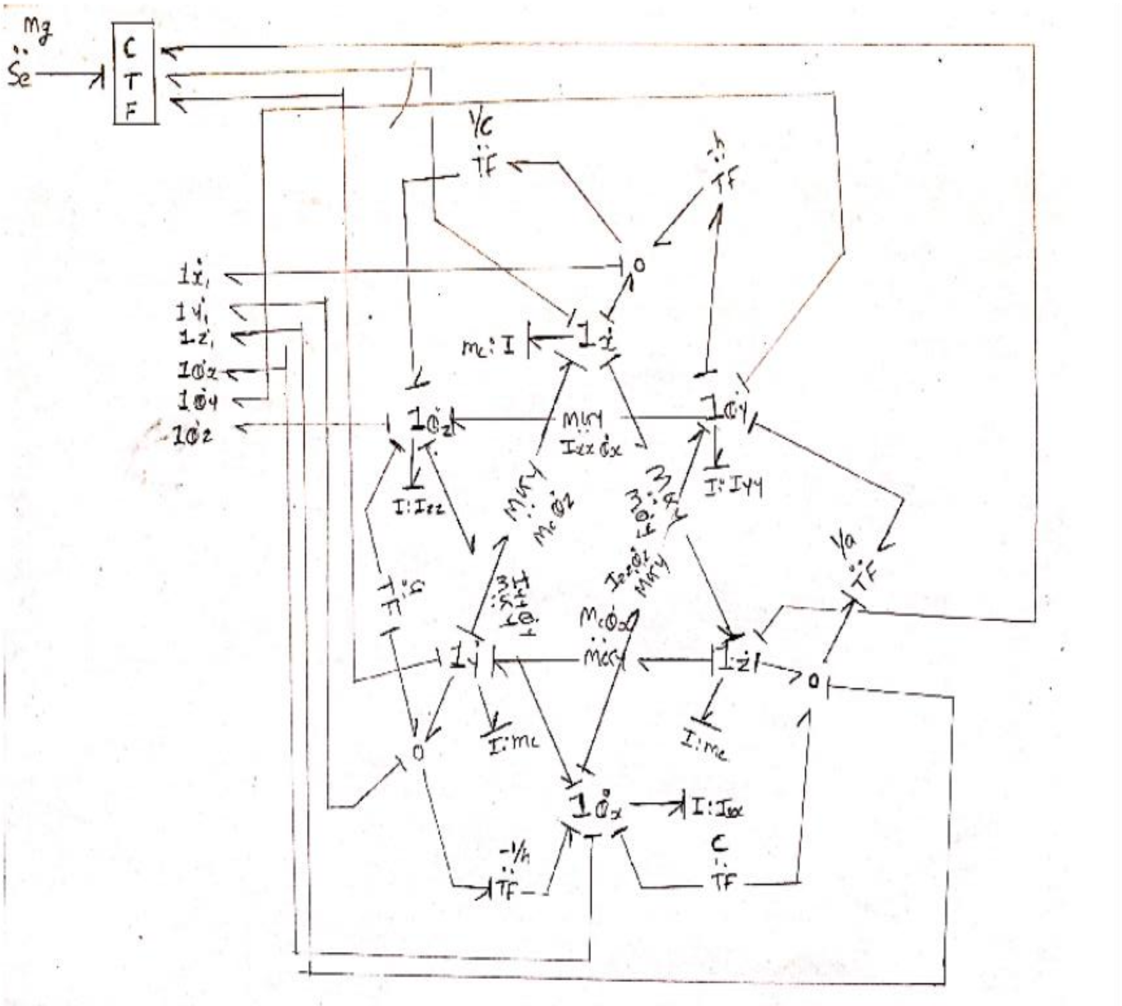

Figure 33 Bond graph model of the car body for the left front suspension.

The above bond graph model shows the six degrees of freedom motion of rigid car body, but only gravitational force and force coming from the front left suspension is considered. To get parameters of car body model is made on inventor program package to define the second moment of inertias, about the axis (Ixx,Iyy,Izz) and important parameters such as wheelbase, the distance between front axel to CG point, the distance between the rear axel to CG point, the width of the car body, and wheel track.

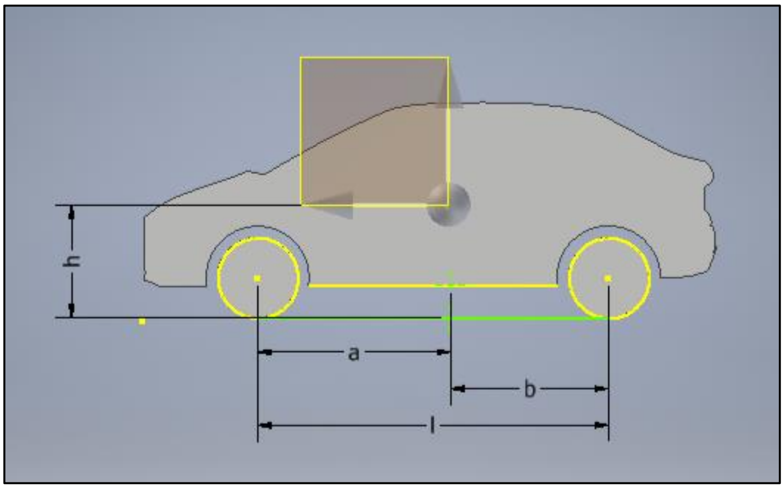

Figure 34 Side view of the car body

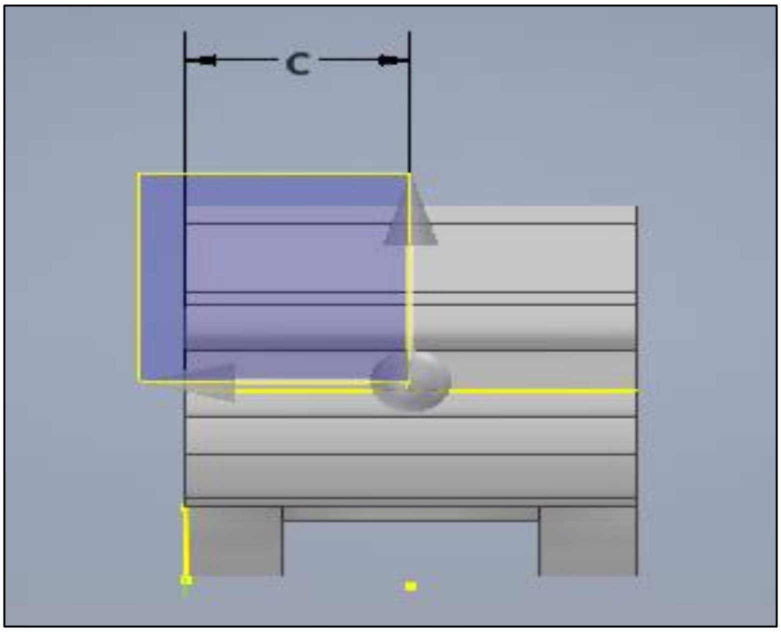

Figure 35 Rear view of the car body

All the values of parameters such as mass, values of the moment of inertias, wheelbase, are presented in appendices for the reference purpose.

5.2 Bond graph model of full car body

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Figure 36 Bond graph model of full car body (Merzouki et al., 2013)

In the car body model, suspension reference points are taken vertically on the car body at the same distance of axel from the CG point. Transformers and Modulated gyrators are used to develop the star model of the rigid body.

In the above car body model, only gravitational forces and forces at the reference point of car body where the suspensions attached are considered in the derivation of equations other forces such as aerodynamic forces and forces coming from the differential are not considered. In the above bond graph model of the car body, angular velocities are modulated as a symbol $\dot{\theta}$ in modulated gyrator element whereas in the state equations instead of $\dot{\theta}$, ω symbols are used, which represents angular velocity of the body about the respective axis of the car body.

All the forces acting on the inertial frame of the body are transferred into the noninertial frame by CTF(coordinate transfer) to get three-dimensional forces acting on the body. Full bond graph model with bond numbers are presented in appendices for the reference purpose.

State equations of six degrees of freedom car body

In this section, state equations are derived for the car body by following the standard procedure to derive state equations by bond graph technique. Six state equations are derived for car body which shows six degrees of motion of the car body with respect to CG Point.

$$\dot{p}_2 = e_2 = e_5 - e_3 - e_{57} - e_{63} - e_{49} - e_4 - e_1$$

$$\dot{p}_2 = m\omega_z(f_{24}) - e_{45} - e_{59} - e_{62} - e_{46} - m\omega_y(f_{10}) - (-mg)$$

$$\dot{p}_2 = m\omega_z\left(\frac{p_{23}}{m}\right) - e_{45} - e_{59} - e_{62} - e_{46} - m\omega_y\left(\frac{p_{11}}{m}\right) - (-mg) \tag{41}$$

$$\dot{p}_2 = m\omega_z(\dot{y}) - e_{45} - e_{59} - e_{62} - e_{46} - m\omega_y(\dot{z}) - (-mg)$$

$$\dot{p}_{23} = e_{23} = e_{20} - e_{107} - e_{72} - e_{80} - e_{21} - e_{22} - e_{24}$$
$$\dot{p}_{23} = m\omega_x(f_{14}) - e_{75} - e_{69} - e_{77} - e_{26} - (-mg) - m\omega_z(f_5)$$
$$\dot{p}_{23} = m\omega_x\left(\frac{p_{11}}{m}\right) - e_{75} - e_{69} - e_{77} - e_{26} - (-mg) - m\omega_z\left(\frac{p_2}{m}\right) \tag{42}$$
$$\dot{p}_{23} = m\omega_x(\dot{z}) - e_{75} - e_{69} - e_{77} - e_{26} - (-mg) - m\omega_z(\dot{x})$$

$$\dot{p}_{11} = e_{11} = e_{10} - e_{84} - e_{111} - e_{12} - e_{109} - e_{13} - e_{14}$$
$$\dot{p}_{11} = m\omega_y(f_4) - e_{83} - e_{97} - e_{16} - e_{101} - (-mg) - m\omega_x(f_{20})$$
$$\dot{p}_{11} = m\omega_y(\dot{x}) - e_{83} - e_{97} - e_{16} - e_{101} - (-mg) - m\omega_x(\dot{y}) \tag{43}$$

$$\dot{p}_{42} = e_{42} = e_{43} + e_{44} - e_{41} - e_{40}$$

$$\dot{p}_{42} = (e_{77} + e_{82} + e_{81} + e_{28}) + I_{yy}\omega_y(f_{34}) - (e_{87} + e_{88} + e_{89} + e_{90}) - I_{zz}\omega_z(f_{39})$$

$$\dot{p}_{42} = \left(\left(-\frac{1}{h}\right)e_{75} + \left(-\frac{1}{h}\right)e_{69} + \left(-\frac{1}{h}\right)e_{77} + \left(-\frac{1}{h}\right)e_{26}\right) + I_{yy}\omega_y(\omega_z) - \left((c)e_{83} + (-c)e_{101} + (c)e_{16} + (-c)e_{97}\right) - I_{zz}\omega_z(\omega_y) \tag{44}$$

$$\dot{p}_{37} = e_{37} = e_{39} + e_{38} - e_{36} - e_{35}$$

$$\begin{aligned}\dot{p}_{37} = {} & \left(\left(\frac{1}{a}\right)e_{97} + \left(\frac{1}{a}\right)e_{16} + \left(-\frac{1}{b}\right)e_{101} + \left(-\frac{1}{b}\right)e_{83}\right) + I_{zz}\omega_z(\omega_x) \\ & - \left((-h)e_{46} + (-h)e_{62} + (-h)e_{59} + (-h)e_{45}\right) - I_{xx}\omega_x(\omega_z)\end{aligned} \tag{45}$$

$$\dot{p}_{32} = e_{32} = e_{33} + e_{31} - e_{34} - e_{30}$$

$$\begin{aligned}\dot{p}_{32} = {} & \left(\left(-\frac{1}{c}\right)e_{46} + \left(-\frac{1}{c}\right)e_{62} + \left(\frac{1}{c}\right)e_{59} + \left(\frac{1}{c}\right)e_{45}\right) + I_{xx}\omega_x(\omega_y) \\ & - \left((a)e_{26} + (a)e_{77} + (-b)e_{69} + (-b)e_{75}\right) - I_{yy}\omega_y(\omega_x)\end{aligned} \tag{46}$$

The above six derived equations are able to show six degrees movement of the car body with respect to the centre of gravity. Bond graph model of the suspension system is not connected to the bond graph model of the car body but in the simulation software 1 DOF model of spring and damper are attached to measure the behaviour of the car body

Coordinate transform used in bond graph model of car body

The coordinate transfer blocks are used to transfer forces and velocities from the noninertial frame to inertial frame, and the corresponding bond graph model of CTF functions are shown below.

