ANALYSIS OF PILE-SOIL INTERACTIONS IN EXPANSIVE SOILS

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DECLARATION

1. I certify that this thesis presented for the degree of Master of Engineering (Civil).
2. I declare that this thesis represents my work and has not been submitted in any previous degree. The work demonstrated is completely my own, except where indicated by reference or acknowledgment.
3. I am aware of the University's policy on plagiarism and strictly comply with the terms of the policy.

I make this solemn declaration following the terms of the University policy about plagiarism and subject to the penalties provided by the University for the making of false statements.

Signed by the Declarant – Khac Tuan Nguyen

In South Australia on the 31st of October 2021

EXECUTIVE SUMMARY

This study provides a simplified model to estimate the pile-soil interaction in expansive soil under axial loading with high accuracy compared with other complicated models.

Three methods can be used to predict the pile-soil interaction under the axial loading. Group 1 is the Simplified Analytical Method. This method is easy to apply. However, the results in terms of the vertical pile movement and the axial load are usually overestimated, and it is impossible to estimate the vertical soil movement. Group 2 is the Boundary Element Method, and Group 3 is the Finite Element Method. Those two methods offer accurate results but the theory behind them is intricate.

The present model uses a combination of theories in Group 1 and Group 2 so that it is easier to apply to interpret the following issues:

- The relationship between load and settlement (t-z curve).
- Vertical movements of pile and soil.
- Settlement of the pile head.
- Axial loads.

There is a good agreement between the present model with RS Pile results and other models. The model has the same interpretations for the pile-soil interaction under the axial loading:

- In the case of under only soil's swelling pressure (no applied load), the pile and the surrounding soil move upwards due to the soil heave.
- As the applied load increases, the vertical pile movements, axial loads, settlement of the pile head, and vertical soil movements also increase.
- The pile-soil interaction is dependent on the distance between the pile axis to the calculated position. The larger distance, the smaller the downward movement of the pile and the surrounding soil. Those movements are inversely proportional to depth.

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CHAPTER I: INTRODUCTION

1.1. Background

1.1.1 Definition of expansive soil and adverse impacts

According to Rajapakse (2016), expansive soil is considered a more problematic clay due to the swelling and shrinking potential when contacting with water. The fluctuation of water content is the main reason causing the development of cracks in the structures. Because of these cracks, the structures become unstable and their bearing capacity is reduced.

The fluctuation of water content is well known as the soil suction represented for the free energy state that can be measured regarding the partial vapour pressure of the soil water. The soil suction includes Matric suction and Osmotic suction. While Matric suction is commonly related to the capillary phenomenon releasing from the surface tension of water, Osmotic suction is associated with the number of salts dissolved in the free pore-water so it entirely depends on the salt concentration and the type of salt. In the reality, the Osmotic suction is much smaller than the matric suction.

Measurement of soil reactivity is represented by Shrinkage Index and Instability Index. Shrinkage Index (I_{ps}) can be calculated from the laboratory test (AS 1289-1998).

Refer to AS 2870-2011, Instability Index (I_{pt}) can be calculated from the Shrinkage Index as the following formula:

$$I_{pt} = \alpha\ I_{ps} \quad (1)$$

Where: $\alpha = 1$ in the cracked zone, and $\alpha = 2 - z/5$ in the uncracked zone (z is the depth).

The characteristic of expansive soils can be classified by the Plasticity Index and the Instability Index described in Table 1.

Table 1. Expansive soil classification (AS 2870-2011)

Description of Plasticity	Plasticity Index, *PI* (%)	Instability Index, I_{pt} (%)
Trace	< 2	< 0.5
Very Low	2 to 5	Approx. 0.5
Low	5 to 10	0.5 to 1
Low to Medium	10 to 20	1 to 1.5
Medium	20 to 25	1.5 to 2
Medium to High	25 to 30	Approx. 2
High	30 to 45	2 to 3.5
Very High	45 to 60	3.5 to 5
Extremely High	> 60	> 5

Refer to AS 2870-2011, the characteristic surface movement due to the soil suction change can be calculated as the following formula:

$$y_s = \sum_{i=1}^{n} y_i = \sum_{i=1}^{n} \left(I_{pt}\overline{\Delta u}h\right)_i \quad (2)$$

Where:

y_s = characteristics surface movement; I_{pt} = Instability Index.

$\overline{\Delta u}$ = the change in total suction; h = thickness of the layer.

n = number of the soil layers within the design depth of suction change.

A study by Li & Guo (2016) shows that expansive soil has caused many damages to lightweight structures, walls, ceilings, road surfaces, basements, pipelines, lateral movements on foundations and retaining walls, and other structures.

Expansive soil constitutes approximately twenty percent of Australia's territory (Richards et al., 1983). The requirement of having thorough studies about this type of soil and solutions for the adverse impacts is necessary to ensure that sustainable foundations are possible to be constructed on this soil.

1.1.2 Ground treatment for expansive soil

A variety of foundation techniques are being used to mitigate the swell-shrink behaviour of the expansive soil. Several measures can be taken to address the adverse impacts as listed below:

- Excavating and removing expansive soil layers: This option should only be applied when the expansive soil areas are not large.
- Using various additives: lime, cement, fly ash, industrial waste products, and polypropylene fiber (Fattah et al., 2010).
- Using pile system: The piles should be embedded in a hard soil layer.

In the construction industry, piles are commonly used in weak soils including expansive soils. Piles are used to transfer the loads into the underlying ground. Piles will be driven through the weak soil and embedded in a hard soil layer. This study investigates the pile-soil interaction in expansive soils. Based on this study, a simplified model is developed to estimate the pile-soil interaction with high accuracy compared with other complicated models.

According to Poulos (1989), the pile-soil interaction in expansive soils can be predicted by three methods as shown in Figure 1.

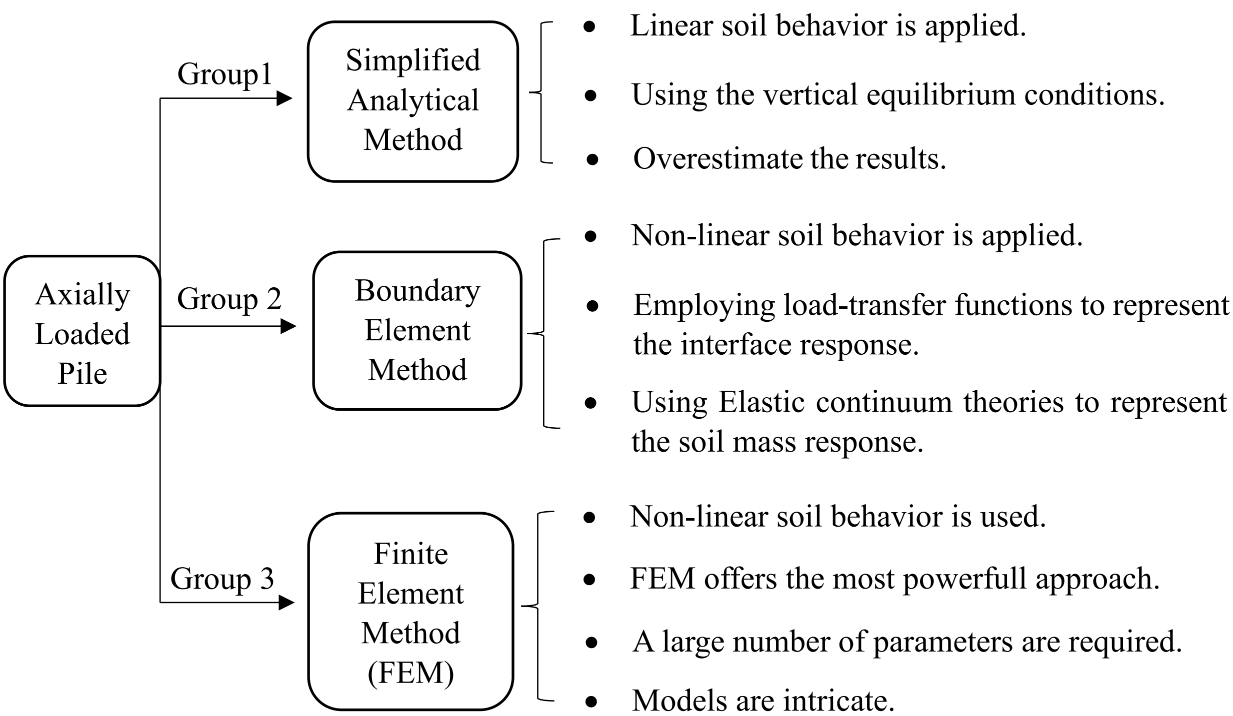


Figure 1. Solutions for axially loaded pile

Figure 1 presents that studies in Group 1 are oversimplified so that the results generated from those models are usually overestimated. Studies in Group 2 and Group 3 are significantly complex to apply. Studies of Crispin et al (2018), Jiang et al (2020), Ju (2015), and other studies which are in Group 2 and Group 3 use the continuum, and Finite Element theories offering complicated equations. For instance, Bessel functions with the non-integer orders require the support of a program, Python or MATLAB for solving those equations and obtaining the results.

This reality requires a model that can harmonize the characteristics of those groups. Further explanation for the necessary and contribution of this research is presented in the next chapter.

1.2. Scope of the thesis

Due to the limitation of the time for the research. This study only investigates the pile-soil interaction under axial loading.

Laterally loaded piles are out of the scope of the project.

Bearing capacity and pile foundation design are also out of the scope of this research.

1.3. The importance of the research

While most studies in Group 1 (Simplified Analytical Method) overestimate the results and are impossible to interpret the soil movement, Group 2 (Boundary Element Method) and Group 3 (FEM) theories are complex to apply.

This study develops a simplified model by combining theories in Group 1 and Group 2. The advantages of the present model are listed below:

- The present model is easy to apply.
- It does not require a large amount of data.
- It offers a highly accurate interpretation of the soil movement that models in Group 1 are impossible.

- It offers an appropriate prediction for other important aspects of the pile-soil interaction: Load-settlement relationship (t-z curve), axial loads, vertical pile movement, and settlement of the pile head.

1.4. Research aims

This study aims to provide a simplified model that can predict the pile-soil interaction in expansive soils with high accuracy compared with complicated models using the Boundary Element Method or Finite Element Method.

This model is established to overcome the drawbacks of the Simplified Analytical Method that are having no function to predict the soil movement and the accuracy is not high.

1.5. The structure of the thesis

This thesis includes the introduction chapter followed by four other chapters described as follows:

- Chapter 2 – Literature review: This chapter provides a comprehensive overview of the pile-soil interaction in expansive soils. Key studies are also presented to see the advantages and disadvantages of each model. Then a thorough evaluation is given for existing studies.
- Chapter 3 – Methodology: This section explains the process of developing the present model including the applied theories and the derivation of all equations used in the model.
- Chapter 4 – Results and Discussion: Key findings are presented in this chapter. The results are compared with previous studies and RSPile results. A Critical interpretation of obtained results is provided.

- Chapter 5 – Conclusion and Future Work: This chapter summarises the key findings including the contributions to the studying field. On the other hand, suggestions are proposed for future studies based on the limit of the present model.
- Appendices demonstrate results from the RSPile software and compare details of the present model with the model of Jiang et al (2020) representing for Boundary Element Method.

CHAPTER II: LITERATURE REVIEW

The previous chapter presented an overview of the three methods in three groups related to the pile-soil interaction under axial loading. This chapter presents details of key studies in those groups and provides evaluations of the existing solutions.

2.1. The Simplified Analytical Method – Group 1

The Simplified Analytical Method assumes the soil behavior is linear, so studies in this group usually overestimate the results. Typical studies are presented in Table 2.

Table 2. Typical studies of Group 1

Studies	Methodology	Remarks
Zhang et al. (1999)	Two techniques were implemented. The first technique is the interaction method, and the remaining is the stiffness method. Those techniques were based on the following methodology: • Finite layer theory was applied for layered soil. • Simple beam theory was used for the piles combining with the influence matrix of the pile and the soil. • The results from the model were compared with existing solutions.	The model was developed to analyse the pile groups which were embedded in both homogeneous and nonhomogeneous soils under the axial and lateral loads and moments. The stiffness method was more accurate than the interaction method. In the case of the piles with fixed heads, the model of Zhang was significantly different from Randolph's results.
Poulos et al. (1980)	In Chapter 12, the interaction of pile and soil in swelling and shrinking soils was presented with several methods as follows: • Simplified Analytical Approach: This study assumed that the full	Chapter 12 of this study provided an overview of the pile-soil interaction in expansive soil with several solutions. However, the application of each method was not detailed.

	slip occurred between the interface of the pile shaft and the expensive soil. • Elastic Theory Approach: This method assumed that the slip occurred along the entire pile shaft. • Pile-Soil Slip method: This method allowed to specify a limiting value of the shear stress, and limiting base pressure.	The Elasitc Theory Approach offered approximate solutions resulting in a certain varatiation with the measured values. The Pile-Soil Slip method must be replicated multiple times until the shear stresses and the base resistance approach the values that are lower than the limiting values.
Motta. (1994)	The model was based on the Simplified Analytical Method to predict the pile-soil interaction. This model assumed the soil shear modulus (G_s), and the shear resistance (K_s) of the t-z curve were constant.	The assumptions of G_s and K_s are only proper with overconsolidated clays. The model overestimated the results compared with the measured values. This model's theories could not offer any prediction for the soil movement under the axial loading.
Butterflield et al. (1971)	The model used the analytical method to analyse rigid under axial loads and the compressible group of piles. Graphs were used to investigate the effect of length, diameter, and the ratio of Young's modulus and shear modulus to the response of pile groups. The results generated by the model were compared with the laboratory tests.	The study displayed an elastic analysis of compressible piles and groups of piles. The model allowed using the extrapolation of the load-displacement data in the case of single piles to predict the pile groups' behaviour. There was a certain difference between the results from the model with the results from the laboratory tests.

Fan et al. (2007)	The pile-soil interaction theory and the shear deformation method were used to derive a set of elastic differential equations of the load-transfer of a single pile under the application of the axial loading and under only soil's swelling pressure. Four assumptions were made: • The cross-section keeps no change along the pile. • The expansive soil behaves elastically. • The heave of the soil varies linearly with depth. • The maximum of the heave occurs at the ground level, and its minimum value is at the active depth. The model was validated by comparing its results with the measured data.	The study presented an analytical method of load-transfer of the single pile in expansive soil to investigate the pile-soil interaction. The load-tranfer law for the single pile was applied to investigate the pile behavior. The model offered the calculation for the maximum tensible force that induced by swelling pressure for the engineering design. The distribtution of the axial force, pile movement, and pile's skin friction along the pile shaft were determined. There was certain difference between the model's result with the measured data.
Xiao et al. (2011)	The model used the different elastic equations of a load transfer on a single pile regarding soil swelling and considering the shear deformation method.	This research investigated the effect of pile length and pile diameter changes on the pile movements and the bearing capacity of a single pile. However, the results generated from the model of Xiao et al had a certain variation with the results of the test.
Zhang et al. (2016)	Analytical equations considering the increase of stiffness with depth and the decrease of expansion along the	The results generated from the model was overestimated when compared with the test results.

	depth were derived by the load transfer method. Calculated results were compared with the field and model test data.	The elastic-plastic stage was not mentioned in this study.

2.2. The Boundary Element Method – Group 2

The Boundary Element Method considers soil behavior is non-linear. Load transfer functions and elastic continuum theories are applied in this method. Typical studies are shown below in Table 3.

Table 3. Typical studies of Group 2

Studies	Methodology	Remarks
Poulos. (1989)	Poulos illustrated several methods including the Boundary Element Method to analysis a single pile, and pile groups in different types of soils. The load-transfer functions were used to represent the response of the interface between the pile shaft and the soil elements. Elastic continuum theory was used to represent the response of the soil mass. The results from different studies were compared.	The study presented the theories and the application of pile behavior under axial loading, and lateral loading. The response of each soil element was considered at a node located at the centre of that element through the pile axis. No assumptions were made. The soil stiffness K_s varies with depth. The increment of the soil movement was caused by two sources, the stresses of the pile-soil interaction, and the free-field soil movements. This method provided a high level of accuracy when predicting the pile-soil interaction.

Chow. (1986)	Load transfer curves and elastic continuum theory were applied to analysis the linear and non-linear responses of the pile groups subjected to the axial loading. The results generated from the model were compared with the Rigorous Boundary Integral Method and results from the field load tests.	This study presented the analysis of vertically loaded pile groups. The soil response to the axially applied load was considered in two cases, linear soil behavior and non-linear soil response. The results from the model had a consistent trend when compared with previous studies. The difference between the model' results and the results from the Rigorous Boundary Integral Method and the field load tests was unnoticeable.
Banerjee et al. (1978)	The three-dimensional solids model was established based on the general form of the Boundary Element Method. Modulus of elasticity was considered to increase linearly with depth. The settlement of the pile groups and the load distribution properties were displayed in a non-dimensional form. Piles were embedded in two layers with different values of Young's modulus. The model was validated by comparing with previous studies conducted by Poulos (1968), Butterfield (1971), and the experimental observations.	Symmetrical pile groups under the vertical load embedded in Gibson soil was described. The load-displacement relationship (t-z curve) was investigated by a set of nondimensional plots for single piles. Interaction factors between the piles in the group were also determined allowing the model to calculate the effect of piles' interaction factors on the pile – soil behaviour. The results from the model were proved to be more realistic predictions of pile groups response than those displayed by the model of Poulos (1968), and Butterfield (1971).

Guo & Randolp. (1997)	In this study, three methods were used to investigate the pile-soil interaction: • The approximate analytical method. • The one-dimensional numerical algorithms. • The full axisymmetric analyses method based on the Boundary or Finite Element Solutions. The results from the three approaches above were compared with the experimental data. The spring stiffness was considered non-linear with depth based on elastic-plastic profile.	The interaction between the pile and the surrounding soi was determined by springs located along the pile shaft and at the pile base. A new closed-form solution was established to predict the response of piles in non-homogeneous soil. The approximate analysis approach offered a significant variation with the experimental data, while results from the one-dimensional numerical algorithms were improved, and the full axisymmetric analyses using the Boundary Element Method provided the closest prediction compared with the experimental data.
Kuwabara. (1991)	A simplified non-linear analysis was performed based on the Boundary Element Method to investigate the vertical pile movement and the surrounding soil under the axial loading. Published laboratory tests data was applied to propose a shear stress and shear strain curve. By considering the shear modulus varying with depth, the settlement of the pile head and the soil movement were obtained. The results from the model were compared with the tests.	This study demonstrated the settlement behavior of the non-linear soil around the single pile subjected to vertical Loads. In this model, the base resistance was neglected. The hyperbolic stress-strain characteristics of soils provided an appropriate shaft load-settlement relationship. There was a good agreement between the results with the tests' values.

Liu et al. (2004)	Soil mechanics, elastic theory, and numerical analysis were applied to determine the axial load, and load-settlement curve (t-z curve). The pile-soil interaction along the pile shaft was solved by the tri-linear softening model. The pile-soil interaction at the pile base was analysed by the tri-linear plastic model. The pile-soil friction test and deep-hole test at the site area were conducted to validate the results of the model.	A series of equations for the load-settlement curve was established. The hyperbolic load-transfer model was reasonable to apply for sand or hard clay along the pile shaft. The tri-linear softening load-transfer model was suitable with the softening clay along the pile shaft. There was a good agreement between the results of the model with the tests' results. The bearing capacity of the manually dug bored pile was less affected by the pile-skin friction.
Seo & Prezzi. (2007)	Explicit elastic solutions were used to identify the single pile response under the vertical loads. The differential equations were derived based on the energy principles. The unknown constants were determined by using Cramer's rule and the recurrence formulas. The load transfer and load-settlement curves were compared with the load tests. The soil was considered a non-linear elastic material. The results generated by the model were compared with previous studies and the load test.	In the reality, piles are rarely embedded in a single layer of soil. This study demonstrated the analytical solutions for a vertically loaded pile in multilayered soil. The solutions were consistent with the boundary conditions. Solutions for piles rested on a rigid material were also obtained by adjusting the boundary conditions. The load-settlement curve and the vertical soil movement were identified by the model. The model offered a high solution compared with the previous studies and the load test results.

Crispin et al. (2018)	The Winkler model based on the load-transfer functions was applied to investigate the pile-soil interaction under the axial loading in inhomogeneous soil. Winkler springs represented the soil stiffness varying as a power function of depth. Differential equations were derived in an exact manner based on Bessel functions with the non-integer orders. Elastic-perfectly plastic Winkler Springs were used for the non-linear range. The ultimate skin friction was calculated using α-method by Skempton (1959). The ultimate resistance of the pile base was identified by the formulas of Skempton (1951). The Winkler modulus used the model of Randolph & Wroth (1978). The model was validated by the load test.	The load-settlement curve has been usually presented in two stages, elastic stage and plastic stage. This study went into more details illustrating four stages of the pile response as follows: Stage 1-Linear elastic response: When the vertical load was small, the pile shaft and pile base behavior were entirely elastic. Stage 2-Non-linear shaft behavior: The increase of the load caused the non-linear behavior of the pile shaft, while the pile base was still in the elastic range. Stage 3-Shaft resistance exhausted: The response of the pile was now only governed by the base response. Stage 4-Base resistance exhausted: The applied load was now equal to the ultimate resistance, and the displacement of the pile head developed without any limit. Bessel functions with non-integer orders were complicated to apply. The validation had a good consistency with the load test.
Costa et al. (2011)	A new stress-wave program was used to calculate the residual driving stress in piles. The program allowed determining the displacement stabilization	The study applied a new stress-wave program to predict the residual driving stresses in piles. The residual stresses were extremely influenced by the ratio of end

	implemented by analysis for a longer time. The effect of base resistance and shaft friction and driving energy on the development of the residual stresses in the pile was identified. The mechanism of hammer blow was applied. When the piles penetrated into the ground, the compression wave began to travel to the pile toe, and the pile shaft was elastically compressed. In the case of no external load being applied, the equilibriums basically involved some residual point load and friction stresses along the pile shaft which balanced each other. Residual stresses did not occur in the end-bearing piles. This was consistent with the study of Briaud & Turker (1984).	bearing capacity to the total resistance. An increase in the percentage of toe resistance resulted in the neutral point movement, closer to the pile toe. The relationship between the toe resistance with the influence of the shaft friction on residual stresses was inversely proportional. The neutral point's position was only significantly affected when the toe resistance was low. The driving energy had a minor effect on the residual stresses, but it influenced the number of loading and unloading cycles. Any changes in driving energy did not cause any change in the position of the neutral point.
Soundara et al. (2017)	This paper aims to investigate the pile uplift in expansive soils using a hyperbolic curve based on the model pile uplift tests, interface shear tests, and consolidation test results. A design procedure was built to predict the uplift forces along the pile shaft in expansive soils. The interface shear resistance on a single pile along the pile shaft during	Analytical studies offered a good correction of the interface shear resistance profile that was explored by laboratory studies. The proposed hyperbolic model can predict those variations. The study displayed a method using the hyperbolic model to estimate the uplift of the pile head under the soil heave. However, the test process

	the swelling was considered to be variable.	was considerably complex to be applied.
Liu. (2019)	Load transfer mechanism was applied to interpret the pile-soil interaction in unsaturated expansive soils.	The pile-soil interaction was presented in two cases, pile only under soil' swelling pressure, and pile only under applied loads. The vertical pile movement and the vertical soil movement could be both estimated by the model with high accuracy compared with the measured values. However, most of the theories are intricate.
Jiang et al. (2020)	This model was based on Boundary Element Method. Elastic theories of the load transfer functions combined with load transfer matrix were applied. The soil shear modulus (G_s), and the shear resistance (K_s) of the t-z curve vary with depth. The suggested methods are confirmed by comparison with the results of the load tests.	The study demonstrated a nonlinear analysis approach comprising of the variation of soil shear modulus with depth to determine the effect of expansive actions on the performance of a single pile. Most equations are related to Bessel functions with non-integer orders which are significantly complicated to apply.

2.3. The Finite Element Method – Group 3

The Finite Element Method assumes soil behavior is non-linear. This is the most powerful analytical approach. However, it requires a large number of parameters, and the models are noticeably complex to apply. Typical studies are displayed in Table 4.

Table 4. Typical studies of Group 3

Studies	Methodology	Remarks
Desai. (1974)	The numerical analysis combined with the Finite Element Analysis was applied to investigate the pile-soil interaction. The numerical procedure was implemented to predict the double curvature in the distribution of the load in the pile observed in plenty of field conditions. Continuum analysis in an approximate manner was also applied to approach the solutions for a wider range of problems in practice. Non-linear soil behaviour was considered to investigate the vertical soil movements under the axial loading. The model was validated by comparing the results with the pile load tests.	A hyperbolic stress-strain response was established. The load-settlement interaction was considerably linear to the load well beyond half the failure load. Softening was possible to be included in the finite element analysis offering a better correlation with the observation. The effect of the installation for the driven piles was available within the range of four times the pile's diameter. The load-deformation behaviour was significantly dependent on the values of the adhesion at the interface, the factor α, and Young's modulus. The results of the model were highly consistent with the pile load tests.
Valliappan. (1974)	The Finite Element Technique was used to predict the settlement of piles in multilayered soil. The discretisation of the continuum was analysed using the two-dimensional axisymmetric approach and isoparametric finite elements. The results generated from the model were compared with the other	Piles are rarely rested in a single layer of soil in the reality. This study demonstrated solutions for an axially loaded pile in multilayered soil. Finite Element Methods provide the most powerful analytical approach so that the model could not only consider non-linear soil response,

	existing models using the Finite Element Method.	but also simulate the processes of the installation, and the subsequent loading of the pile. The model was consistent with the theory of (Nystrom, 1986), and (Trautmann et al., 1983). There was a good agreement between the model with the other existing models.
Ju. (2015)	The 3D Finite Element Analysis was implemented to predict the settlement of the vertically loaded pile group. Three different types of analysis related to the characteristics of the soil were displayed: • A linear elastic analysis was applied when the soil was considered to be linear elastic. • A complete non-linear analysis was used in the case of the soil surrounding the pile shaft was hardening soil. • A non-linear and linear elastic analysis was conducted when the soil close to the pile shaft was hardening soil, while the soil was an elastic material in the remaining areas. The results from the Finite Element Analysis were compared with the analytical method.	By assuming that the soil close to the pile shaft was nonlinear, while the soil in the remaining areas was linear elastic provided a more reasonable prediction of the interaction factor, and thus lead to higher accuracy of estimation for the pile group settlement. This assumption had a good agreement with the study by (Ottaviani, 1975). Half of the distance from the pile axis to the edge of the non-linear zone was sufficient to capture the load transfer mechanism for the pile group in the case of using the Finite Element Method. The Finite Element Method using the load-transfer functions and continuum theory offered a high level of accuracy compared with the analytical approach.

Cheng et al. (2007)	A simple and useful controlled model based on the Finite Element Method was applied to investigate the pile-soil interaction in the pile foundation of a tunnel. The model simulated tunneling by using displacements to the tunnel boundary based on the convergence patterns measured in the field and centrifuge test results. Forces in terms of a fraction of the initial stress-state to the nodes on the boundary of the tunnel were applied. ABAQUS software was used to validate the model.	This model improved the drawbacks of previous studies, the model of Dasari (1996), Attewell & Woodman (1982), O'reilly & New (1982), Withiam & Kullhawy (1978), and Smirnoff (1989). During the analysis, the invert movements were assumed to be zero. The displacement-controlled model offered an accurate prediction for the tunneling-induced ground movements. The shape and values of the displacement profiles were consistent with the ABAQUS software.
Jardine. (1986)	This study focused on the measurement of the non-linear stress-strain properties of a low plasticity clay based on the Finite Element Method used for footings, excavations analysis. The Finite Element Analysis was applied to assess the influences of the small non-linear strain, then compared the linear elastic behavior. The set of analyses were conducted to assess the effects of the non-linear soil responses observed in the recent laboratory tests.	The non-linear response resulted in the concentration of deformation and strain regarding the loading boundaries. Linear elastic analysis was a convenient tool to express the measurements of the stiffness of the soil, however, this method led to the misleading of the pile-soil interaction. Finite Element Method allowed to detail the undrained responses of the soil when carrying loads by footings, piles, cavity expansion, and other external loads.

Gennaro & Said. (2008)	The Finite Element Method was used to establish a model predicting the pile-soil interaction under the axial loading in sands. The interface model MEPI-2D conducted by Gennaro & Frank (2002) was applied in this study to describe the response of the interface between the granular soil and a rigid structure. The numerical investigations were validated by pile loading tests using the calibration chamber proposed by Gennaro et al., (2004), Chow & Jardine (1996), and Jardine et al., (2005). Numerical simulations based on the Finite Element Method were implemented using empirical correlations via field data.	The results of the series of the numerical analysis regarding two compression tests for piles installed in the sand were demonstrated. In numerical calculations, the overestimation of the pile shaft friction resulted in the unreasonable prediction of the dilatancy at the interface. When the soil was considered elastic material, a coupling between the volumetric response of the interface and its surrounding soil might occur. This combination might cause changes in normal stress on the pile shaft. The model will become more accurate if the soil below the pile tip can be taken into consideration.
Mohamedzein et al. (1999)	The Two Dimensional Axisymmetric Finite Element Model was established to predict the pile-soil interaction in expansive soils. The pile was considered a linearly elastic material, while the soil was assumed as a non-linear material. The model was validated by comparing the results with the two field experiments.	The developed software was reasonable for soil layer thickness up to 12 m, and the pile length up to 6 m. Pile length had a significant effect on the vertical pile movements. The relationship between those factors was inversely proportional. The heave decreased to zero when the pile length was 4 m, and pile diameter was 0.25 m, and the applied load was larger than 90 kN.

	Forces causing the soil's swelling applied the formulas conducted by Chen (1975). The software calculating the displacement was developed by Mohamedzein (1989). The pile was represented by ten 8-noded isoparametric elements, while the soil was divided into 95 8-noded isoparametric elements. The pile-soil interaction finished at the interface when approaching the small movements (< 40 mm). The field and laboratory tests were conducted by Mohamed & Sharief (1989) at two sites in Sudan.	The tensible stress decreased when the applied load increased. The maximum of tensible stress occurred around the mid-height of the pile. The developed program based on the Finite Element Method needs to be modified to account for the interface response between the pile and the soil. The study provided a good agreement compared with the test results. However, most used theories were complex to apply.
Abhishek & Sharma. (2019)	A numerical study was presented to predict the movement of the uplift of the Granular Pile Anchors (GPA) in expansive soils with the support of 3D Finite Element PLAXIS software. An upward displacement of 10% of pile diameter was applied at the center top of GPA and the corresponding uplift was measured. Different values of length and diameter of the pile were applied to assess their effects on the pile-soil interaction.	The uplift of the pile head was calculated using the numerical method using 3D Finite Element PLAXIS software for the Granular Pile Anchors in expansive soil related to many cases of changing parameters such as the pile length, pile diameter, elastic modulus of soil, elastic modulus of the pile, and spacing of piles in a pile group. Granular Pile Anchors (GPA) reduced the settlement and increased the stability of retaining walls, embankments, and other structures founded on poor soils.