Coordinate transform from moving frame to fixed frame

$$\begin{Bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{Bmatrix} = \mathbf{T}_{\psi,\theta,\phi} \begin{Bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{Bmatrix} \text{ and } \begin{Bmatrix} \omega_X \\ \omega_Y \\ \omega_Z \end{Bmatrix} = \mathbf{T}_{\psi,\theta,\phi} \begin{Bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{Bmatrix}$$

Where,

$$\mathbf{T}_{\psi,\theta,\phi} = \underbrace{\begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{T}_\psi} \underbrace{\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}}_{\mathbf{T}_\theta} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}}_{\mathbf{T}_\phi},$$

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Figure 37 Bond graph model of coordinate transform from rotating to the fixed frame

Coordinate transform from fixed to rotating frame

The coordinate transfer from fixed to the rotating frame can be achieved by the following matrices.

$$\begin{aligned} \mathbf{T}^{-1}_{\psi,\theta,\phi} &= \mathbf{T}^{-1}_{\phi}\mathbf{T}^{-1}_{\theta}\mathbf{T}^{-1}_{\psi} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}. \end{aligned}$$

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Figure 38 Bond graph model of coordinate transform from fixed to the rotating frame.

Transformation of angular velocities into Eulers angle

The angular velocities of the body about fixed body axes can be represented by $\omega_x\omega_y\omega_z$

Angular velocities of car body transferred to the Eulers angle by following matrices

$$\begin{Bmatrix} \omega x \\ \omega y \\ \omega z \end{Bmatrix} = \begin{matrix} 1 & 0 & -sin\theta \\ 0 & cos\varphi & cos\theta\ sin\varphi \\ 0 & -sin\varphi & cos\theta\ cos\varphi \end{matrix} \begin{Bmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\Psi} \end{Bmatrix}$$

6 Results and discussion

6.1 Introduction

In this chapter Simulink model of the car body proposed and results obtained by simulation are discussed. Six state equations derived by the bond graph technique for the car body, are modelled in MATLAB Simulink software to obtain results of six state equation. Simulation results show the three translational displacements of the CG point of the car body and three angular displacements of the car body. All the results are obtained for the inertial and non-inertial frame of the car body. The translational positions of the car body in the inertial and non-inertial frame of the body are obtained by integration of velocities in the inertial and non-inertial frame of the car body. The angular velocities of the car body are obtained by simulation in the inertial and non-inertial frame of the car body, which shows the angular displacement of the car body. The angular positions of the car body are obtained by Eulers angle in the inertial frame of the car body. The Simulink model codes are presented in the appendices for the reference purpose.

6.2 Simulink model of the car body

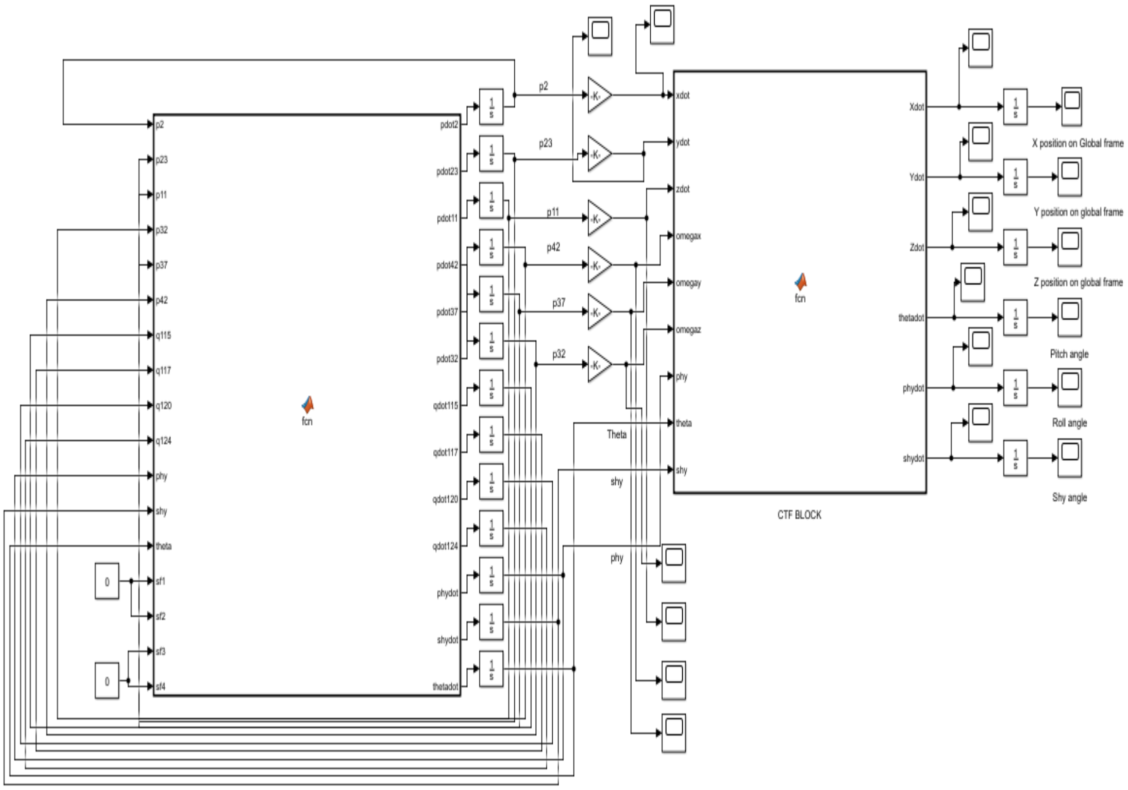

Figure 39- Simulink model of car body

6.2.1 Results for the non-inertial frame of car body

Simulation has been done to obtain the results in non-inertial (body-fixed frame) and inertial frame of the car body. Results are obtained for the two scenarios. Where in the first scenario only gravitational force acting on car body has been considered.

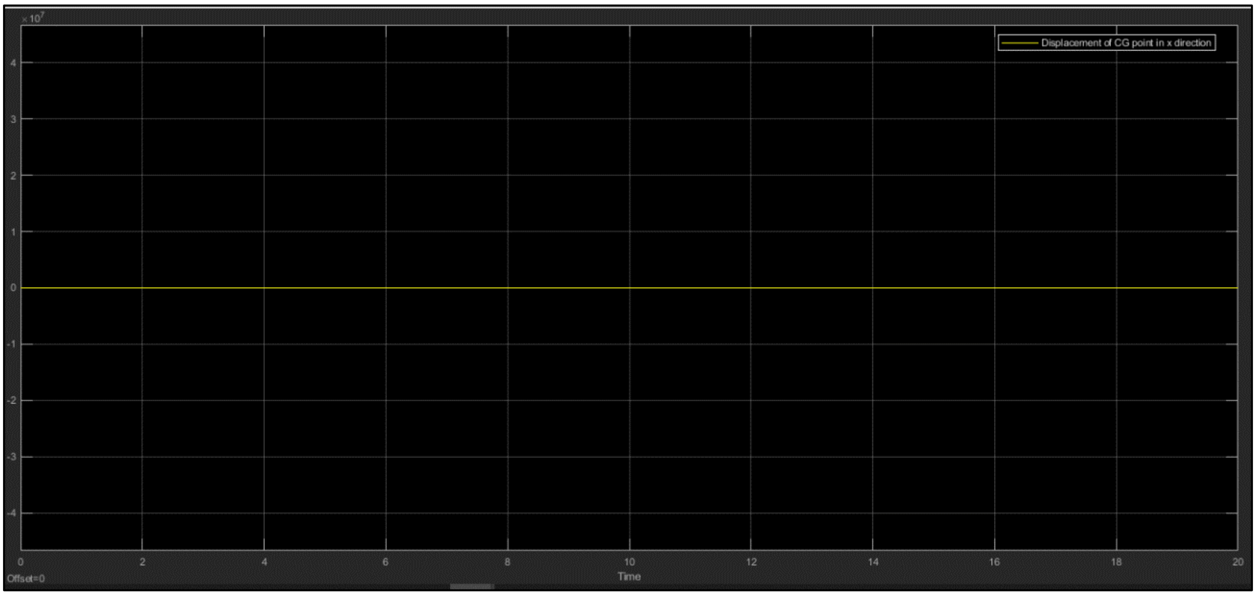

Figure 40 Displacement of CG point in x-direction for the non-inertial body frame

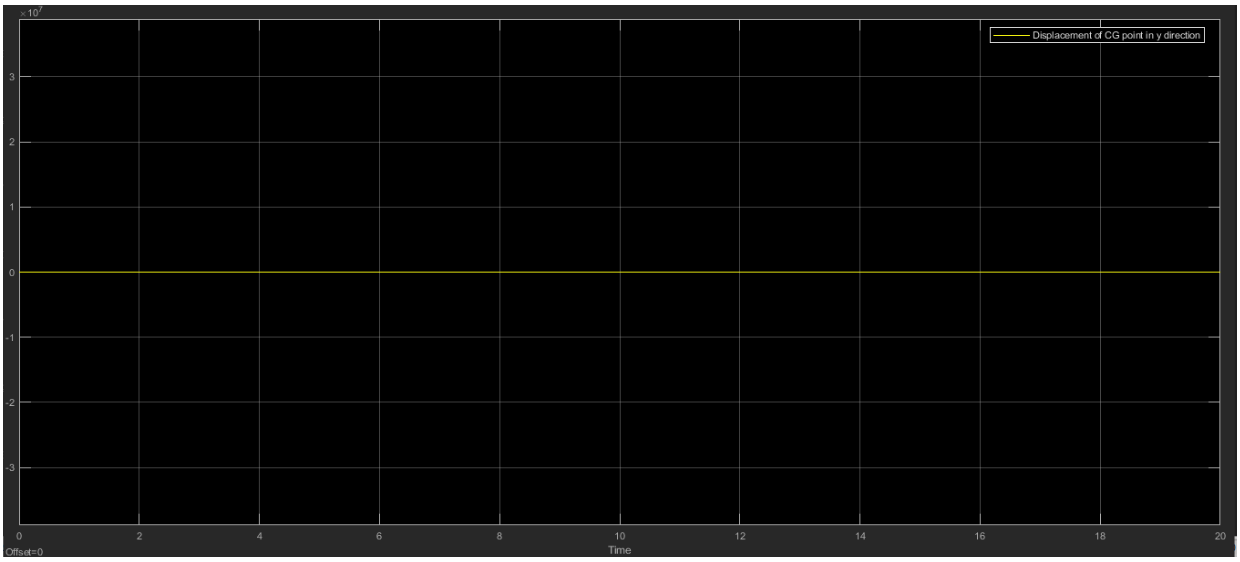

Figure 41 Displacement of CG point in y-direction for the non-inertial body frame