2.4. Literature review discussion

Tables 2,3, and 4 present that most studies conducted in Group 1 (Simplified Analytical Method) usually overestimate the results and are impossible to interpret the soil movement. While Group 2 (Boundary Element Method) and Group 3 (FEM) theories are considerably complex to apply. Crispin et al (2018), Jiang et al (2020), Ju (2015) applied Python for programming Bessel functions with non-integer orders, and other intricate equations to obtain the results. Most theories relate to the complicated theories involving the assumption of a non-linear soil model. With those theories the complete pile's history, the installation processes, the soil's reconsolidation following the installation are possible to be simulated offering high accuracy of results, however, this quires a large number of geotechnical data for supporting. This also requires a thorough consideration of the allowable budget spending for the foundation design, the scale of the project, and the complexity of the geotechnical profile and loading conditions.

Each method has advantages and disadvantages. The evaluation of existing solutions is demonstrated in Table 5.

Table 5. The evaluation of existing solutions

Solutions	Advantages	Disadvantages
Group 1 – Simplified Analytical Method	The theories are not complex. It is convenient to apply the models using this method.	Overestimate the result. It impossible to interpret soil movements.
Group 2 – Boundary Element Method	The results are rather close to the measured values. It is possible to predict soil movements.	Theories are complicated. It is difficult to apply.
Group 3 – Finite Element Method	This is the most powerful analytical approach. High accurate in predicting the soil movements.	It requires a large number of parameters. Theories are significantly intricate to apply.

This study develops a simplified model combining theories in Group 1 and Group 2. Hence, the present model has the potential to overcome the drawbacks of the Simplified Analytical Method.

Furthermore, because of the combination of theories between those two groups, the complicated formulas in Group 2 become easier to apply.

Therefore, the contribution of this study to the pile-soil interactions in expansive soils can be summarised as follows:

- Improving the accurate level of a Simplified Analytical Method compared with the Boundary Element Method and Finite Element Method.
- Supplementing a function of the vertical soil movement prediction for the Simplified Analytical Method.
- Developing equations to solve the pile-soil interaction issue by hand calculations or Microsoft Excel without any support of a computer program including Python, MATLAB, and the like.

CHAPTER III: METHODOLOGY

The major aim of this section is to explain how this model was developed and demonstrate the validation for the model. The process of the model's development is shown in Figure 2.

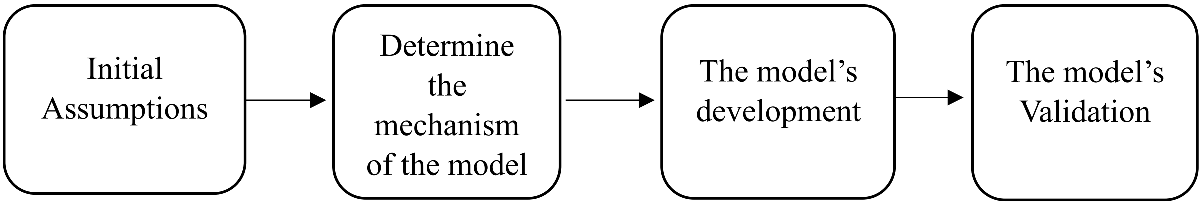


Figure 2. The process of building the model

3.1. Assumptions

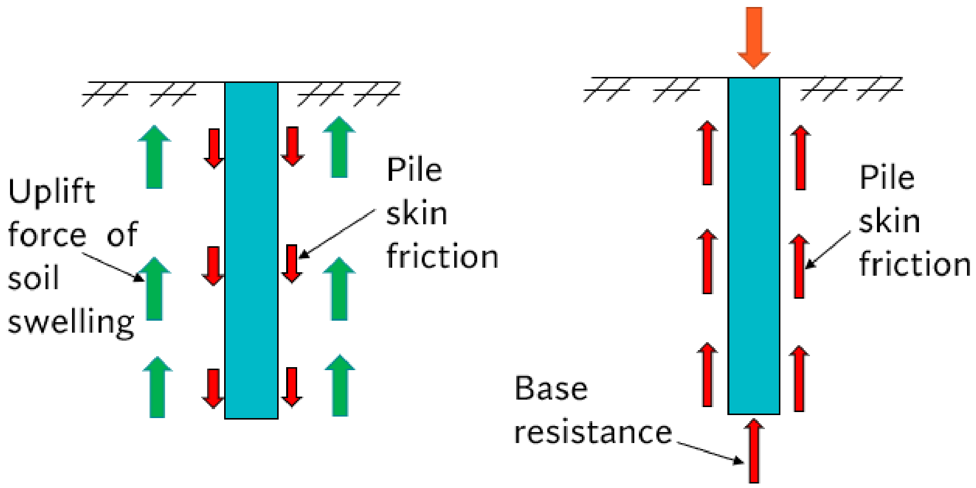
To avoid the complication for the model, initial hypotheses are suggested as follows:

- The load-settlement curve (t-z curve) is elastic perfectly plastic. Initial behavior is elastic, then plastic soon after reaching the yield stress.
- The cross-section of the pile remains unchanged over the entire length of the pile.
- The pile reacts elastically.
- There is no significant load distributed to the pile tip.

3.2. Pile-Soil interaction mechanism in expansive soil

The potential of soil's swelling due to the heave of the expansive soil has a significant effect on the pile-soil interaction. This characteristic distinguishes an expansive soil from other types of soils.

The mechanism of pile-soil interaction in expansive soil is presented in Figure 3. The mechanism was divided into 2 cases. Case 1 presented the pile only under soil's swelling pressure, and Case 2 illustrated the pile only under applied load.



a. Case 1 - Pile only Under soil's swelling pressure

b .Case 2 - Pile only under applied load

Figure 3. Pile-soil interaction mechanism

In Case 1, Pile only under soil's swelling pressure, uplift forces are released due to the swelling of the soil. Those forces cause the upward movements of the pile and its surrounding soils. In reverse, pile skin frictions have a downward direction and resist the movement of the pile.

On the other hand, the heave of the soil may counteract the pile skin friction and be affected by it. The nearer the distance between the pile axis and the soil's position, the larger the counteraction.

In Case 2, Pile only under applied load, the pile will move downwards due to the applied load. In this case, pile skin friction and base resistance of the pile resist the displacement of the pile. However, with Assumption 4 (there is no significant load distributed to the pile tip), the base resistance of the pile can be neglected. This assumption does not affect noticeably on the results because the pile is long enough to transfer the load to its surrounding soil by the friction between the pile and the soil.

Based on the mechanism above, the model was developed to estimate the pile-soil interaction in expansive soil.

3.3. Developments of the model

The present model is based on the elastic perfectly plastic solution for the axially loaded pile shown in Figure 4. There are two stages. Stage 1 is the elastic stage (Zone 2), and Stage 2 is the plastic stage (Zone 1). Z_P is the extent of the plastic zone. The larger the applied load, the larger the extent of the plastic zone. The red curve represents the vertical pile movement, and the blue curve represents the vertical soil movement.

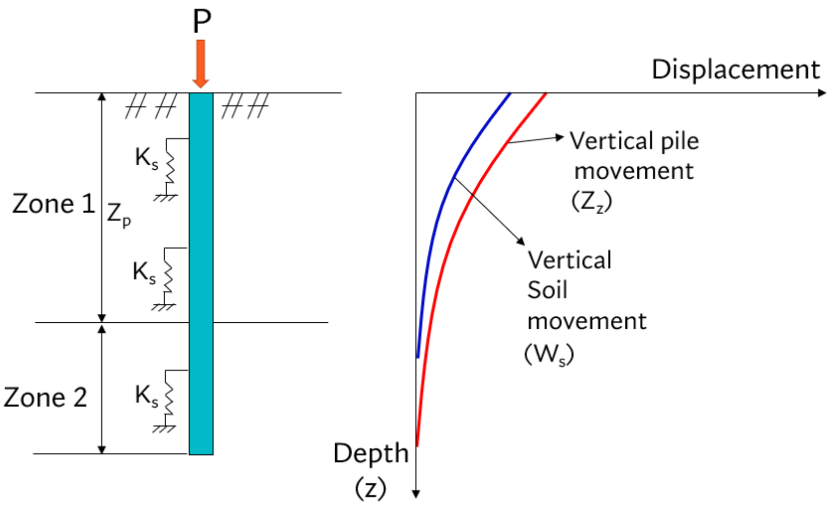


Figure 4. The present model

K_s is the soil's stiffness which is the slope of the t-z curve. The model was based on the elastic perfectly plastic assumption, and K_s is the slope of the t-z curve shown in Figure 5.

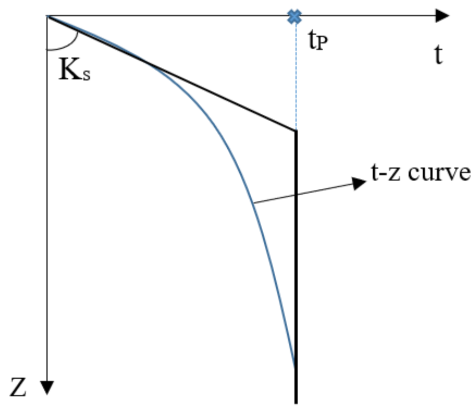


Figure 5. K_s and t-z curve

Where: t_p is the plastic value of the t-z curve

W_p (z) is the pile movement

$W_p(z) = t_p / K_s$ (3)

Applying Hooke law: EA $\frac{d^2_{w_{p(z)}}}{d^2 z}$ = t(z) (4)

Where: E is the Young modulus of the pile

A is the area of the cross-section of the pile

t-z curve is the load-settlement curve.

Assumes the shear strength along the pile shaft is linear with depth, then we have:

$t(z) = t_o + \beta z;$ for $z < z_P$ (5)

Where t_o is the skin friction of the pile shaft at z = 0.

β is the slope of the skin friction.

Z_p is the extent of the plastic zone.

Z is the calculated depth.

$t(z) = K_s W_p(z);$ for $z > z_P$ (6)

Where K_s is the soil's stiffness as shown in Figure 5.

W_p (z) is the pile movement.

- **Case 1: $z < z_P$: Plastic zone**

By Substituting (5) into (4), the equation then becomes:

EA $\frac{d^2_{w_{p(z)}}}{d^2 z}$ = $t_o + \beta z;$ for $z < z_p$ (7)

Rearranging the equation (7), then:

$$\frac{d^2_{w_{p1}(z)}}{d^2 z} = \frac{t_o}{EA} + \frac{\beta\, z}{EA} \qquad (8)$$

After integrating (8), it then becomes:

$$W_{p1}(z) = \int \left(\frac{t_o}{EA} z + \frac{\beta}{2EA} z^2 + c_1\right) d_z = \frac{\beta}{6EA} z^3 + \frac{t_o}{2EA} z^2 + C_1 z + C_2\ (0 \le z \le z_p) \qquad (9)$$

Where C_1 and C_2 are constants and can be obtained by boundary conditions.

At Z = 0 m: $\frac{d_{w_p(z)}}{d_z} = -\frac{P}{EA}$ (P is the axial load at depth z = 0) (10)

Substitute (10) into (9) to have: $\frac{t_o}{EA} z + \frac{\beta}{2EA} z^2 + c_1 = -\frac{P}{EA}$ (11)

By substituting z = 0 into (11), C_1 can be determined, $C_1 = -\frac{P}{EA}$ (12)

On the other hand, substitute equation (9) into (3), the equation then becomes:

$$\frac{\beta}{6EA} z^3 + \frac{t_o}{2EA} z^2 + C_1 z + C_2 = \frac{t_p}{K_S} = \frac{t_o + \beta Z_P}{K_S} \qquad (13)$$

Substitute z = 0 into (13), C_2 can be determined , $C_2 = \frac{t_o + \beta Z_P}{K_S}$ (14)

After substituting (12) and (14) into (9), the vertical pile movement $W_{p1}(z)$ becomes:

$$W_{p1}(z) = \frac{\beta}{6EA} (z^3 - z_p^3) + \frac{t_o}{2EA} (z^2 - z_p^2) - \frac{P}{EA} (z - z_p) + \frac{t_o + \beta z_P}{K_S} \qquad (15)$$

Where $Z_P = \left(\frac{t_o^2}{\beta} + \frac{1}{K_S/EA} + \frac{2P}{\beta}\right)^{1/2} - \frac{t_o}{\beta} - \left(\frac{EA}{K_S}\right)^{1/2}$ (Motta, 1994) (16)

Furthermore, According to Crispin et al., (2018),

$$K_S = \frac{2\pi G_s}{\ln\left(\frac{2r_m}{D_S}\right)} \qquad (17)$$

Where: G_s is the soil shear modulus at depth z

r_m is the empirical radius

$$r_m = \frac{2.5\,L\,(1-V_s)G_s(L/2)}{G_s(L)} \quad (18)$$

D_s is the pile diameter

Vs is the Poisson's ratio of the soil

G_s (L/2) is the soil shear modulus at depth z = L/2

G_s (L/2) is the soil shear modulus at depth z = L

L is the length of the pile

By Substituting (17) into (15), the equation then becomes:

$$W_{p1}(z) = \frac{\beta}{6EA}(z^3 - z_p^3) + \frac{t_o}{2EA}(z^2 - z_p^2) - \frac{P}{EA}(z - z_p) + \frac{\frac{t_o+\beta z_P}{2\pi G_S}}{\ln\left(\frac{2rm}{D_S}\right)} \quad (19)$$

After rearranging (19), the vertical pile movement $W_{p1}(z)$ becomes:

$$W_{p1}(z) = \frac{\beta}{6EA}(z^3 - z_p^3) + \frac{t_o}{2EA}(z^2 - z_p^2) - \frac{P}{EA}(z - z_p) + \frac{t_o+\beta z_P}{2\pi G_S}\ \ln\left(\frac{2r_m}{D_S}\right) \quad (20)$$

Axial force can be derived from Hooke's law:

$$\frac{d_{w_p(z)}}{d_z} = -\frac{P(z)}{EA} \quad (21)$$

By rearranging (21), the aixal force is as the following equation:

$$N_1(z) = P(z) = -EA\,\frac{d_{w_p(z)}}{d_z} \quad (22)$$

After substituting (20) to (22), the axial force becomes:

$$N_1(z) = -EA\left(\frac{\beta}{2EA}z^2 + \frac{t_o}{EA}z - \frac{P}{EA}\right) \quad (23)$$

By executing the multiplication in equation (23), the axial force can be obtained:

$$N_1(z) = \frac{-\beta}{2}z^2 - t_o z + P \quad (24)$$

For the settlement of the pile head, substitute z = 0 into equation (20),

Now: $\Delta = \frac{\beta}{6EA}(-z_p^3) + \frac{t_o}{2EA}(-z_p^2) - \frac{P}{EA}(-z_p) + \frac{t_o+\beta z_P}{2\pi G_s}\ln\left(\frac{2r_m}{D_S}\right)$ (25)

- **Case 2: z > z_p: Elastic zone**

By substituting (6) into (4), the equation then becomes:

$$EA\,\frac{d^2_{w_{p2}(z)}}{d^2z} = K_s\,W_p(z); \qquad \text{for } z > z_p \tag{26}$$

After rearranging, the equation (26) becomes:

$$W''_{p2z} - \frac{K_s}{EA}W_{p2}(z) = 0 \tag{27}$$

(27) has the characteristic equation as follow:

$$r^2 - \frac{K_s}{EA} = 0 \tag{28}$$

Solutions for the equation (28) is presented below:

$$r = -\left(\frac{K_s}{EA}\right)^{1/2} \text{ or } r = \left(\frac{K_s}{EA}\right)^{1/2} \tag{29}$$

Based on (29), the solution for the differential equation (27) can be derermined:

$$W_{p2}(z) = C_3\,e^{-z\left(\frac{K_s}{EA}\right)^{1/2}} + C_4\,e^{z\left(\frac{K_s}{EA}\right)^{1/2}} \tag{30}$$

Where C_3 and C_4 are constants.

Assume the extent of the plastic zone is sufficiently large, then C_4 can be ignored without any appreciable error.

Now, (30) becomes: $W_{p2}(z) = C_3\,e^{-z\left(\frac{K_s}{EA}\right)^{1/2}}$ (31)

By substituting the equation (3) into (31), the equation then becomes:

$$C_3\, e^{-z\left(\frac{K_S}{EA}\right)^{1/2}} = \frac{t_p}{K_S} = \frac{t_o+\beta Z_P}{K_S} \tag{32}$$

After substituting z = 0 into (32), C_3 can be obtained:

$$C_3 = \frac{t_o+\beta Z_P}{K_S} \tag{33}$$

Then, substitute this C_3 to (31) to get:

$$W_{p2}(z) = \frac{t_o+\beta Z_P}{K_S}\, e^{(z_p-z)\left(\frac{K_S}{EA}\right)^{1/2}} \tag{34}$$

Follow the same steps with Case 1 to get the axial force:

$$N_2(z) = \frac{t_o+\beta Z_P}{\left(\frac{K_S}{EA}\right)^{1/2}}\, e^{(z_p-z)\left(\frac{K_S}{EA}\right)^{1/2}} \tag{35}$$

Substitute z = 0 into (35) to get the settlement of the pile head:

$$\Delta = \frac{t_o+\beta Z_P}{K_S}\, e^{(z_p)\left(\frac{K_S}{EA}\right)^{1/2}} \tag{36}$$

The solutions for pile-soil interactions: the vertical pile movement (Equation 20 and 34), the axial load (Equation 24 and 35), and the settlement of the pile head (Equation 25 and 36) are summarized in Table 6.

Table 6. Pile-soil interaction solutions

Outputs	Plastic Zone ($Z < Z_p$)	Elastic Zone ($Z > Z_p$)
Vertical pile movement	$W_{p1}(z) = \frac{\beta}{6EA}(z^3 - z_p^3) + \frac{t_o}{2EA}(z^2 - z_p^2) - \frac{P}{EA}(z - z_p) + \frac{t_o+\beta z_P}{2\pi G_s}\ln\left(\frac{2r_m}{D_S}\right)$	$W_{p2}(z) = \frac{t_o+\beta Z_P}{K_S}\, e^{(z_p-z)\left(\frac{K_S}{EA}\right)^{1/2}}$
Axial load	$N_1(z) = \frac{-\beta}{2}z^2 - t_o z + P$	$N_2(z) = \frac{t_o+\beta Z_P}{\left(\frac{K_S}{EA}\right)^{1/2}}\, e^{(z_p-z)\left(\frac{K_S}{EA}\right)^{1/2}}$
Settlement of the pile head	$\Delta = \frac{\beta}{6EA}(-z_p^3) + \frac{t_o}{2EA}(-z_p^2) - \frac{P}{EA}(-z_p) + \frac{t_o+\beta z_P}{2\pi G_s}\ln\left(\frac{2r_m}{D_S}\right)$	$\Delta = \frac{t_o+\beta Z_P}{K_S}\, e^{(z_p)\left(\frac{K_S}{EA}\right)^{1/2}}$

3.4. Vertical soil movements

As explained in the previous sections, the Simplified Analytical Method (Group 1) cannot interpret the vertical soil movement, while The Boundary Element Method (Group 2) and the Finite Element Method (Group 3) are considerably complex to apply.

The study used the existing formulas of studies in Group 2, then adjusted them to become more simple to apply.

- **Case 1: Under only soil's swelling pressure**

Jiang et al., (2020) presented:

$$W_{s1} = s_o\left(\frac{z}{h_o} - 1\right) + \frac{\tau_{1z} r_o}{G_s} \ln\left(\frac{r_m}{r}\right) \quad (37)$$

Where: W_{s1} is the vertical soil movement under only soil's swelling pressure

s_o is the soil heave at z = 0 m

h_o is the active depth

Where: G_s is the soil shear modulus at depth z

r_m is the empirical radius

r is the distance from the pile axis to the calculated position.

τ_{1z} is the pile's skin friction

Furthermore, $\tau_{1z} = \frac{G_s}{r_o \xi}\left(s_o\left(1 - \frac{z}{h_o}\right) + \mathrm{Wp(z)}\right)$ (38)

Where r_o is the radius of the pile

ξ is the effective parameter related to the pile radius

$\xi = \ln (r_m / r_o)$ (39)

By substituting (38) into (37), the equation then becomes:

$$W_{s1} = s_o\left(\frac{z}{h_o} - 1\right) + \frac{s_o\left(1 - \frac{z}{h_o}\right) + w_P(z)}{\xi} \ln\left(\frac{r_m}{r}\right) \quad (40)$$

Where W_{pz} is determined from Table 5

- **Case 2: Under only applied load**

$$W_{s2} = \frac{\tau_{2z} r_o}{G_s} \ln\left(\frac{r_m}{r}\right) \quad \text{(Jiang et al., 2020)} \tag{41}$$

τ_{2z} is the pile's skin friction determined as following equation

$$\tau_{2z} = \frac{G_s}{r_o \xi} W_P(z) \tag{42}$$

After substituting (42) into (41), the vertical soil movement becomes:

$$W_{s2} = \frac{W_p(z)}{\xi} \ln\left(\frac{r_m}{r}\right) \tag{43}$$

Solutions for vertical soil movement, The Equation (40) and (43) are summarized in Table 7. Those formulas are easier to apply compared with the original ones.

Table 7. Vertical soil movements

Under only soil swelling pressure	Under only applied load
$W_{s1} = s_o\left(\frac{z}{h_o} - 1\right) + \frac{s_o\left(1 - \frac{z}{h_o}\right) + w_P(z)}{\xi} \ln\left(\frac{r_m}{r}\right)$	$W_{s2} = \frac{W_p(z)}{\xi} \ln\left(\frac{r_m}{r}\right)$

3.5. The model's validation

After developing the model, four case studies of previous studies which are in Group 2 and Group 3 were applied, then the results generated from the present model are compared with the results from the previous studies and results from the RSPile software.

The process of the model's validation is displayed in Figure 6.

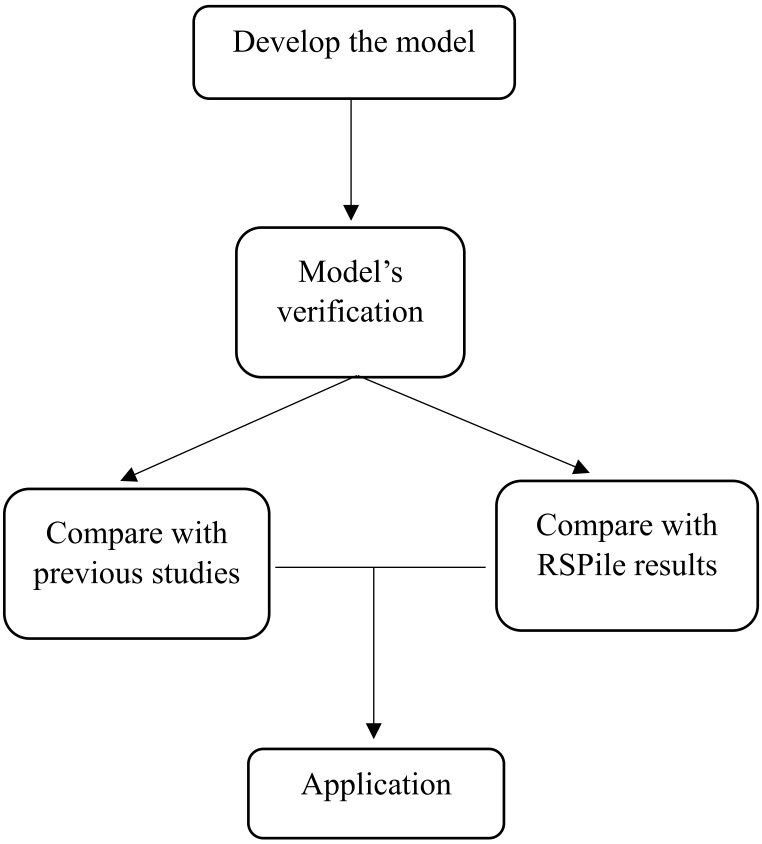


Figure 6. The flow chart of the model development

The input parameters remained unchanged in four case studies. Then, those parameters were simulated by the model. Besides, those parameters were also be used to model in RSPile Software. Ultimately, the results generated from the present model were compared with the results from other complicated models and RSPile software. A good agreement between the model with previous studies, and RSPile indicates that the model can be contributed to estimate pile-soil interaction in expansive soils.

3.6. RSPile software

RSPile is a program developed by Rocscience. RSPile has been widely used to analyse the pile-soil interaction under axial loading and lateral loading. In this report, RSPile results generated from the RSPile models were used to validate the present model.

The Load-Settlement curve (t-z curve)

RSPile using Finite Element Analysis estimates the t-z curve through three loading mechanisms: the axial deformation, soil skin friction along the shaft, and the soil end-bearing resistance as shown in Figure 7.

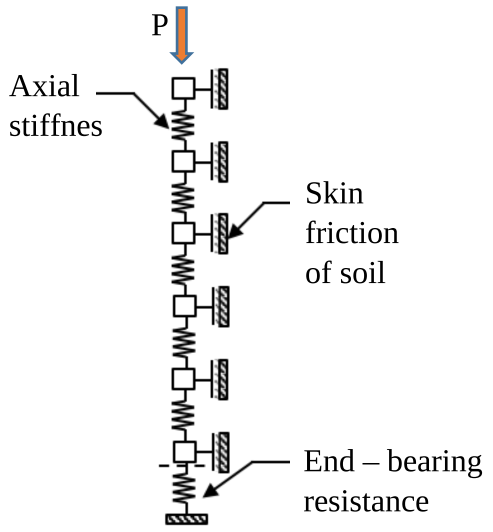


Figure 7. The concept of RSPile model (Rocscience, 2021)

A Spring-mass model based on Finite Element Analysis allows RSpile to simulate the non-linear stiffness curve to approach the non-linear stress-strain behavior of the soil. In this model, Springs represent the stiffness of the pile. Pile's stiffness can be modeled as a function that varies with depth. Besides, numerical techniques are employed to determine solutions for the t-z curve. Hence, RSPile has the ability to provide high accurate interpretations of the pile-soil interaction in the expansive soil including axial loads, vertical pile movements, and settlement of the pile head.

Vertical soil movements

The vertical soil movement and the pile depth are measured along the pile axis considering the rotation because of the batter angle. RSPile assumes non-displacement under the sliding depth. The original displacement and defined displacement profile are shown in Figure 8.

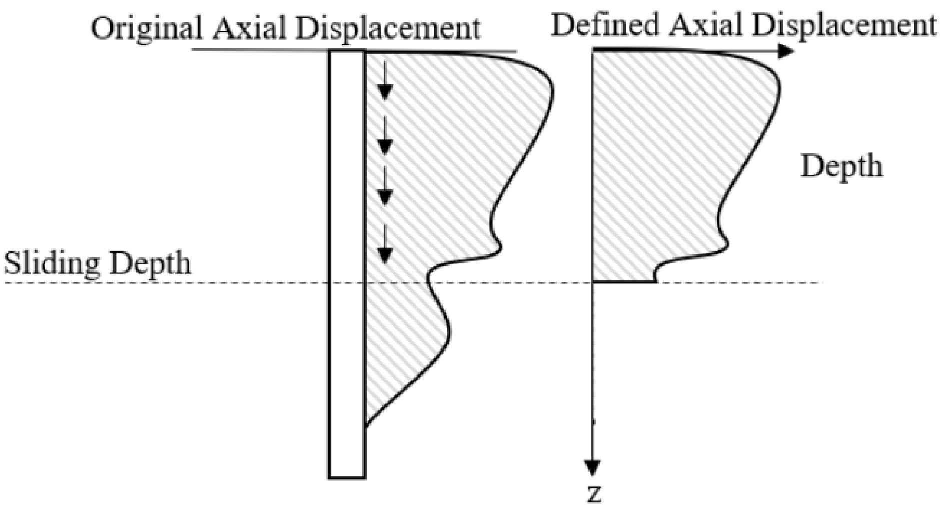


Figure 8. Axial displacement profile (Rocscience, 2021)

The pile was divided into segments including two pile elements and one soil shear element demonstrated in Figure 9. Each soil shear element represents the skin friction's effect between the pile and soil.

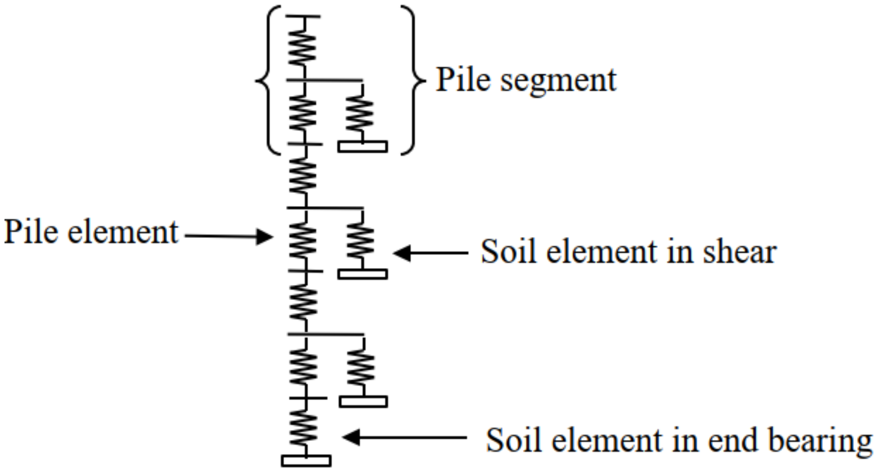


Figure 9. Pile emlements and soil elements (Rocscience, 2021)

To calculate the vertical soil movement, four assumptions are applied.

- The pile's geometry remains unchanged along the depth.
- There are no eccentric loads.
- The pile deformation is not significant during the simulation.
- The pile material is homogeneous.

In the case of the soil displacement profile is uniform, RSPile simulates the relationship between the soil displacement and sliding depth is as Figure 10.