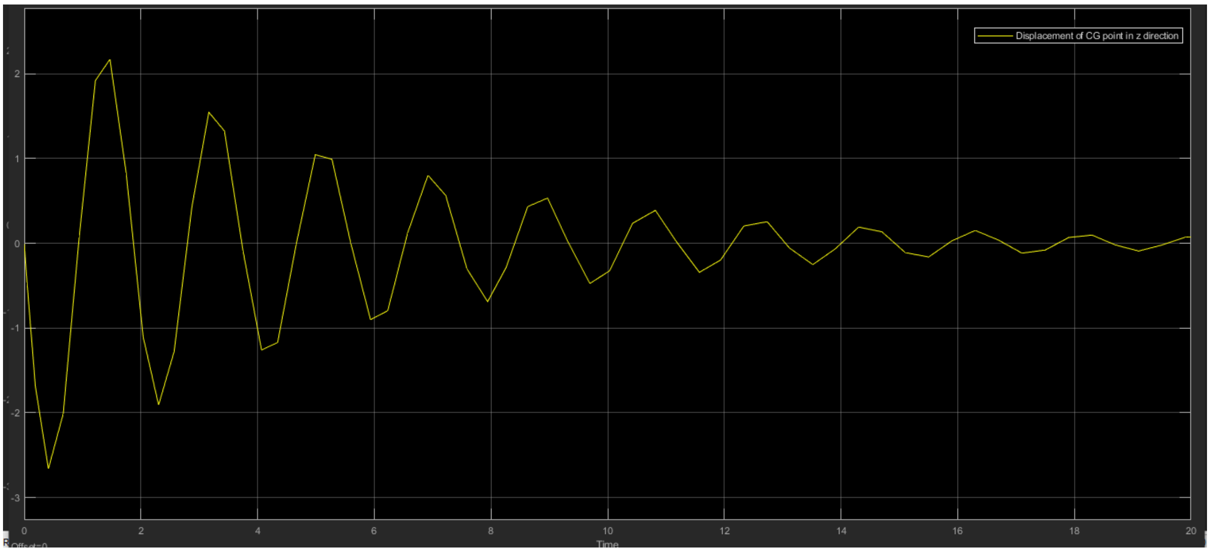

Figure 42 Displacement of CG point in z-direction for the non-inertial body frame

Figure 40, 41, and 42 Represent displacement of the CG point in body-fixed axis (x, y, z, direction) over time. The results shown on the above figure shows particular behaviour because of in this research, the car body is considered as a rigid body which is symmetric from the principal axis of the car body. In figure 40 and figure 41 graph shows zero movement of CG point in x and y direction, but it shows vibration in z-direction because, in the scenario-1, the only gravitational force acting on the body is considered which is acting on z-direction whereas in x and y direction body does not have any forces due to symmetric nature. In the figure 42, body fluctuated in between 2.7 m/s (lower limit) to 2.1 m/s (upper limit), and at the time after 20 sec, it becomes zero.

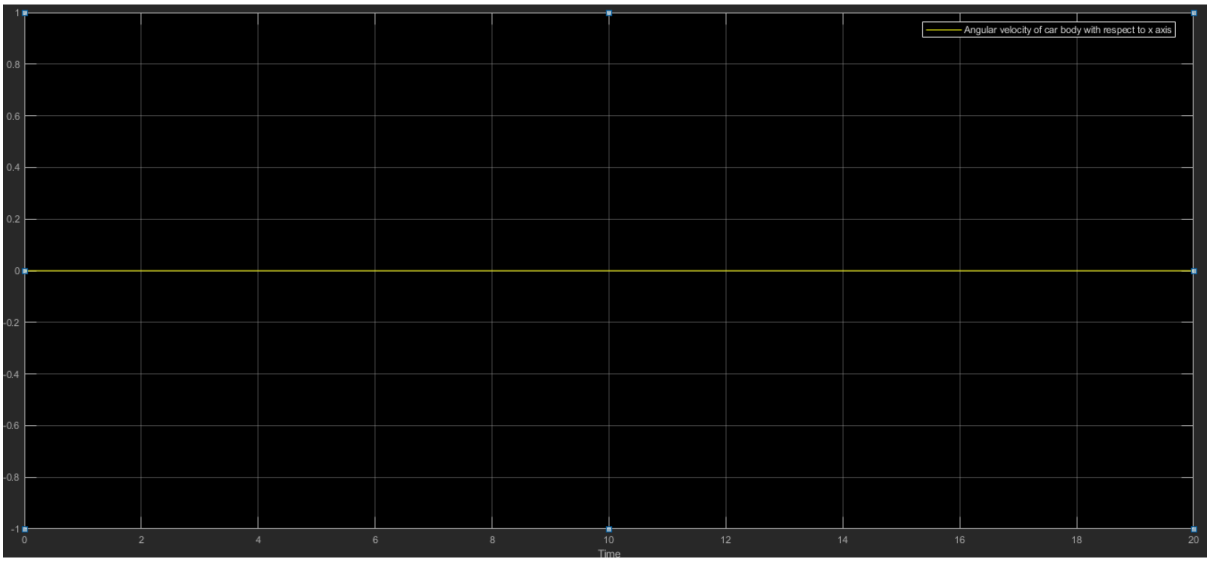

Figure 43 Angular velocity about x-axis of the car body for the non-inertial body frame

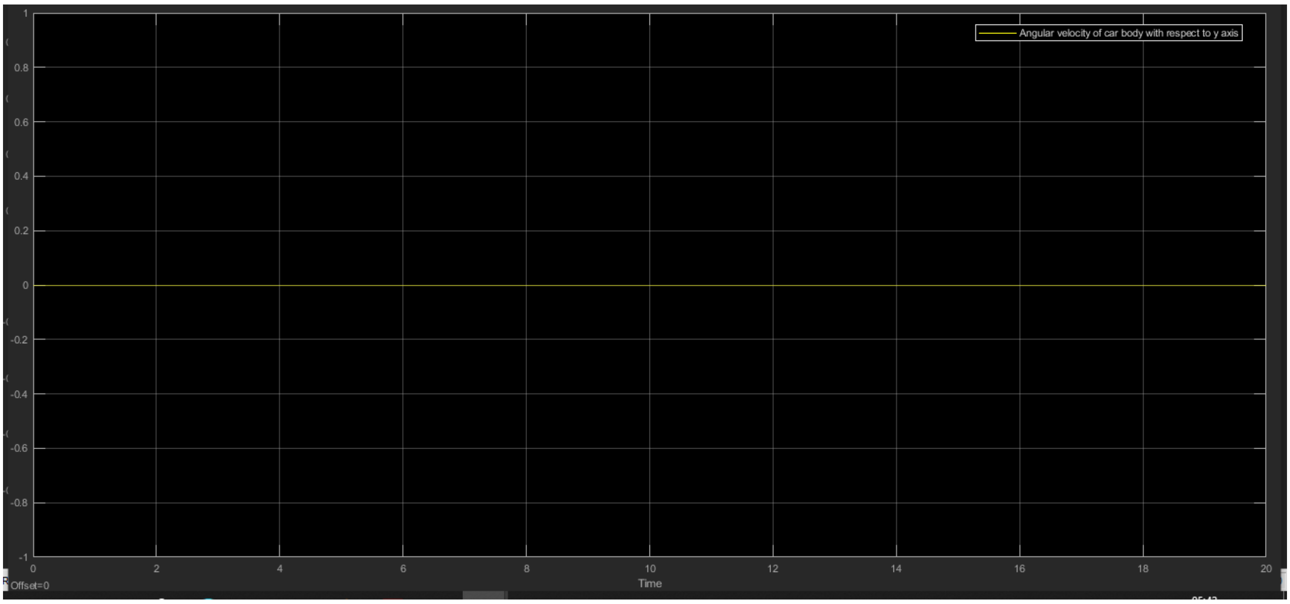

Figure 44 Angular velocity about y- axis of the car body for the non-inertial body frame

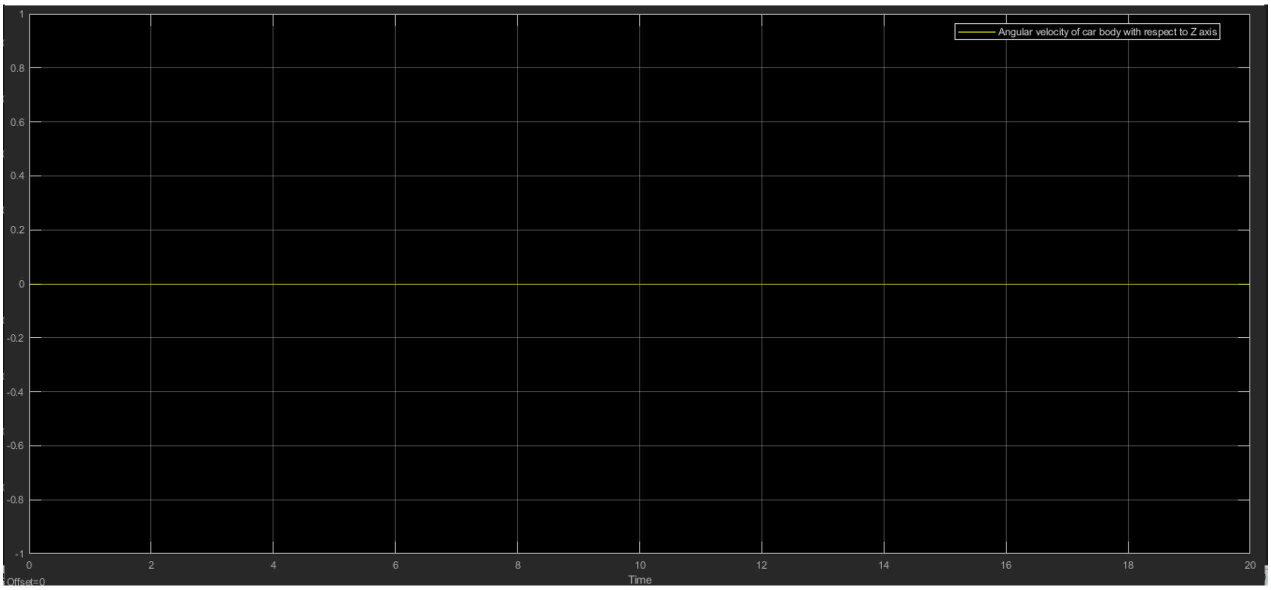

Figure 45 Angular velocity about z-axis of the car body for the non-inertial body frame

Figure 43, 44 and 45 represent graphs for angular velocities over time with respect to the principal axis of the car body. Due to the car body is symmetric from principal axis figure 43,44, and 45 does not have any vibration it becomes zero for all the time for the scenario-1