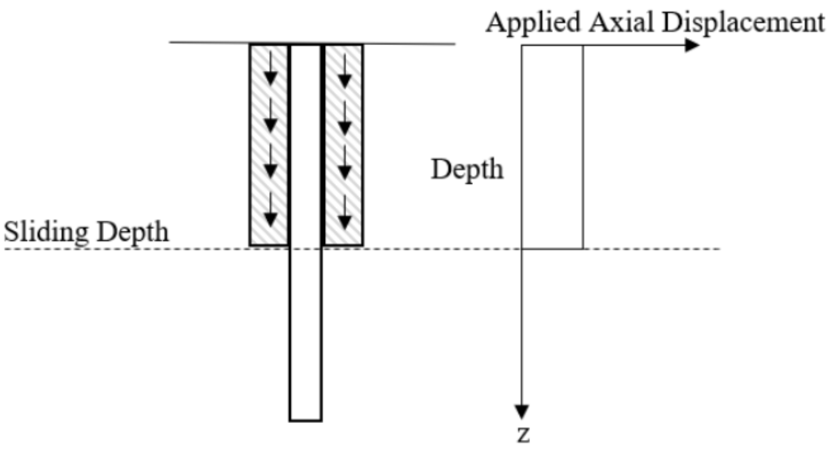


Figure 10. Uniform soil displacement profile (Rocscience, 2021)

Free body diagram of the pile segment

Equations for the load transfer of the externally applied loads to the skin friction between the pile shaft and the surrounding soil, and the pile deformation can be obtained from the free body diagram of the pile segment illustrated in Figure 11.

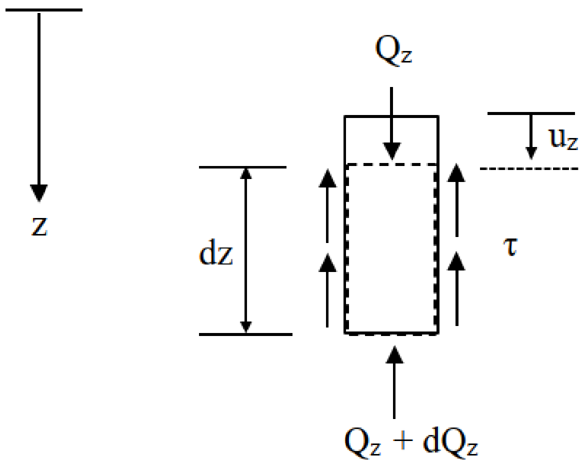


Figure 11. Free body diagram of the pile segment (Rocscience, 2021)

3.8. Criteria for good results

This research aims to offer a Simplified Analytical Method to analyse the pile-soil interaction in expansive soil within an acceptable range compared with Boundary Element Method and Finite Element Method. Therefore, expected results including the vertical pile movements, vertical soil movements, settlement of the pile head, axial loads should satisfy the following criteria.

- Criterion 1: Results are produced based on simple theories.
- Criterion 2: Results can be produced with the support of Excel without any further program.
- Criterion 3: The trends of the graphs generated by the present model are consistent with the graphs determined by Boundary Element Method and Finite Element Method.
- Criterion 4: Results generated by the present model are less than 15% difference compared with Boundary Element Method, Finite Element Method, and RSPile's model.
- Criterion 5: Results generated by the present are within the lower bound and upper bound results come from the Finite Element Method.

Boundary Element Method and Finite Element Method require the support of a program. For instance, MATLAB or Python are commonly used in those methods to solve the Bessels functions and other complicated theories.

Formulas developed in this research were applied the simplified analytical method, and formulas can be easily proved by using differential equations and double integrals. Hence, Criterion 1 and 2 are met for all the results presented in Chapter IV.

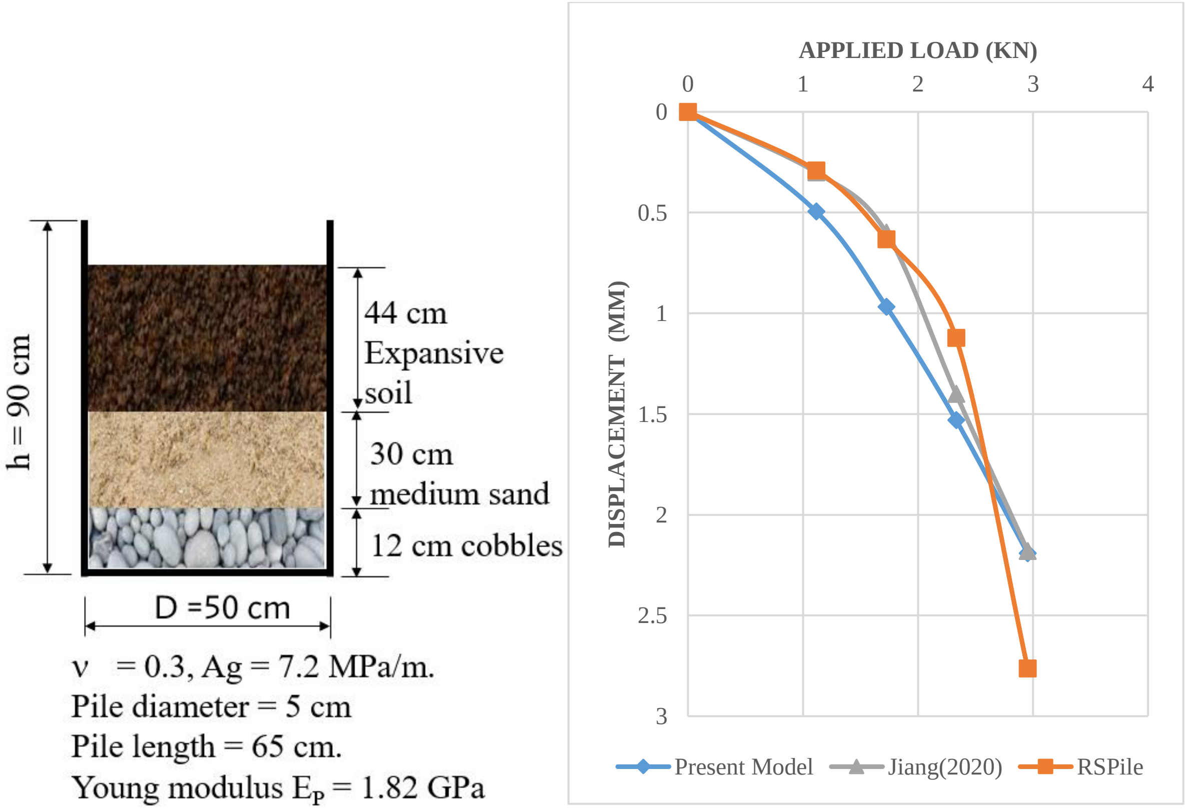
Thereupon, good results should meet criteria 3, 4, and 5. The next chapter demonstrates four case studies, all of whose results are compared with those criteria.

CHAPTER IV: RESULTS AND DISCUSSION

The model is used to analyse the pile-soil interaction under axial loading applying four case studies conducted by previous researchers, then the results from the model are compared with the results of previous studies and RSPile software. RSPile' models are presented in the Appendices.

4.1. Case Study 1

This study used input parameters in a test carried out by Jiang et al (2020) in Nanning expansive soil. A cylinder jerrican with a diameter of 50 cm, and a height of 90 cm contains three layers of soil. The PVC pipe was sticked 10 pieces of strain-gauges. The input data and results are shown in Figure 12 and Table 8 below.



a. Inutput parameters *b. Results*

Figure 12. Case Study 1-Input parameters and results

Table 8. Results-Case Study 1

Present model		(Jiang, 2020)	RSPile
P	Δ	Δ	Δ
kN	mm	mm	mm
0	0	0	0
1.1	0.4942	0.3	0.291
1.7	0.9673	0.6	0.6325
2.3	1.53	1.4	1.1225
2.9	2.1921	2.18	2.7638

Table 8 and Figure 12 present the results of the t-z curve of the present model, Jiang's model, and the RSPile's model. Figure 12 demonstrates that the present model is rather close to the model of Jiang (2020), and RSPile software, implicating that the present model can be used to estimate the pile-soil interaction in Nanning expansive soil with a high level of accuracy compared with those models.

All models describe that the increase of the applied load causes the increase of the settlement of the pile head. When the applied load develops from 0 kN to 2.3 kN, the settlement of the pile head also increases from 0 mm to 1.12 mm (RSPile), 0 mm to 1.4 mm (Jiang, 2020), and 0 mm to 1.53 mm (the present model). The variation between the present model with the model of Jiang is 8% in this case (less than 15% compared with the initial criteria).

When the applied load continues rising to 2.9 kN, the settlement of the pile head approaches 2.7 mm (RSPile), 1.18 mm (Jiang, 2020), and 2.19 mm (the present model). The difference between the present model with the model of Jiang is 1% in this case (less than 15% compared with the initial initial criteria).

Those results have met the criteria presented in Chapter III. The results of axial loads vary with depth generated by the present model are presented in Figure 13.

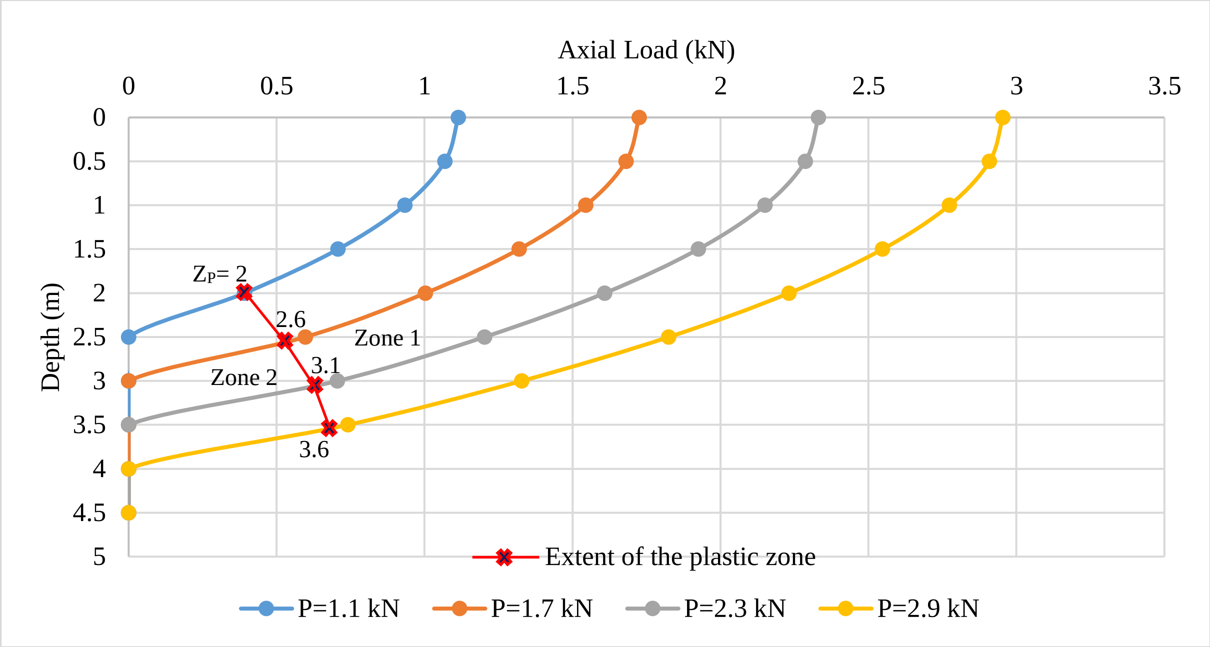


Figure 13. Axial loads vary with depth-Case Study 1

Figure 13 shows that the axial loads of the pile decrease with the depth. The red line illustrates the limit between plastic and elastic behaviour. Zone 2 represents the elastic zone, and Zone 1 represents the plastic zone.

The increase of applied axial load (P) causes the increase of the extent of the plastic zone (Z_p), and the pile axial load (N).

The extent of the plastic zone, Z_p = 2 m regarding the applied load P = 1.1 kN. The increment of the applied load from 1.1 kN to 1.7 kN, 2.3 kN, and 2.9 kN resulted in the proportional increase of the extent of the plastic zone from 2 m to 2.6 m, 3.1 m, and 3.6 m, respectively.

At the ground level, the axial loads (N) approach the maximum value. Those values are equal to the applied load (P = N).

The axial loads decrease with depth, and get the minimum value (N = 0) at the depth z = 2.5 m (P = 1.1 kN), z = 3 m (P = 1.7 kN), z = 3.5 m (P = 2.3 kN), and z = 4 m (P = 2.9 kN).

In Zone 2 (Elastic zone), the pile behaves elastically. On the other hand, the pile response to the applied load was plastic in Zone 1 (Plastic zone).

The vertical soil movements are investigated in 3 cases: Under only soil swelling pressure, Under different applied loads, and Under different values of the calculated position. Results are presented in Figures 14, 15, and 16, respectively.

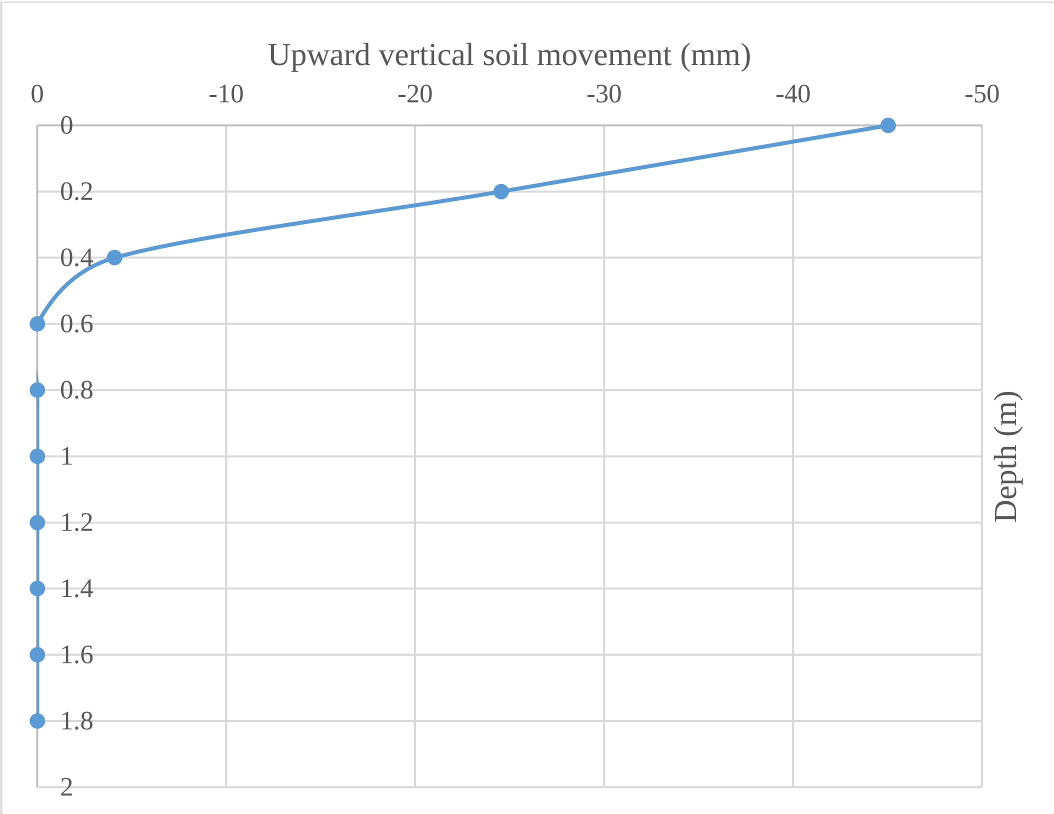


Figure 14. Vertical soil movements under only soil swelling pressure-Case Study 1

Figure 14 shows that under the pressure of the soil's swelling, the pile moves upwards, and is approximately linear with the depth.