6.2.2 Results for the inertial frame of car body

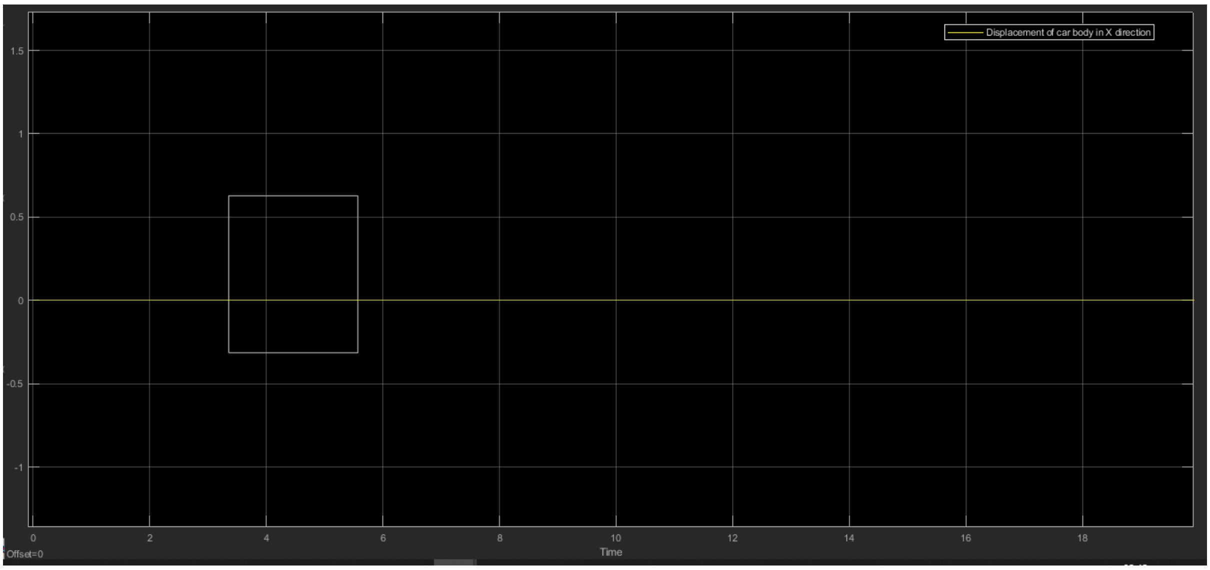

Figure 46 Displacement of CG point in x-direction for the inertial body frame

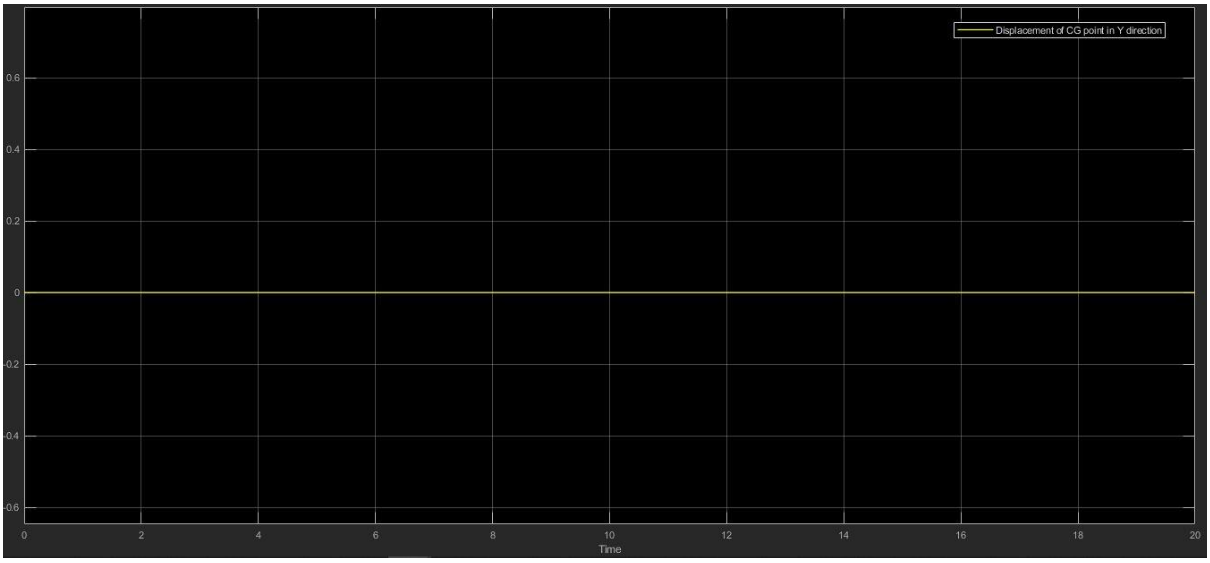

Figure 47 Displacement of CG point in y-direction for the inertial body frame

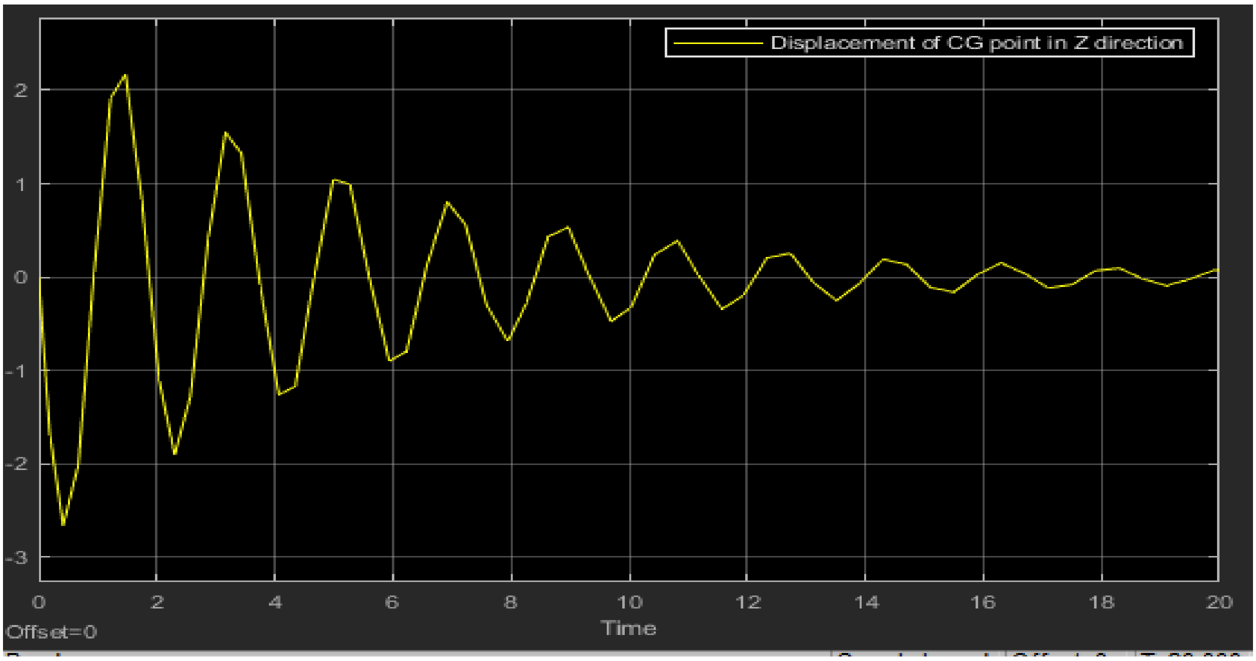

Figure 48 Displacement of CG point in z-direction for the inertial body frame

Same as for non-inertial frame of body, figure 46,47 and 48 represent the displacement of the CG point over time in the inertial frame of reference. The inertial frame of reference is nothing but a reference frame or global frame from which movement of the car body measured. In the inertial frame, results are showing the same behaviour for the scenario-1 as results shown in the non-inertial frame.