The upward movement of the soil has the greatest value at the ground level (W_s = - 45 mm). The minus sign (-) represents the upward movement.

The relationship between the upward movement with the depth is inversely proportional. With the increase of the depth from 0 m to 0.2 m, and 0. 4 m, the upward movement decreases from -45 mm to -24.5 mm, and -4 mm, respectively. This movement approaches the value of 0 mm at the depth of 0.6 m.

In this case, the soil reacts elastically from the depth of 0 m to 0.4 m. Under this region, the soil behaves plastically until getting the value of 0 mm of the upward movement.

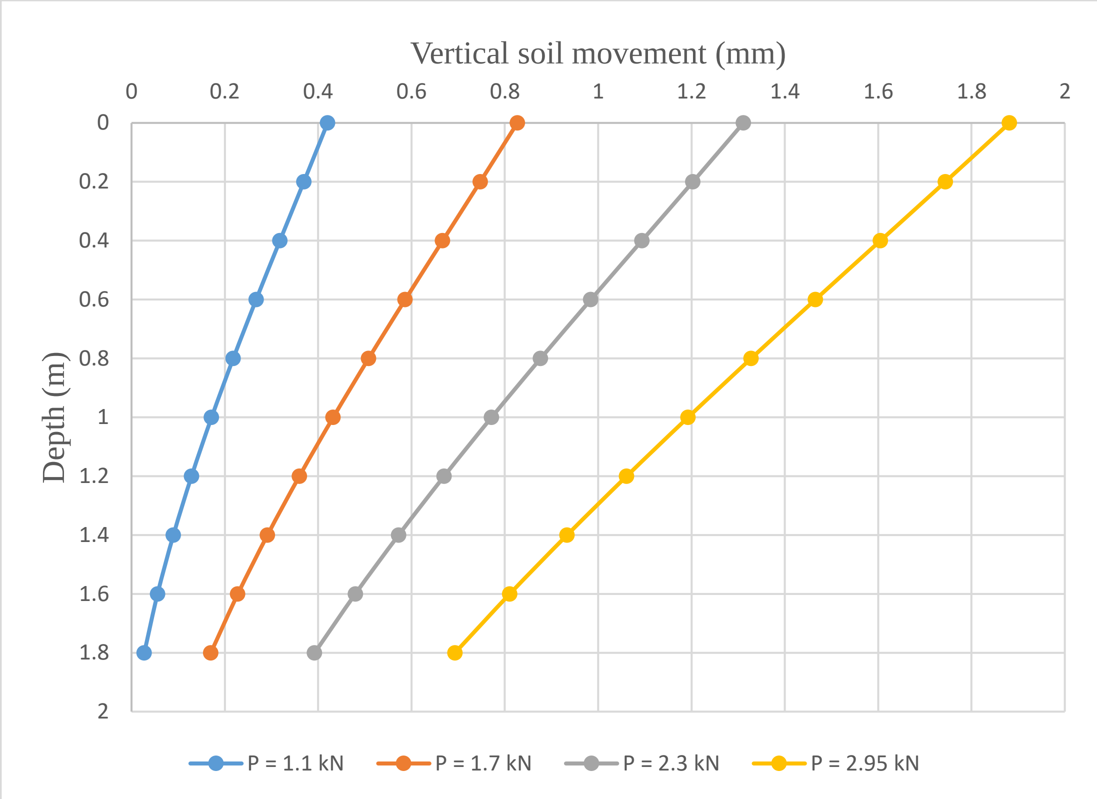


Figure 15. Vertical soil movements under different applied loads-Case Study 1

Figure 15 illustrates that under the applied loads, the soil moves downwards. This movement approaches the maximum value at the depth, z = 0 m. When the applied load, P = 1.1 kN, the maximum vertical soil movement is 0.42 mm (Ws-max = 0.42 mm). This value increases with the increase of the applied load, Ws-max = 0.83 mm (P = 1.7 kN), Ws-max = 1.31 mm (P = 2.3 kN), and Ws-max = 1.9 mm (P = 2.95 kN).

When the applied load is not large, the soil displacement around the pile tip is trivial. However, when the applied load increases to 2.3 kN, and 2.95 kN, those soil elements' movement increases to 0.4 mm, and 0.7 mm, respectively.

Within the range of applied load from 0 kN to 2.95 kN, the soil still responses elastically in approximation.

On the other hand, the vertical soil movements are heavily dependent on the distance from the pile axis to the calculated position. This relationship is demonstrated in Figure 16.

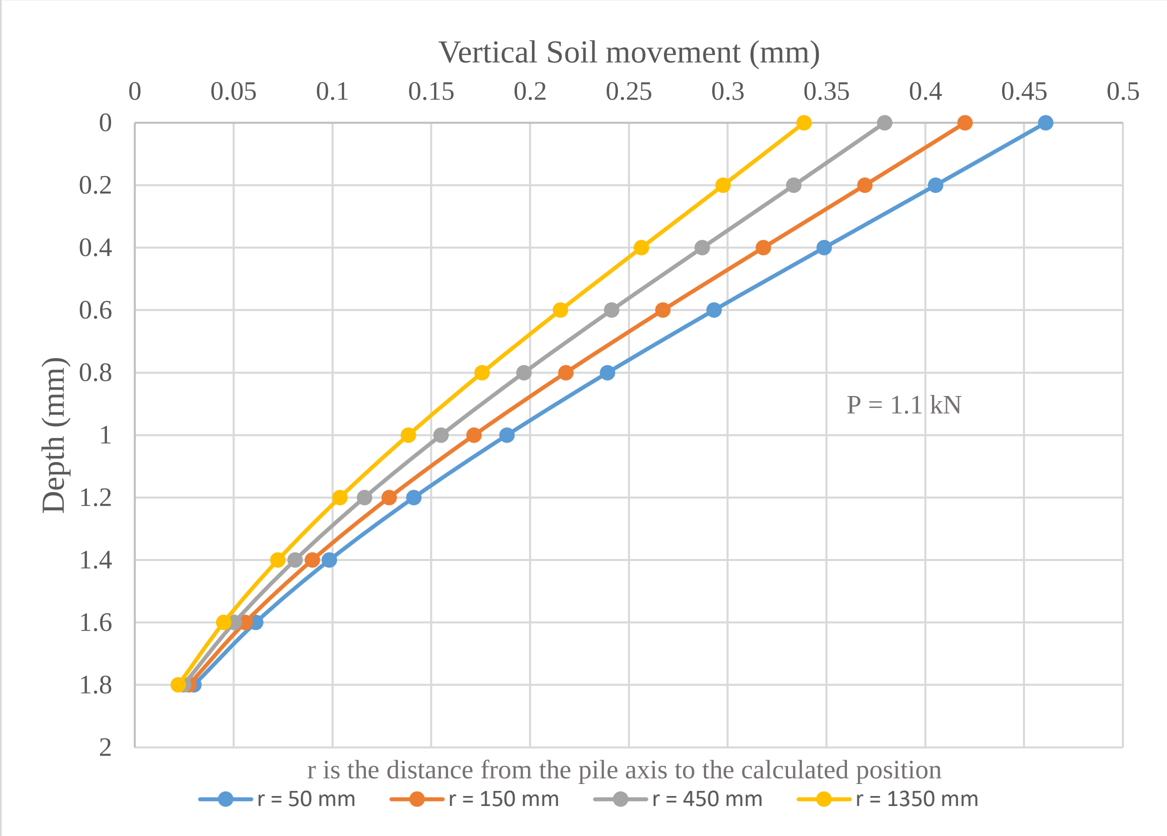


Figure 16. Vertical soil movements under different values of r-Case Study 1

Figure 16 illustrates that soil elements that are closer to the pile axis move deeper compared with the farther elements. Most differences in the vertical soil movements occur at the ground level. When r = 50 mm, the vertical soil movement, Ws = 0.46 mm. This value decreases to 0.42 mm, 0.38 mm, and 0.34 mm when the distance r increases to 150 mm, 450 mm, and 1350 mm, respectively.

The differences of the vertical soil movement in terms of the changes of the distance r reduce with the depth. At the depth of 1.8 m, those differences are extremely minor.

4.2. Case Study 2

Case Study 2 used input parameters in a test conducted in Queensland by Poulos (1989). The pile length, diameter, and Young's modulus are 12.2 m, 0.275m, and 35000 MPa, respectively. Other input parameters are shown in Table 9. The results are shown in Table 10 and Figure 17.

Table 9. Input parameters-Case Study 2

Depth (m)	Soils	Cone Resistance (MN/m^2)	E_s (MPa)	C_u (kPa)
0-4	Soft Clay	1	25	10
4-8	Lose sand	2	50	20
8-21	Soft Clay	2	50	70
21-26	Stiff Clay	3	75	110
26-31.5	Dense sand	14	350	150

Table 10. Results-Case Study 2

	Upper Bound Poulos model	Lower Bound Poulos model	Present model	RSPile
P (kN)	Δ (mm)	Δ (mm)	Δ (mm)	Δ (mm)
0	0	0	0	0
1000	3.00	7.50	7.54	7.70
2000	15.00	22.00	21.32	23.00
2500	31.00	50.00	29.80	31.00

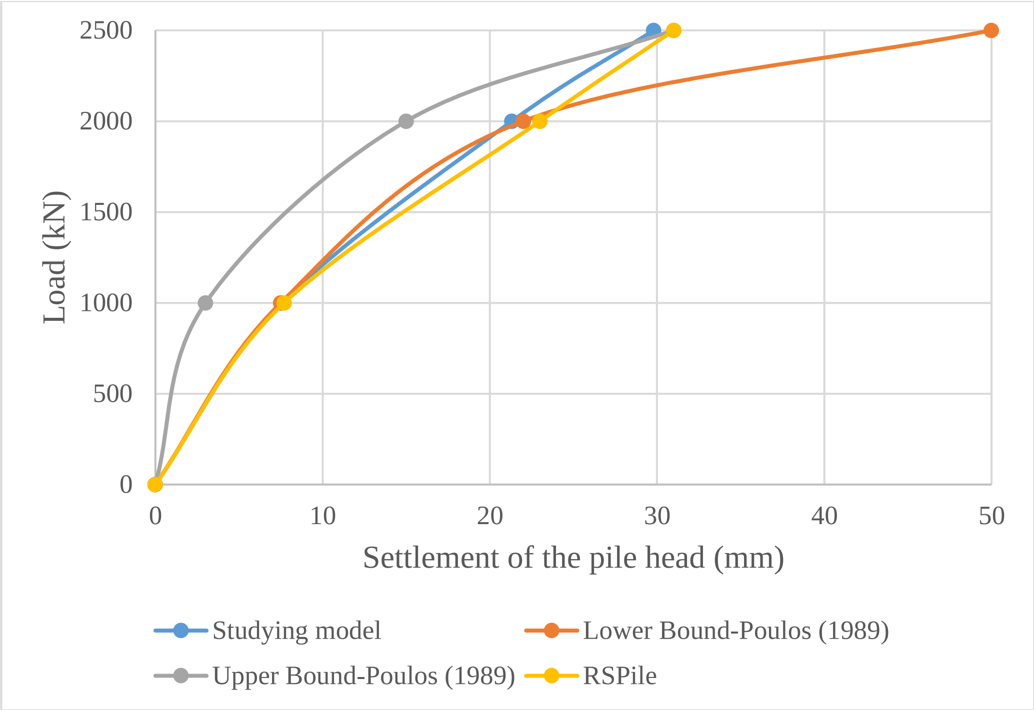


Figure 17. Load-settlement curve-Case Study 2

Table 10 and Figure 17 demonstrate that the present model is within the range of the lower bound and the upper bound of Poulos' model, and it is almost identical to the RSPile results.

When the applied load increases to 1000 kN (P = 1000 kN), the settlement of the pile head generated by the present model is 7.54 mm (Δ = 7.54 mm), while this value is 7.5 mm in the case of the lower bound of Poulos's model giving the variation of 0.5%, and Δ = 7.7 mm generated by RSPile providing a difference of 2% (satisfied the initial criteria)

With P = 2000 kN, Δ = 21.32 mm (the present model), Δ = 22 mm (Lower bound of Poulos), and Δ = 23 mm (RSPile). The variation between the present model with Poulos's model, and RSPile's model is 3%, and 8%, respectively (satisfied the initial criteria).

If the applied load continues increasing to 2500 kN, the settlement of the pile head increases proportionally, Δ = 29.8 mm (the proposal model), Δ = 31 mm (Upper bound of Poulos), and Δ = 31 mm (RSPile). The variation between the present model with Poulos's model, and RSPile's model is both 4% (satisfied the initial criteria).

The results proved that the t-z curve can be estimated by the present model, which is easy to apply and does not require a large number of geotechnical parameters.

4.3. Case Study 3

Case Study 3 is another study that was originally conducted by Poulos (1989). This Case Study is about vertical pile movements that vary with different pile diameters. The input parameters and results are shown in Figure 18, Table 11, and Figure 19.

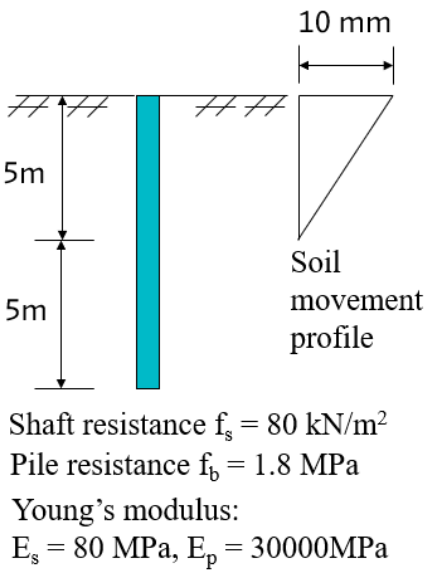


Figure 18. Case Study 1-Input parameters

Table 11. Results-Case Study 3

Diameter	Upper Bound Poulos-model	Lower Bound Poulos-model	Present model	RSPile
D (m)	W_p (mm)	W_p (mm)	W_p (mm)	W_p (mm)
0	0	0	0	0
0.5	2.5	1.8	2.27	2.1
1	2.1	1.8	1.88	2.1
1.5	2.1	1.8	1.87	2.1
2	2	1.9	2	2.2

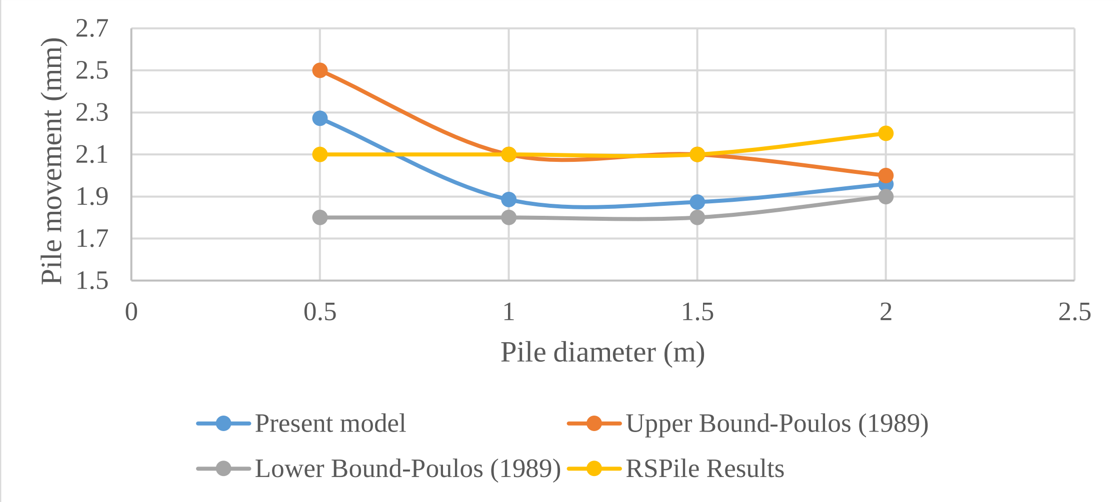


Figure 19. Results-Case Study 3

Table 11 and Figure 19 present that the present model is within the range of minimum and maximum predictions of Poulos's model. It is also similar to the RSPile results.