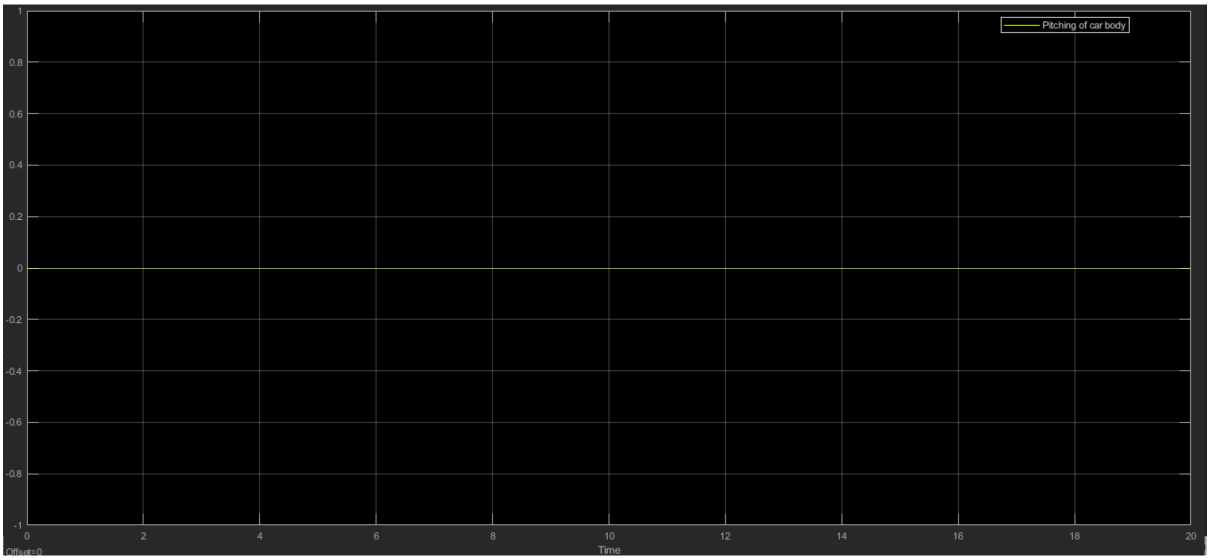

Figure 49 Pitch movement of the car body for the inertial body frame

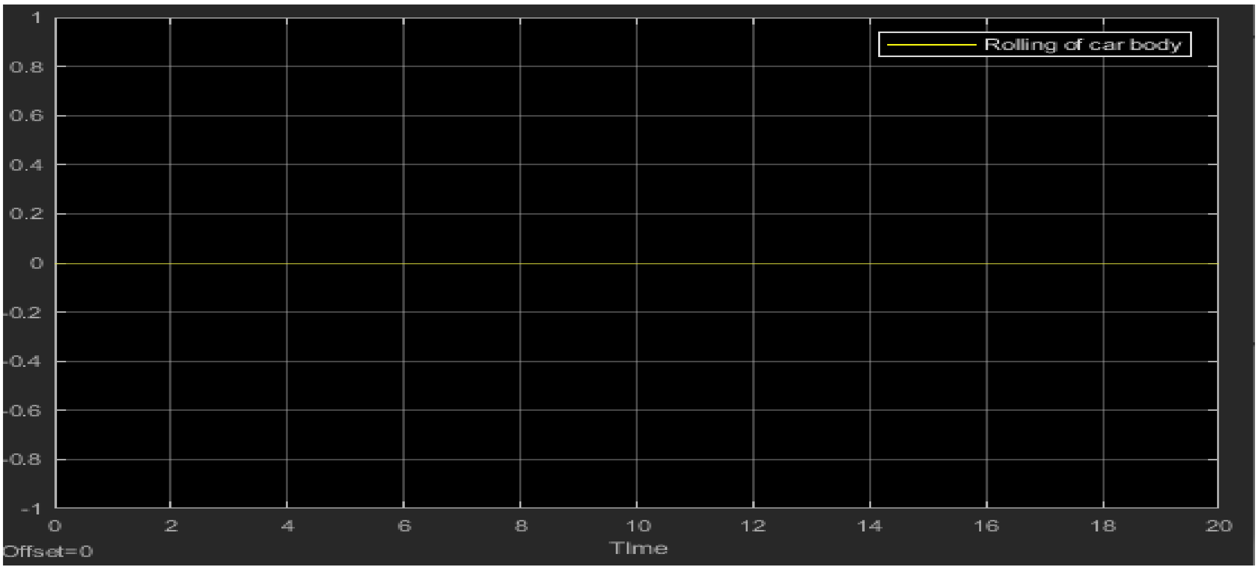

Figure 50 Rolling movement of the car body for the inertial body frame

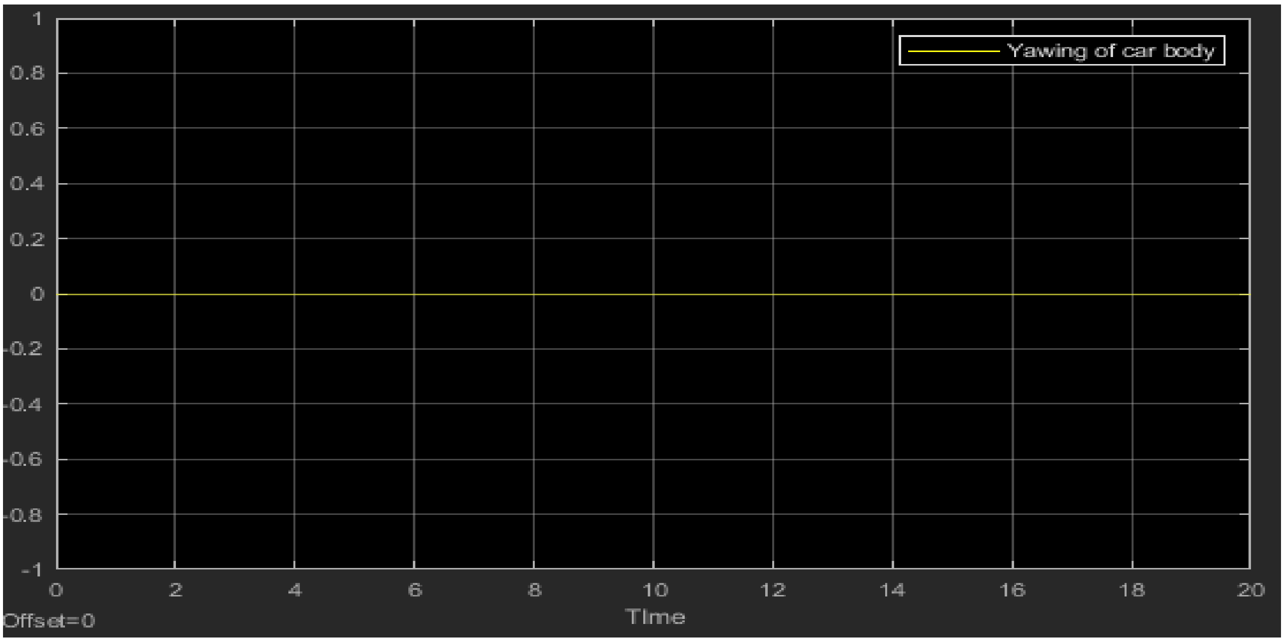

Figure 51 Yawing movement of the car body for the inertial body frame

Figure 49, 50, and 51 showing the pitching, rolling and yawing movement of car body over time in the inertial frame. Because of the symmetric geometry of the car body, the car body does not experience any kind of angular movement.

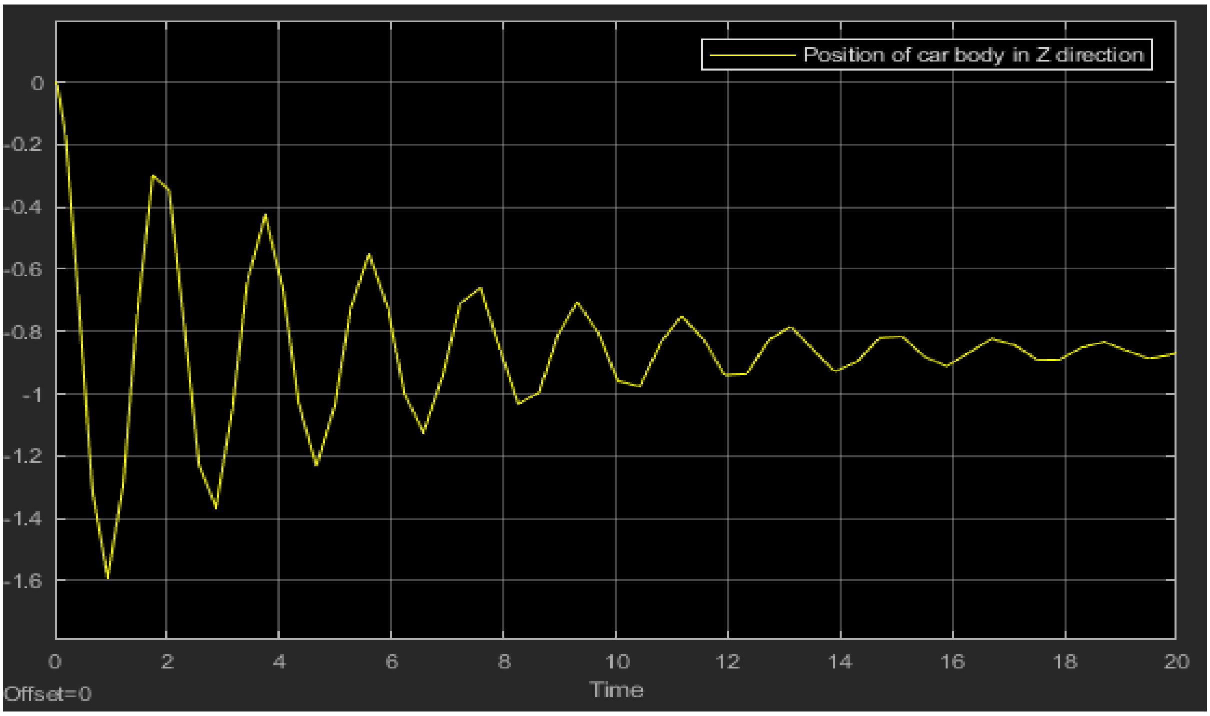

Figure 52 Position of the car body in the Z direction for the inertial body frame

Positions of the car body in the X and Y direction(inertial frame) have the same behaviour as velocities behaving in the above results, so the results of the positions in X and Y directions are not included. In the figure-52 position of the car body in the Z-direction (inertial frame) is shown which represent due to gravitational force acting on the car body position of the car body fluctuate in between 0 to -1.6 mm and steady at the value near -0.8 mm. To define the real positions of the car body or to evaluate the real behaviour of the car body, always the inertial frame of the car body considered.

Scenario 2

In scenario 2, the external force applied on the front left suspension components with the help of the sine wave where the real situation of road conditions are considered. Due to applied external force to the left front suspension components from road excitation created by sinewave the car body experiences several motions with respect to its CG point. For three-dimensional car body translational and rotational motion of the car body about its principal axis shown in below figures which shows six degrees of freedom motion of the car body under external forces generated by suspension components through road excitation. Results are obtained for the inertial and non-inertial frame of the car body.

6.2.3 Results for the non-inertial frame of car body

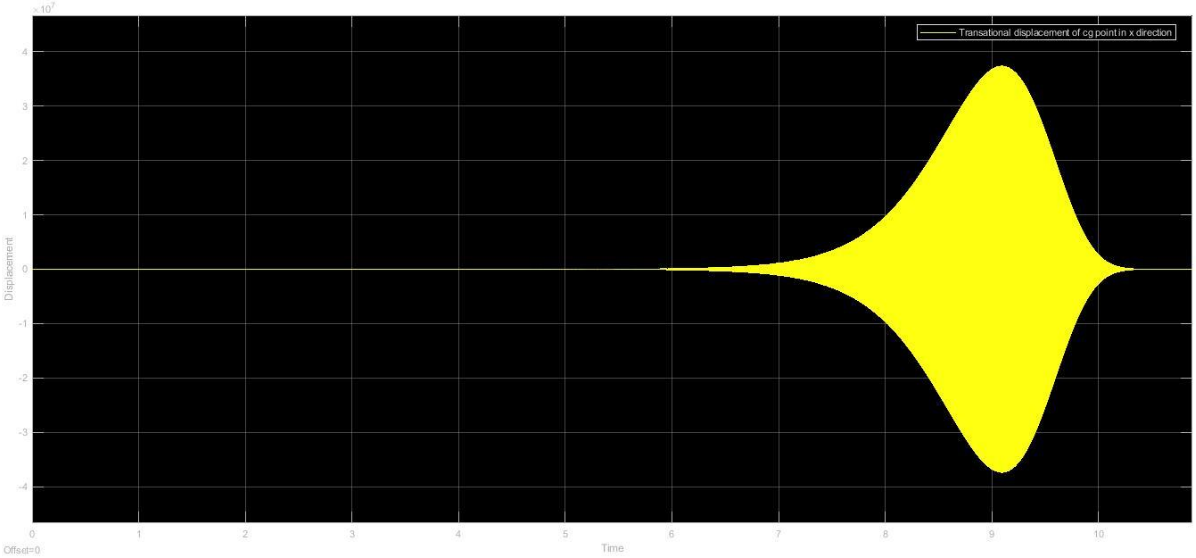

Figure 53 Displacement of CG point in the x-direction for non-inertial frame

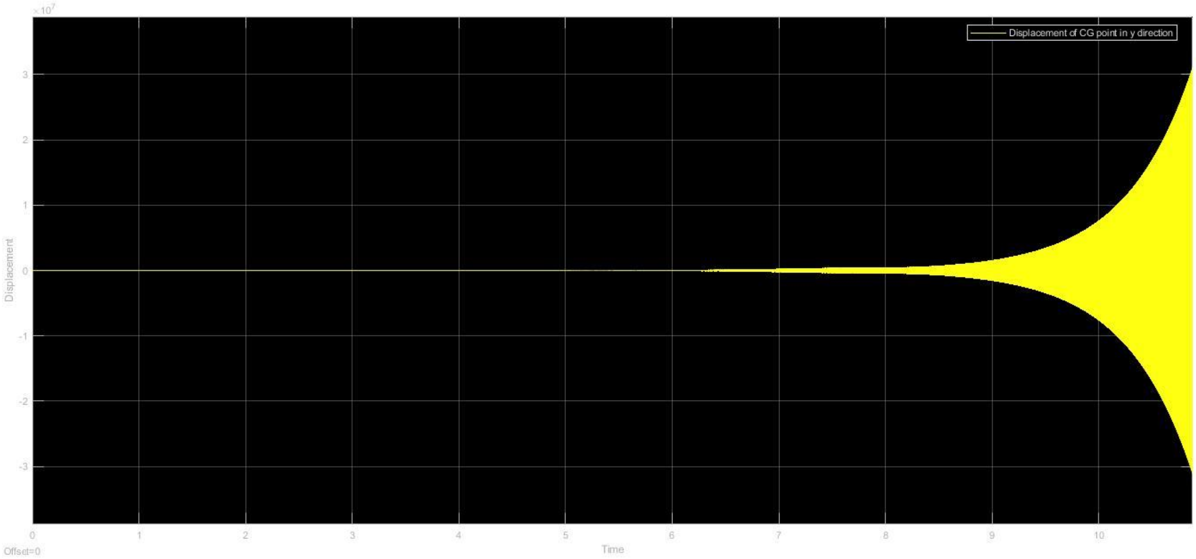

Figure 54 Displacement of CG point in the y-direction for non-inertial frame

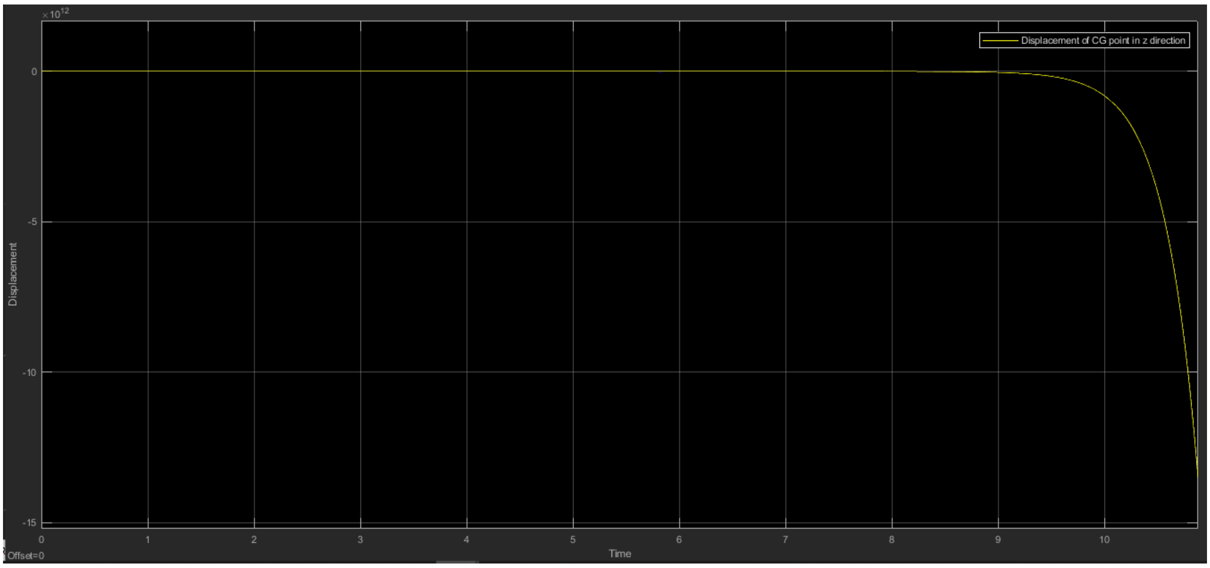

Figure 55 Displacement in the z-direction for non-inertial frame

Figure 53, 54 and figure 55 shows the translational displacements of the CG point in x,y and z-direction in the non-inertial frame when the external force applied on the suspension components at the front left wheel from road excitation. Because of the road excitation, forces are generated in all the three directions of the body, due to forces generated in x,y and z-direction CG point displaced in all three direction which is shown in the fig 53,54 and 55.

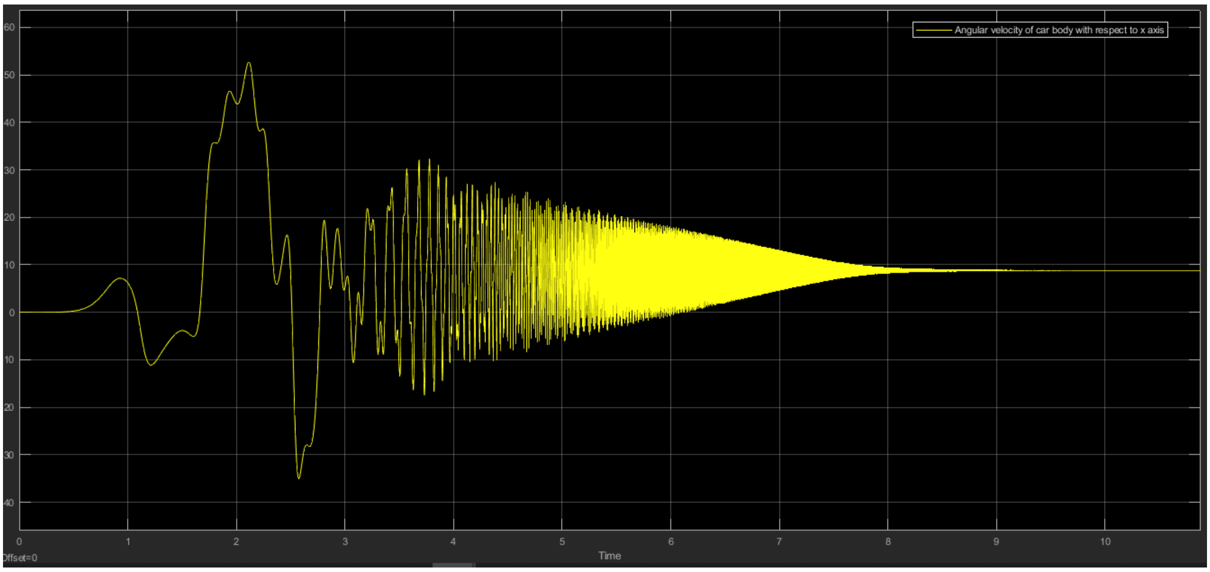

Figure 56 Angular velocity about x-axis of the car body

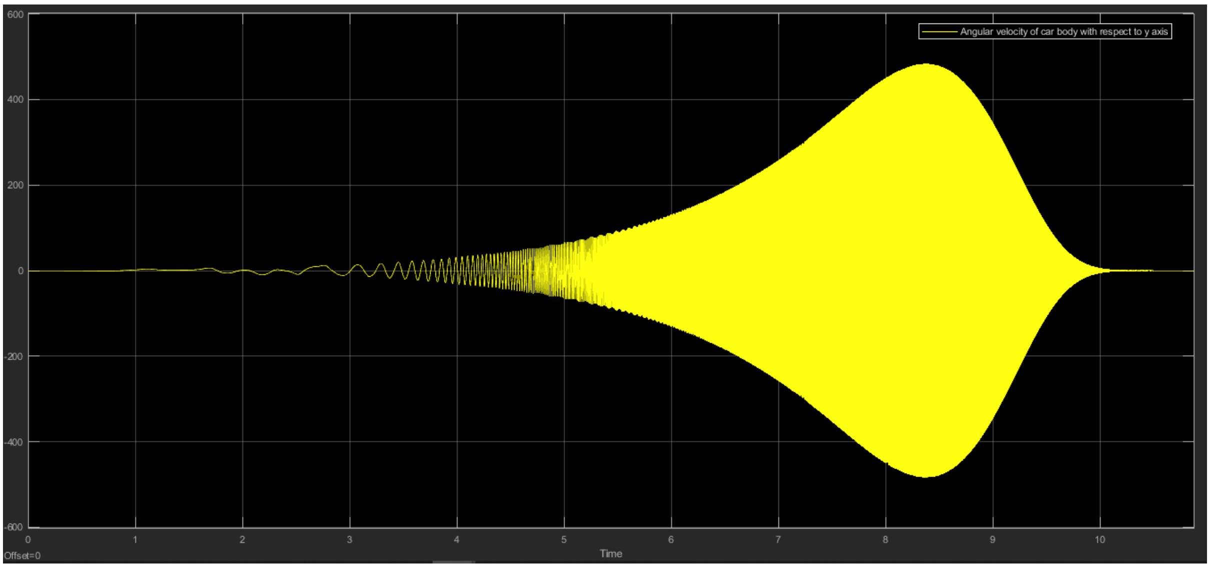

Figure 57 Angular velocity about y-axis of the car body

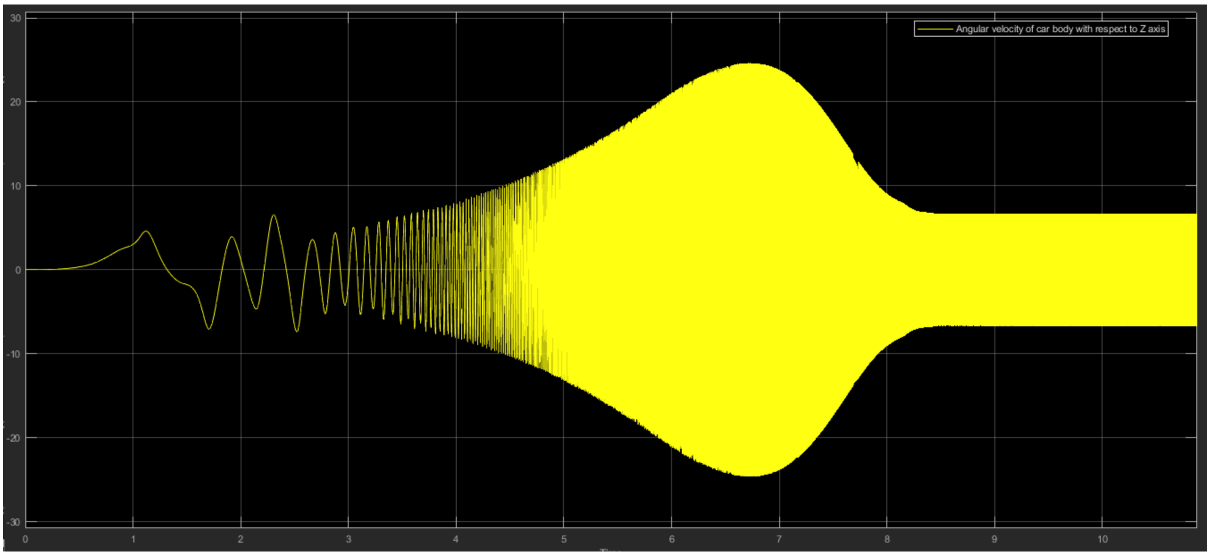

Figure 58 Angular velocity about z-axis of the car body

Figure 56, 57 and figure 58 shows the angular displacement of the car body with respect to the principal axis of the car body for the non-inertial frame of the car body. Because of the road excitation forces generated in x, y and z-direction torque applied on the car body, so the car body experienced several angular movements with respect to the principal axis of the car body. The behaviour of the car body under road excitation shown in figure 56, 57, and 58. In the above figures, angular movements of the car body increase at such points and then steady at zero value or a constant value.