When the diameter of the pile is 0.5 m (D = 0.5 m), the vertical pile movement generated by the present model is 2.27 mm ($W_p = 2.27$ mm) while this value is 2.5 mm ($W_p = 2.5$ mm) in the case of the upper bound of Poulos's model giving the variation of 10%, and $W_p = 2.1$ mm generated by RSPile providing a difference of 7% (satisfied the initial criteria).

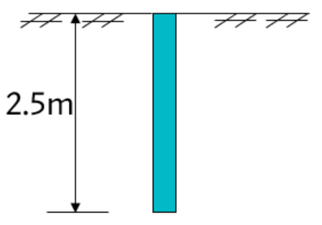
With D = 1m, $W_p = 1.9$ mm (the proposal model), $W_p = 1.88$ mm (Lower bound of Poulos), and $W_p = 2.1$ mm (RSPile). The variation between the present model with Poulos's model, and RSPile's model is 5%, and 11%, respectively (satisfied the initial criteria).

If the pile's diameter continues increasing to 1.5 m, the vertical pile movement has a minor change, $W_p = 1.87$ mm (the proposal model), $W_p = 1.8$ mm (Upper bound of Poulos), and $W_p = 2.1$ mm (RSPile). The variation between the present model with Poulos's model, and RSPile's model is 5%, and 11%, respectively (satisfied the initial criteria).

The model demonstrates that diameter has a minor effect on the vertical pile movement which is consistent with the model of Poulos and RSPile.

4.4. Case Study 4

Case Study 4 used the input data from a case study conducted by Kuwabara (1991) was about vertical soil movement. The input data is shown in Figure 20; The results are shown in Table 12, Figure 20, and Figure 21.



$E_p/G = 3000$; $v = 0.5$

Young's modulus:

$E_p = 27000$MPa

$D = 0.1$ m; $r_o = 0.05$m

Figure 20. Input parameters-Case Study 4

Table 12. Results-Case Study 4

r/r_o	Present model	Kuwabara (1991)	RSPile
	W_s/D	W_s/D	W_s/D
1	0.78	0.8	0.88
2	0.65	0.6	0.7
5	0.47	0.46	0.54
10	0.34	0.33	0.38
20	0.21	0.21	0.25
50	0.04	0.1	0.1

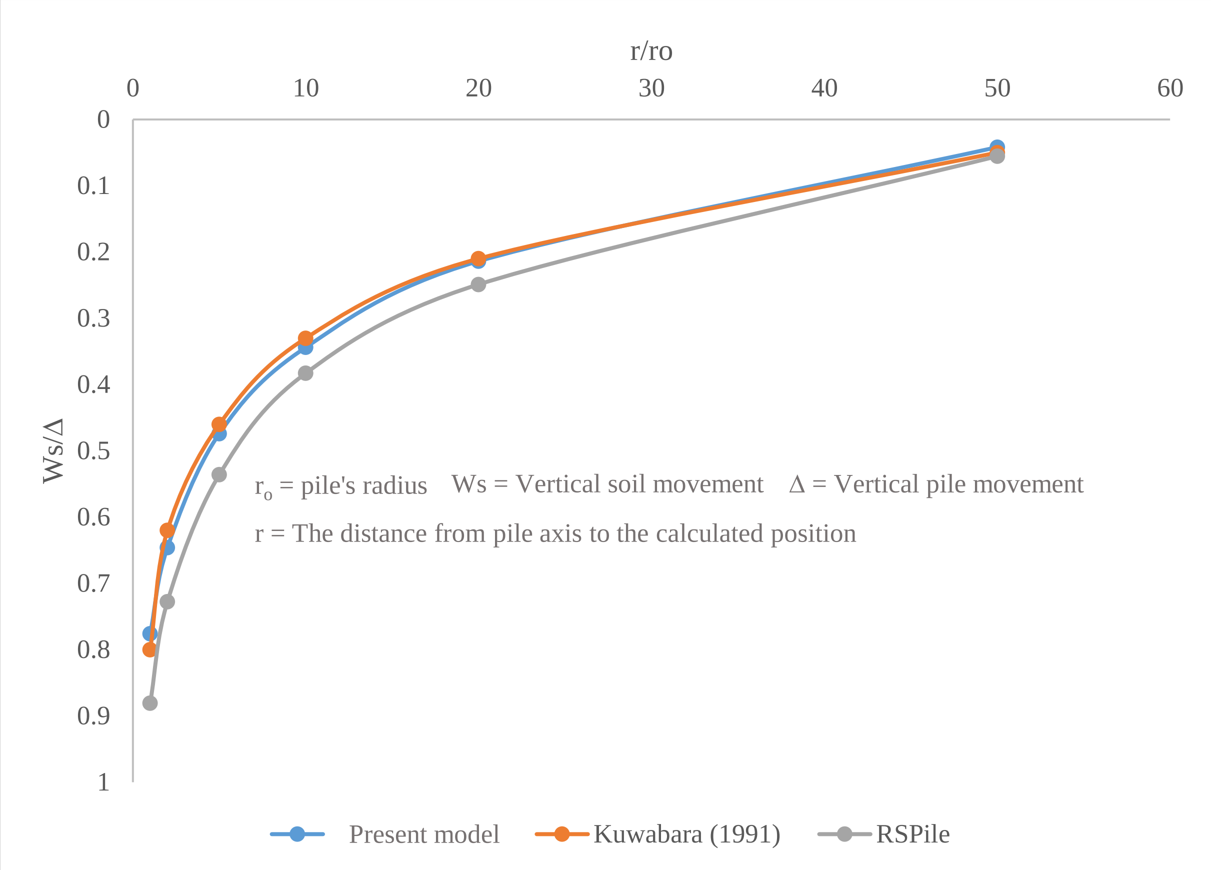


Figure 21. Vertical soil movements-Case Study 4

Table 12 and Figure 21 present that the vertical soil movements are interpreted with a high level of accuracy compared with (Kuwabara, 1991) and RSPile results so that the model can be used to estimate the vertical soil movements, and other solutions demonstrated in the previous case studies.

All the models in Figure 21 illustrate that the vertical soil movement is inversely proportional with the ratio r/r_o implicating that soil's elements that are closer to the pile will move deeper than the further elements. When the ratio $r/r_o = 1$, $Ws/\Delta = 0.78$ (The present model), $Ws/\Delta = 0.8$ (Kuwabara's model), and $Ws/\Delta = 0.88$ (The RSPile's model). The increase of the ratio r/r_o resulted in a decrease of the vertical soil movent. The average difference between the present model with the model of Kuwabara, and the model of RSPile is 3%, and 13%, respectively (satisfied the initial criteria).

CHAPTER V: CONCLUSIONS AND FUTURE WORK

5.1. Conclusion

This study has developed a simplified model to predict the pile-soil interaction in expansive soil with the results being close to other intricate models generated by the Boundary Element Method and the Finite Element Method. Results from the present model have met the initial criteria.

The developed model can account for the following issues:

- The relationship between load and settlement.
- Vertical pile movements.
- Vertical soil movements.
- Settlement of the pile head.
- Axial loads.

The present model overcomes the drawbacks of a Simplified Analytical Method:

- The results are not close to the measured values.
- It is impossible to predict the vertical soil movement

The advantages of the model are listed below:

- Easy to apply.
- Does not require a large number of data.
- Offers accurate results compared with the Boundary Element Method, and the Finite Element Method.

There is a good agreement between the present model with RSPile results and previous models. The results from the model have the same interpretations for the pile-soil interaction under the axial loading described as follows:

- In the case of under only soil's swelling pressure (no applied load), the soil heave will be produced resulted in an upward movement for the pile and its surrounding soil.
- An increase in the applied load causes the increment of the settlement of the pile head, the vertical pile movement, axial load, and vertical soil movement.
- The settlement of the pile head, vertical pile movement, axial load, and vertical soil movement decrease with the depth.
- The extent of the plastic zone develops with the increase of the applied load.
- The vertical pile movement, and the axial load approach the maximum value at the ground level and get the minimum value at the pile tip.
- Under the applied loads, the diameter of the pile has a minor effect on the vertical pile movement. However, the pile-soil interaction is significantly dependent on the distance between the pile axis to the calculated position. The larger distance, the smaller the downward movement of the pile and its surrounding soil. Those movements are inversely proportional to depth.

5.2. Future Work

Due to the limitation of the time for the research. This study only investigated the pile-soil interaction under axial loading.

Prospective studies should focus on the pile-soil interaction in expansive soil under the lateral loading, and the combination of two types of loads.

In addition, this study did not study the effect of the group of piles on the pile-soil interaction in expansive soil. Further studies should determine this effect.

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APPENDICES

1. Appendice 1: Detailed comparison of models

As presented in the main sections, the Boundary Element Method and Finite Element method are significantly complex to apply. This part of the Appendice demonstrated in detail the comparison between the present model with other complicated models. The model of Jiang et al (2020) which was presented in Case study 1 is selected to compare with the present model. Details of the comparison are shown in Table 13.

Table 13. Detailed comparison of models

Solutions	The present model	Jiang et al (2020)
Vertical pile movement	$W_{p1}(z) = \frac{\beta}{6EA}(z^3 - z_p^3) + \frac{t_o}{2EA}(z^2 - z_p^2) - \frac{P}{EA}(z - z_p) + \frac{t_o+\beta z_P}{2\pi G_s} \ln\left(\frac{2r_m}{D_S}\right)$ Where β is the slope of the skin friction E is Young's modulus of the pile. A is the cross-section area of the pile Z_p is the extent of the plastic zone $Z_P = \left(\frac{t_o^2}{\beta} + \frac{1}{K_s/EA} + \frac{2P}{\beta}\right)^{1/2} - \frac{t_o}{\beta} - \left(\frac{EA}{K_s}\right)^{1/2}$ K_s is the soil stiffness. G_s is the shear modulus at depth z	$W_{p1}(z) = \sqrt{z}\left[A_2 I_{1/3}(x) + B_2 K_{1/3}(x)\right]$ Where x = 2/3 $\sqrt{K_s}\, z^{3/2}$ K_s is the soil stiffness. $I_{1/3}$ (x) and $K_{1/3}$ (x) are the 1/3th-order Bessel functions of the first and second kind, respectively. $I_{-2/3}$ (x) and $K_{-2/3}$ (x) are the -2/3th-order Bessel functions of the first and second kind, respectively. $A_2 = \left[A_1 C_3 + \frac{\sqrt{3}\Gamma\left(\frac{1}{3}\right)So}{3^{\frac{2}{3}}\sqrt{K_s}^{\frac{1}{3}}\pi h_o} Ks_{\frac{1}{3}}(x(ho))\right] \times \frac{Ks_{-\frac{2}{3}}(x(L))}{C_1}$

	P is the applied load t_o is the skin friction of the pile shaft at z = 0	$C_3 = I_{\frac{1}{3}}(x(ho) + \frac{\sqrt{3}}{\pi} Ks_{\frac{1}{3}}(x(ho)) +$ $I_{\frac{1}{3}}(x(ho))Ks_{-\frac{2}{3}}(x(L))$ Γ is the gamma function h_o is the effective depth S_o is the soil heave
Axial Load	$N_1(z) = \frac{-\beta}{2} z^2 - t_o z + P$ Where P is the applied load. Z is the calculation depth. t_o is the skin friction of the pile shaft at z = 0 β is the slope of the skin friction	$P_1(z) = -E_p A_p \left\{ \sqrt{Ks}\, z \left[A_1 I_{-\frac{2}{3}}(x) \right.\right.$ $\left.\left. - B_1 Ks_{-\frac{2}{3}}(x) \right] \right\}$ Where: $A_1 = \left\{ \frac{\sqrt{3}\Gamma\left(\frac{1}{3}\right)So}{3^{\frac{2}{3}}\sqrt{K_s}^{\frac{1}{3}}\pi h_o} \left[Ks_{\frac{1}{3}}(x(ho)C_2 - \right.\right.$ $\left.\left. Ks_{-\frac{2}{3}}(x(ho)C_1 \right] + \frac{So}{\sqrt{Ks}\, h_o^2} C_1 \right\} /$ $\left\{ -I_{-\frac{2}{3}}(x(ho)C_1 - I_{-\frac{1}{3}}(x(ho)C_2 - \right.$ $\left. \frac{\sqrt{3}}{\pi} \left[Ks_{\frac{1}{3}}\left(x(ho)\right)C_2 - Ks_{-\frac{2}{3}}(x(ho)C_1 \right] \right\}$ Where: $C_1 = K_{\frac{1}{3}}(x(ho)I_{-\frac{2}{3}}(x(L)) + I_{\frac{1}{3}}(x(ho)$

Vertical soil movement	Under applied load: $W_{s2} = \frac{W_p(z)}{\xi} \ln\left(\frac{r_m}{r}\right)$ Where $w_p(z)$ is the vertical pile movement r is the distance from the pile axis to the calculated postion $r_m = \frac{2.5\,L\,(1 - V_s)G_s(L/2)}{G_s(L)}$ $\xi = \ln(r_m / r_o)$ Under soil swelling pressure: $W_{s1} = s_o\left(\frac{z}{h_o} - 1\right) + \frac{s_o\left(1 - \frac{z}{h_o}\right) + w_P(z)}{\xi} \ln\left(\frac{r_m}{r}\right)$	Under applied load: $W_{s2} = \frac{\tau(z) r_o}{Gs} \ln\left(\frac{r_m}{r}\right)$ Where τ (z) is the pile's skin friction. $\tau_{2z} = \frac{G_s}{r_{o\xi}} W_P(z)$ $r_m = \frac{2.5\,L\,(1 - V_s)G_s(L/2)}{G_s(L)}$ G_s is the shear modulus at depth z r_o is the pile diameter r is the distance from the pile axis to the calculated postion Under soil swelling pressure: $W_{s2} = -S(z)\frac{\tau(z) r_o}{Gs} \ln\left(\frac{r_m}{r}\right)$ Where S(z) is the soil heave.
Remarks	All parameters and formulas used in the present model are not difficult to apply. The model's ability to predict the pile-soil interaction in expansive soils is less accurate than the model of Jiang et al (2020) and other complicated models in the	The model of Jiang et al (2020) provided a highly accurate prediction of the pile-soil interaction in expansive soils. However, most of the formulas are significantly complex to apply. As presented above, the Bessel functions with non-integer orders are

	Boundary Element Method, and Finite Element Method. However, the difference can be accepted. In reality, the easily applicable characteristic of the present model would be an advantage. In the case of little information about the input parameters, the model will be an optical choice to predict the pile-soil interaction.	considerably intricate. The solutions for those functions need support from the program, MATLAB, Python, etc. In the paper, Jiang used support from the Python program to solve those equations. Other studies regarding the Boundary Element Method and the Finite Element Method offered good predictions for the pile-soil interaction. However, the theories are considerably intricate.