6.2.4 Results for the inertial frame of car body

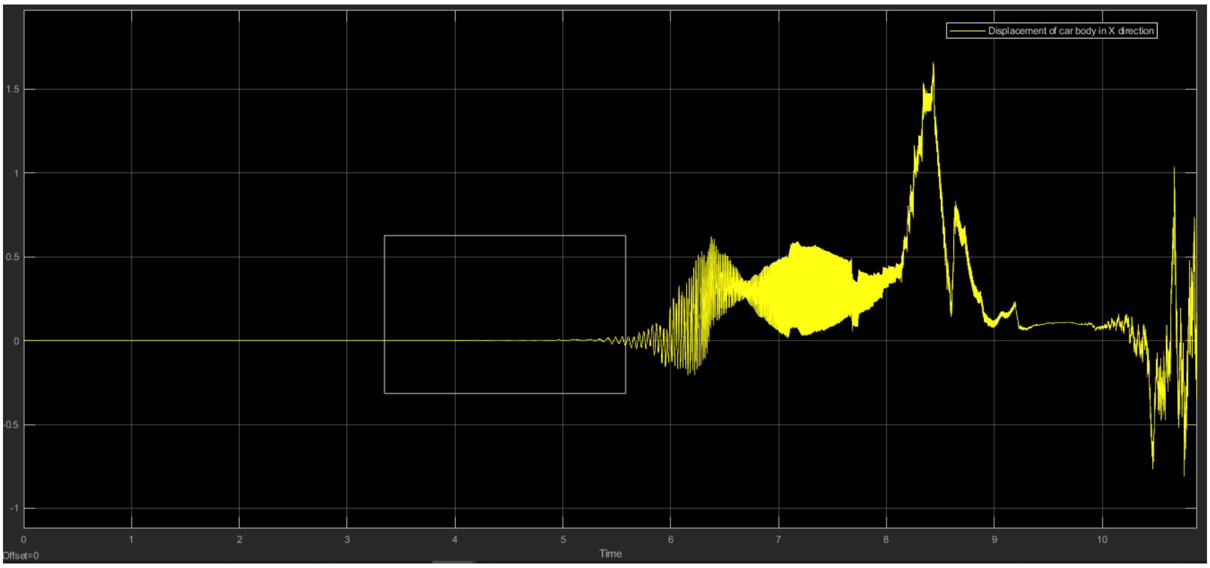

Figure 59 Displacement in the X-direction from the inertial frame

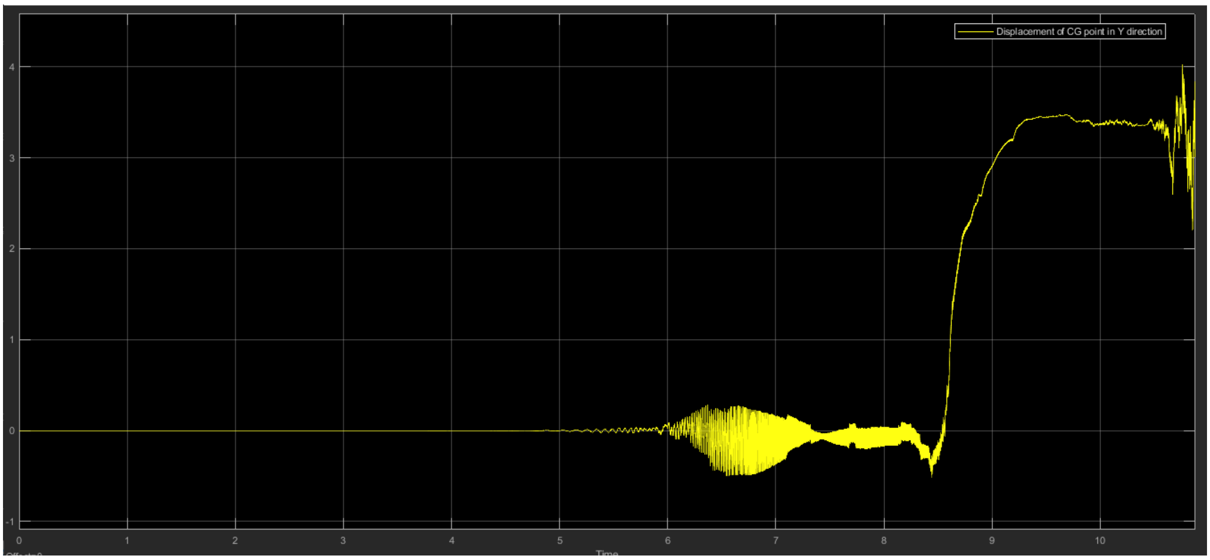

Figure 60 Displacement in the Y-direction from the inertial frame

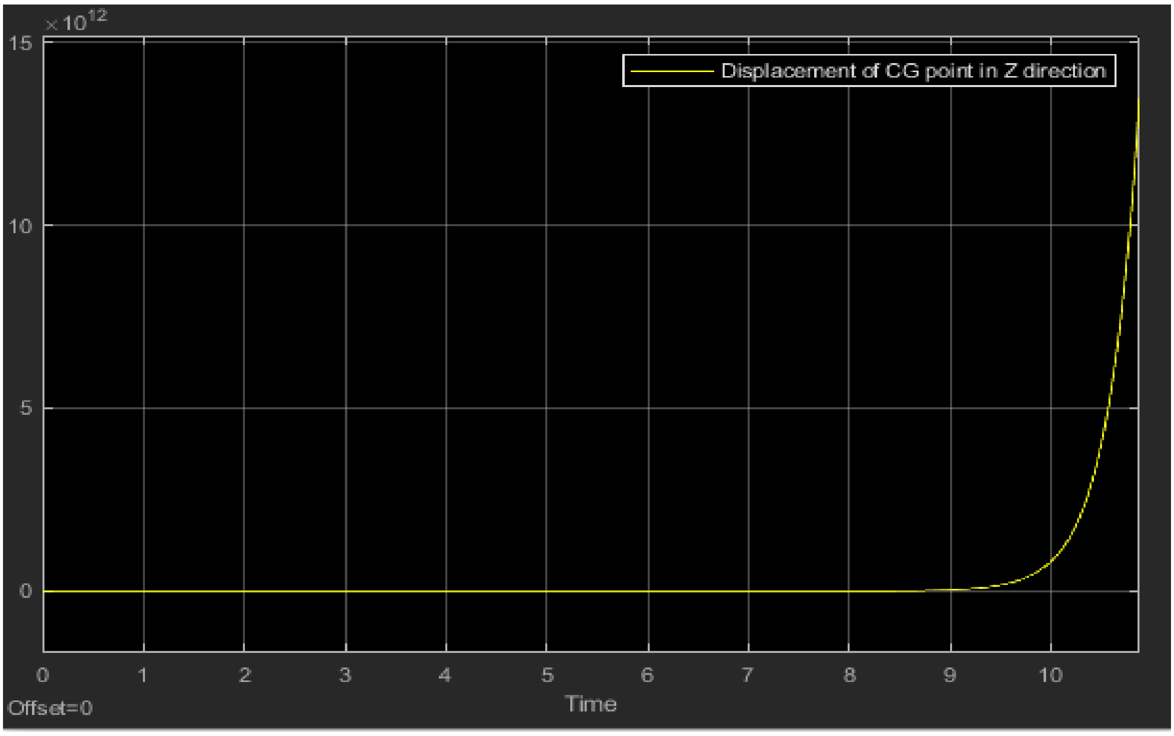

Figure 61 Displacement in the Z-direction from the inertial frame

Figure 59, 60 and 61 shows the displacement of the CG point in X, Y, and Z direction for the inertial frame over time.

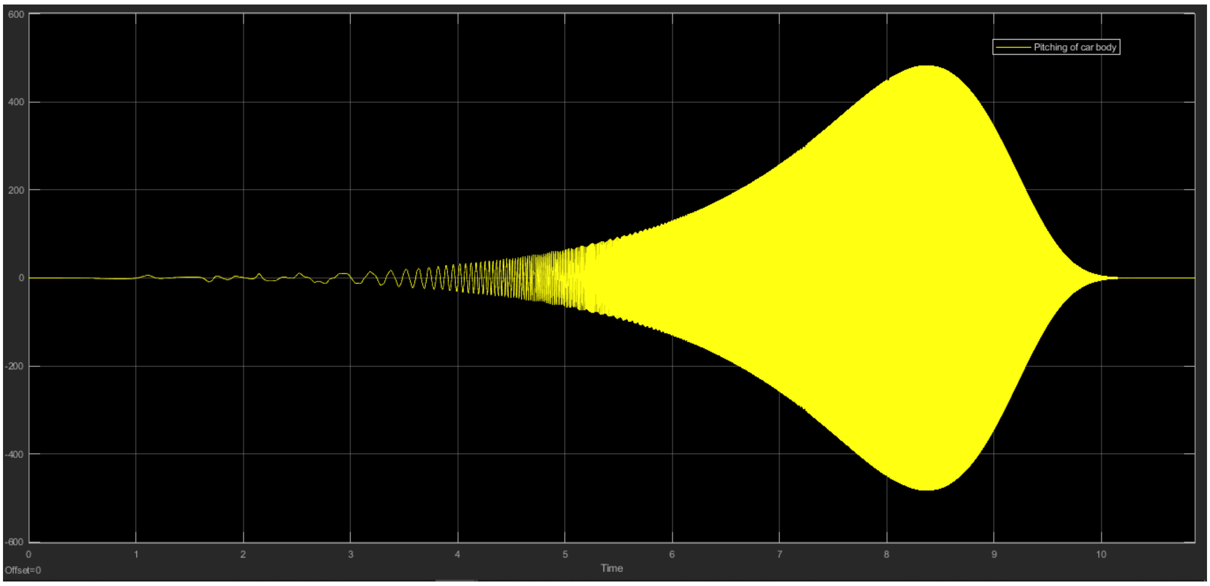

Figure 62 Pitching of the car body from the inertial frame

Figure 62 shows the pitching of the car body in the inertial frame which shows it fluctuated in between 430 Nm to -430 Nm after some time pitching movement become zero which reflect the pitching behaviour of the car body which is useful for the analysis of the car body under forces generated by suspension components.

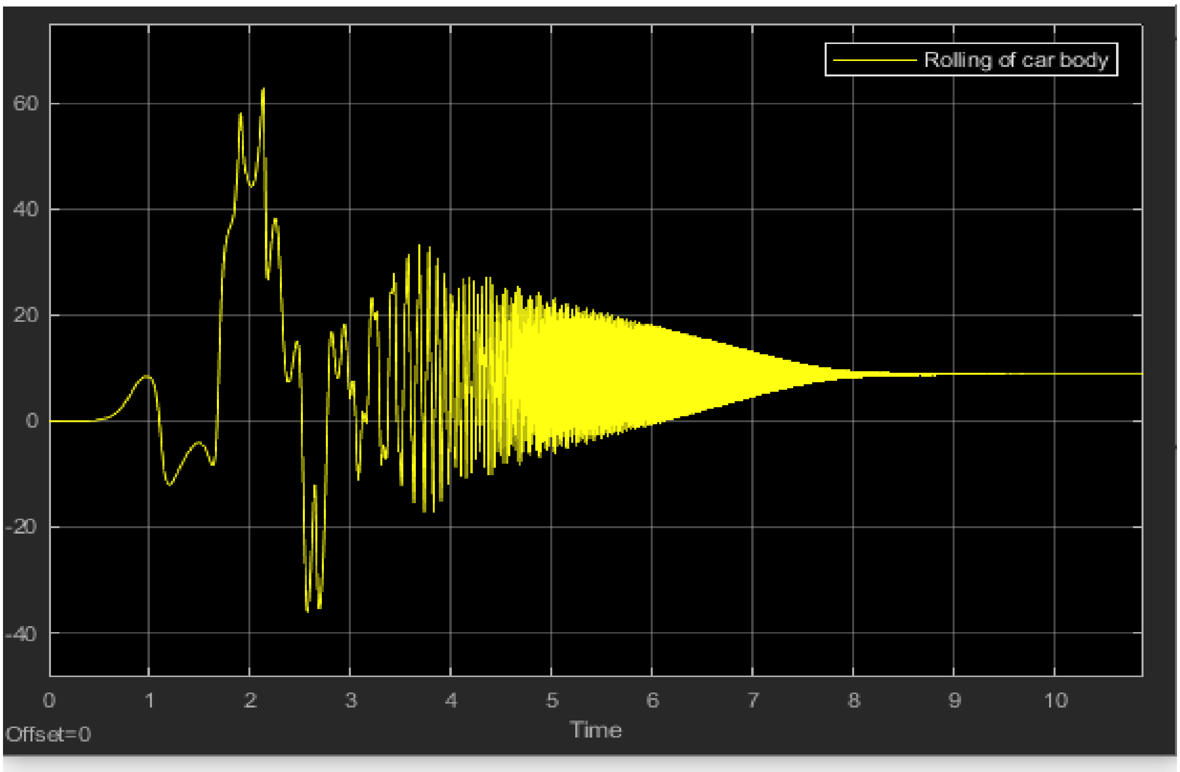

Figure 63 Rolling of the car body from the inertial frame

Figure 63 shows the rolling movement of the car body over time under road excitation applied on the front left wheel. The graph shows several fluctuations at the beginning and then it becomes zero as time goes.

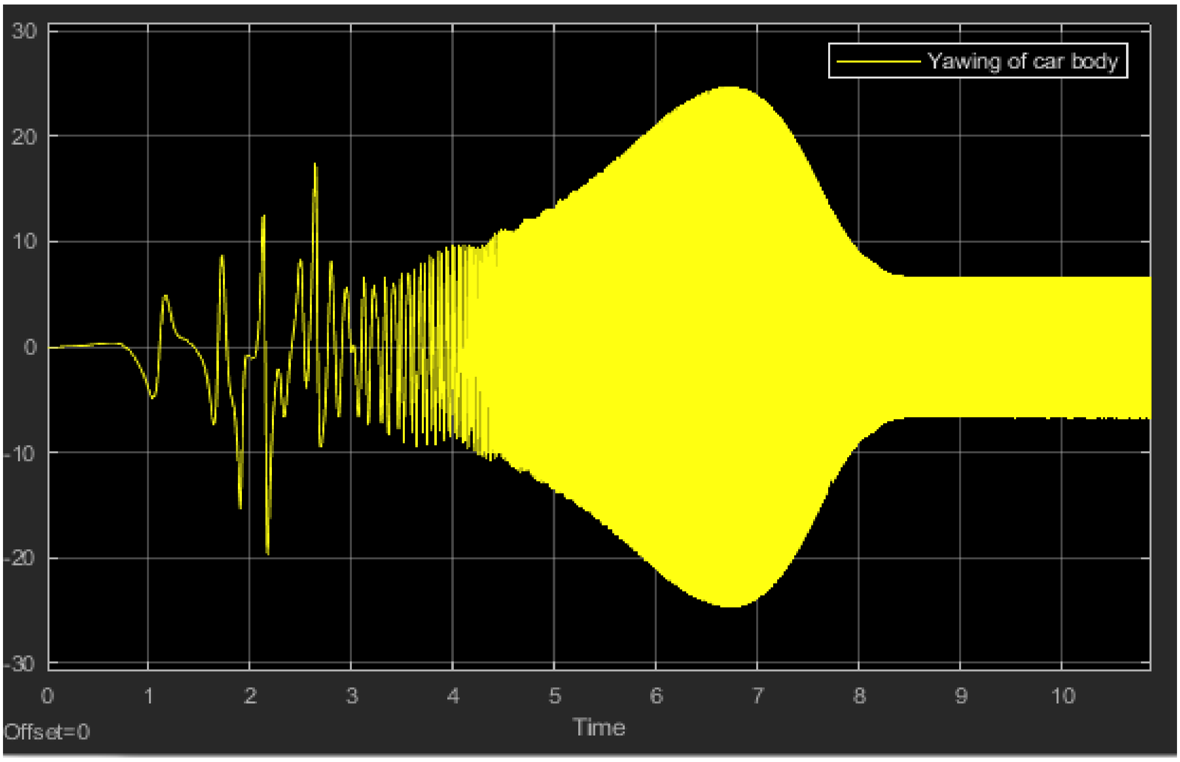

Figure 64 Yawing of the car body from the inertial frame

Figure 64 shows the yawing of the car body with respect to time in the inertial frame of the body.

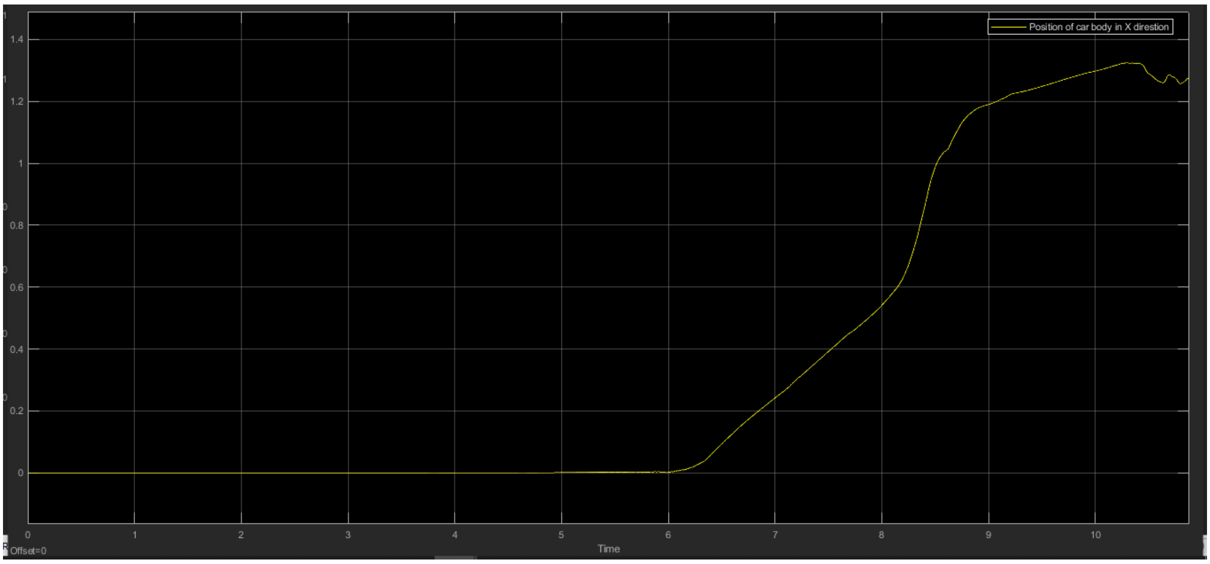

Figure 65 Position of the car body on the x-axis in the inertial frame

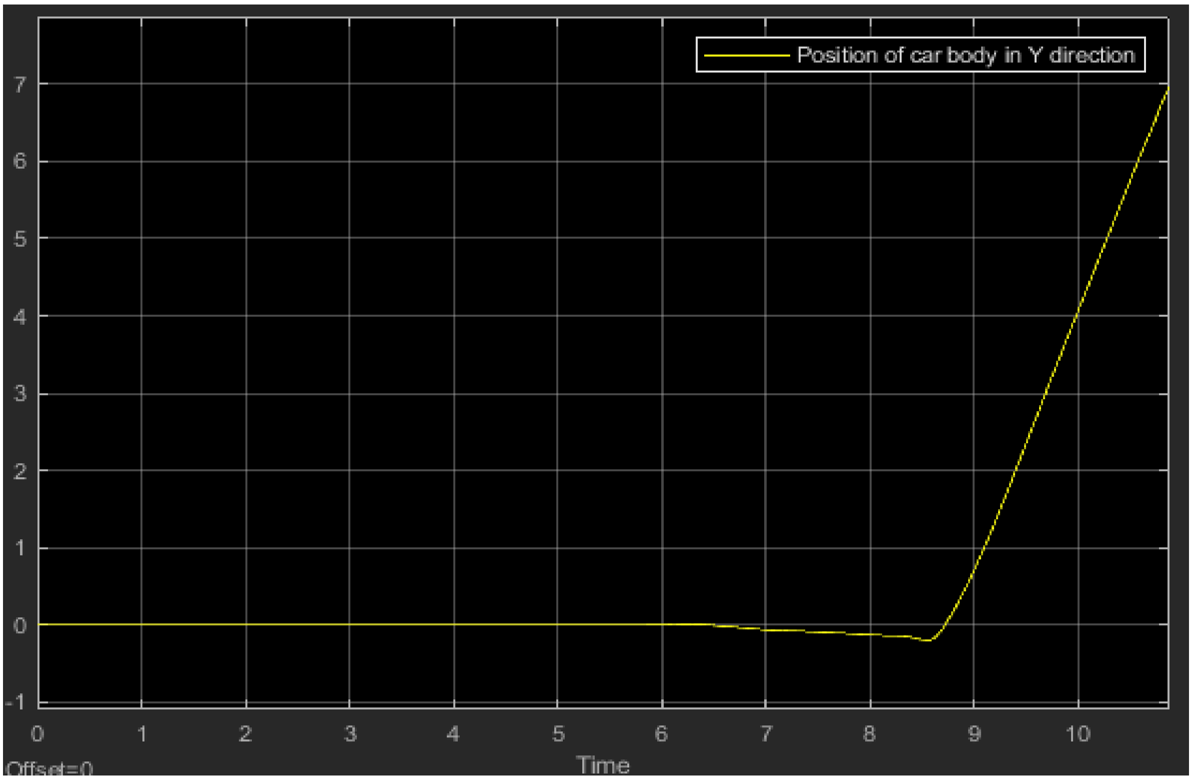

Figure 66 Position of the car body on the y-axis in the inertial frame

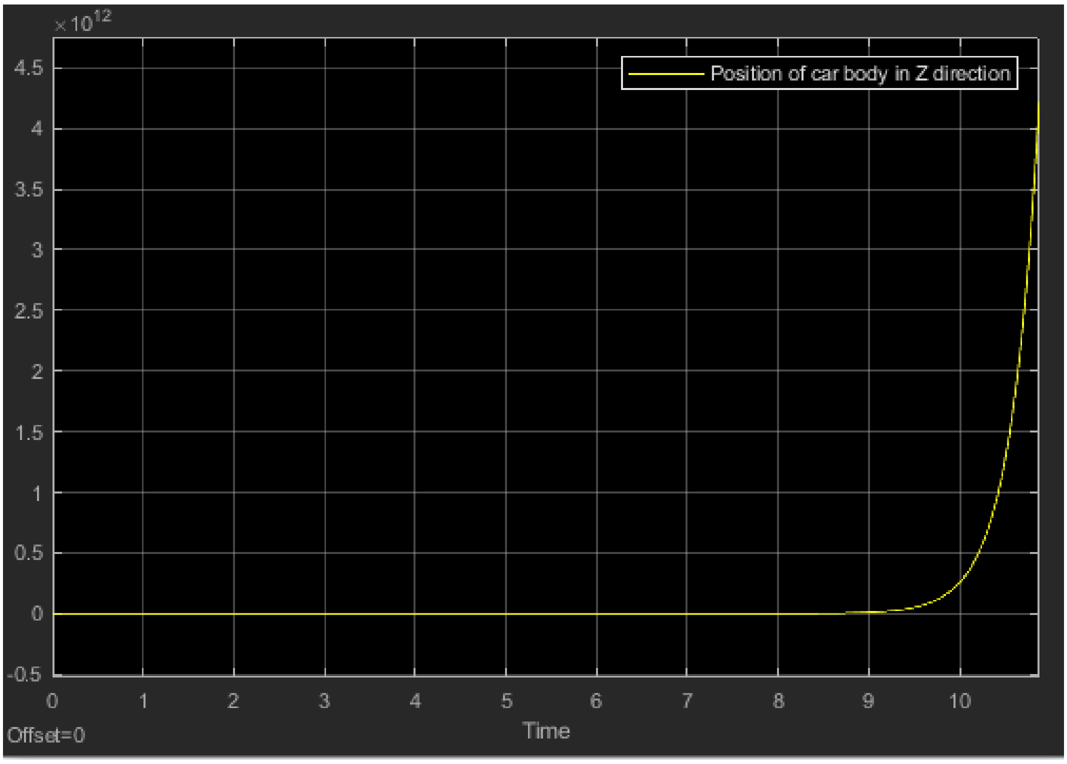

Figure 67 Position of the car body on the z-axis in the inertial frame

Figure 59-64 shows the six degrees of freedom motion of the car body for the inertial frame. And figure 65- 67 shows the position of the car body with respect to the CG point of the car body. Above obtained results show the behaviour of car body under forces generated by suspension components due to road excitation. Above results help to investigate the suspension system by simply investigating the behaviour of the car body under forces generated by the suspension components in different road conditions. For the above results, the clockwise direction of the car body considered as a positive with respect to the principal axis of the car body, whereas anticlockwise direction is taken as negative. The behaviour of the car body helps to understand how the suspension system works for the vehicle body. Critical parameters for suspension system such as spring constant and damping coefficient can be analysed by simple investigation of the car body behaviour, which is helpful in the development of any suspension system.

7 Conclusion and Future works

7.1 Conclusion

In the field of engineering, model development of any system is a continuous endeavour for the researchers because the model is the system on which experiments are being done to gather data which shows the behaviour of that particular system. Vehicle dynamics is the major subject in the automobile sector, where researchers are continually improving the model development process to investigate the behaviour of the vehicle. The suspension system is the most crucial system of vehicle dynamics, which directly incorporate with the ride quality of the automobile where ride, comfort and handling characteristics of the vehicle are considered. In order to analysis of suspension system, various models are used to investigate the behaviour of suspension system by considering the 2DOF model of the system components. This research project aims to develop six degrees of freedom model of the car body, which is used as a reference for suspension system analysis in future by considering the behaviour of the car body under suspension forces generated by suspension components. Various modelling techniques are reviewed in the literature, and with the correct approach of physical modelling technique, six degrees of freedom model of car body developed which shows the behaviour of car body under suspension forces generated by road shocks coming from road excitation. The behaviour of the model is simulated in Simulink environment for analysis. In conclusion, In this research work, mathematical modelling of the car body presented, which shows six degrees of freedom motion of the car body, which can be used in the analysis of the suspension system. This research work helps to find the effective parameters of the suspension components such as spring constant for the spring and damping coefficient for the damper. The mathematical model developed in this research work is also used to investigate the advance suspension system such as active suspension system as the active suspension system controls the suspension forces generated by road shocks in accordance with the movement of the car body. With this mathematical model, the active suspension system can also be analysed in the early stage of development without building the real prototype model of the active suspension system by simply connect the mathematical model of the active suspension system with the model of the car body. In short, the mathematical model developed in this research work can be used for the investigation of any suspension systems in future by considering car body behaviour under suspension forces generated by suspension components by considering different road conditions.

7.2 Limitations and future work

In this model of car body components such as unsprung masses and suspension structure such as wishbone structure or Macpherson structure are not incorporated due to limited time and pandemic situation arising in the last few months. In near future bond graph model of the suspension structures, will be directly connected with the bond graph model of the car body to investigate the behaviour of these structures on the car body.

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Appendices

Matlab Simulink model codes

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Figure 68 Bond graph model with assigned bond numbers