2. Appendice 2: RSPile models

❖ **Case study 1:** RSPile was used to simulate the test conducted by Jiang et al (2020).

There are 4 cases of the applied load, P =1.1 kN, P = 1.7 kN, P = 2.3 kN, and P = 2.9 kN. Those loads were simulated to RSPile software. The models and results of each case are presented below.

- **The applied load: P = 1.1 kN**

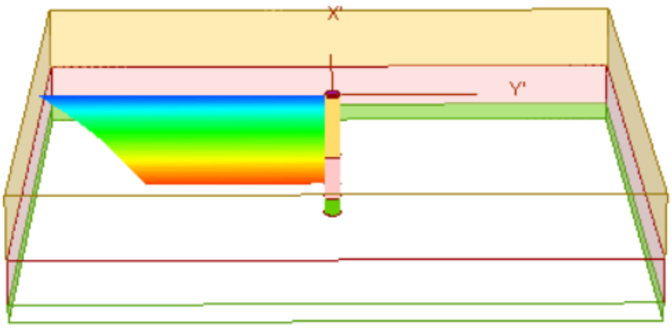

Figure 22. RSplie model-Case study 1 (P =1.1 kN)

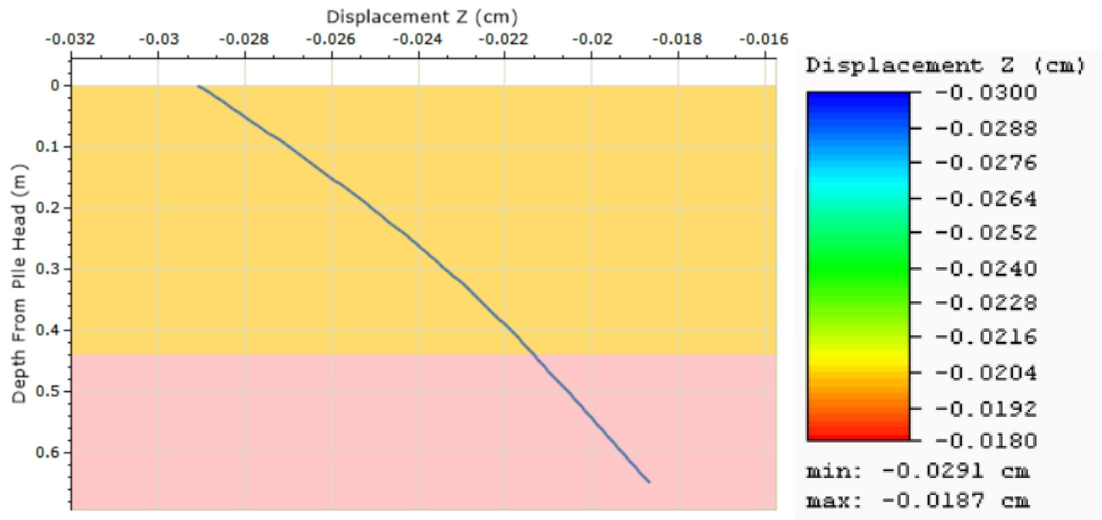

Figure 23. RSplie model-Case study 1 (P = 1.1 kN)-Displacement

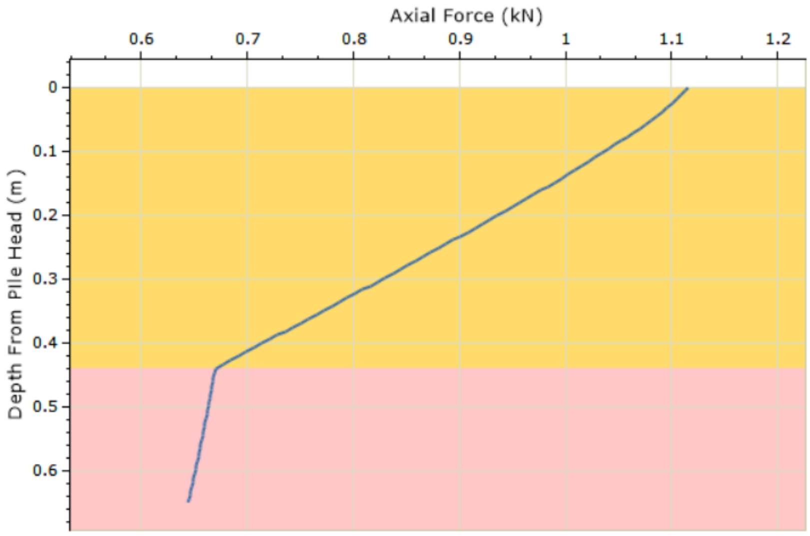

Figure 24. RSplie model-Case study 1 (P = 1.1 kN)-Axial load

- **The applied load P = 1.7 kN**

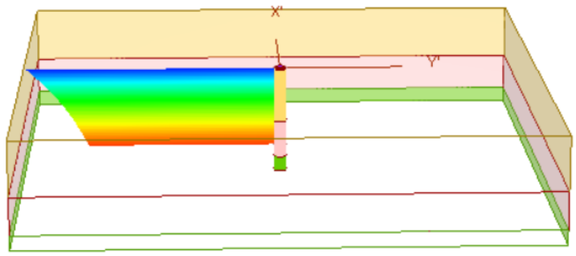

Figure 25. RSplie model-Case study 1-P =1.7 kN

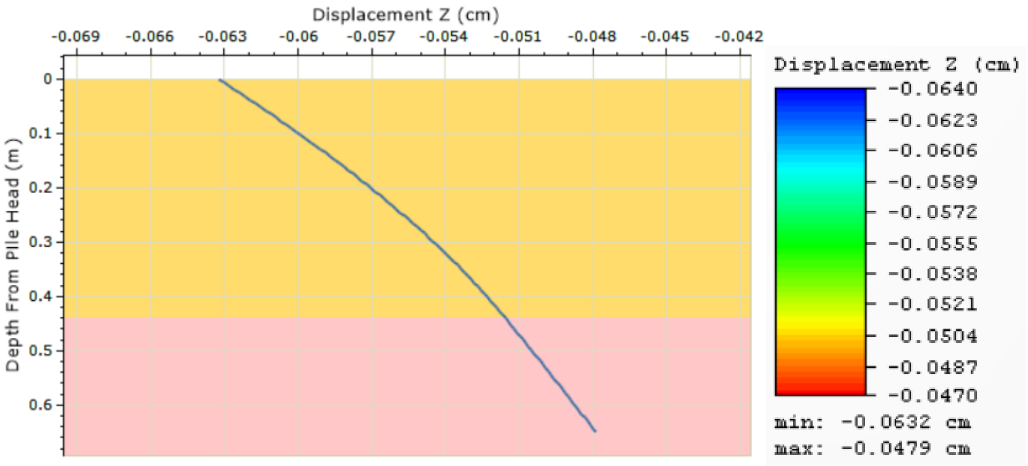

Figure 26. RSplie model-Case study 1 (P = 1.7 kN)-Displacement

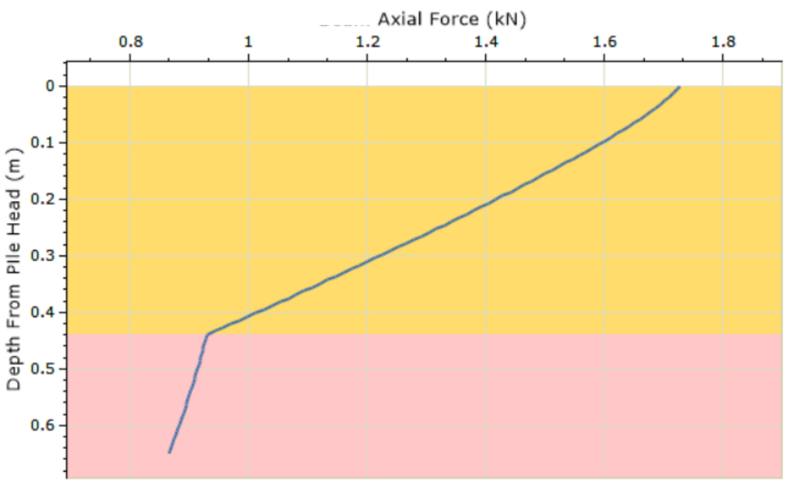

Figure 27. RSplie model-Case study 1 (P = 1.7 kN)- Axial load

- **The applied load: P = 2.3 kN**

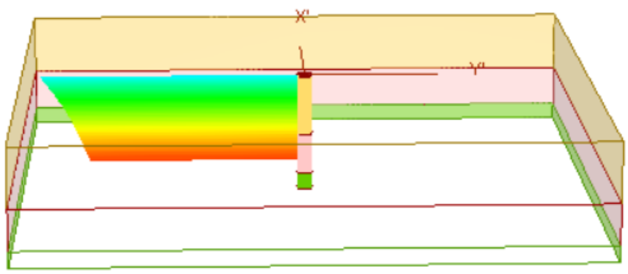

Figure 28. RSplie model-Case study 1 (P =2.3 kN)

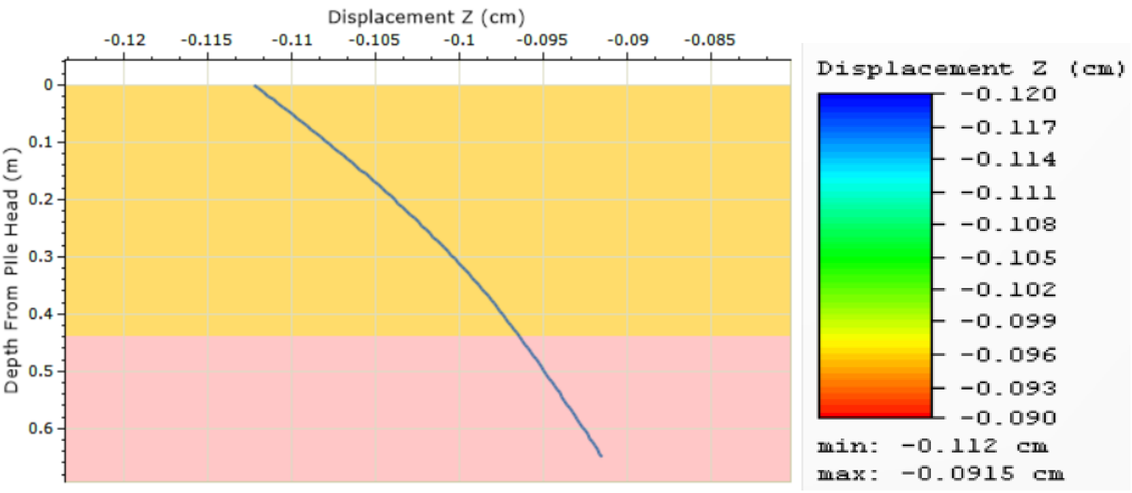

Figure 29. RSplie model-Case study 1 (P = 2.3 kN)-Displacement

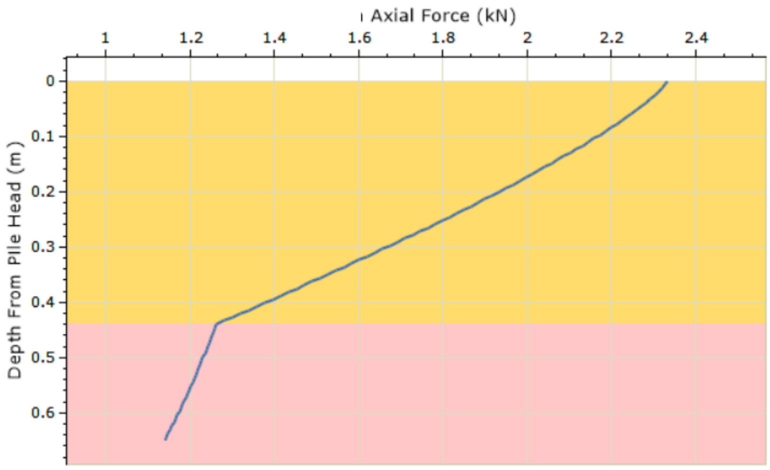

Figure 30. RSplie model-Case study 1 (P = 2.3 kN)-Axial Force

- **The applied load: P = 2.9 kN**

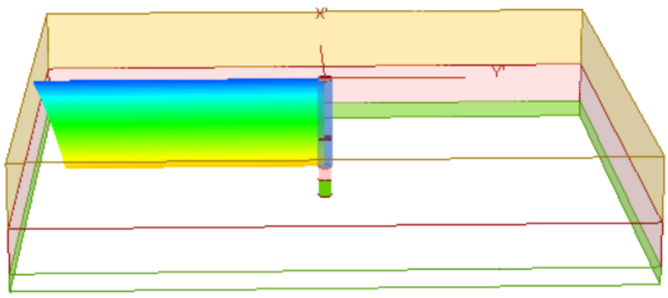

Figure 31. RSplie model-Case study 1 (P = 2.9 kN)

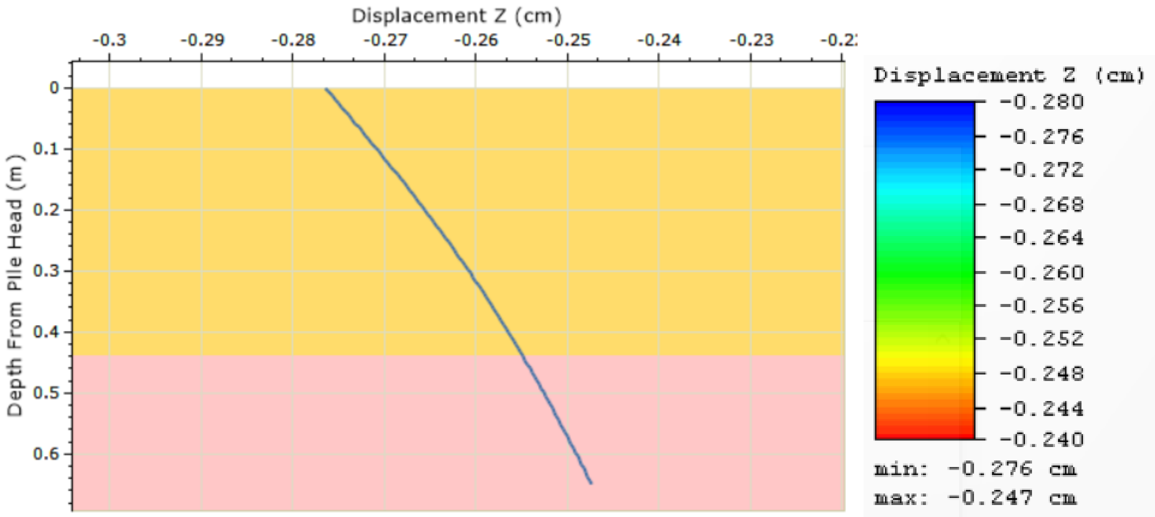

Figure 32. RSplie model-Case study 1 (P = 2.9 kN)-Displacement

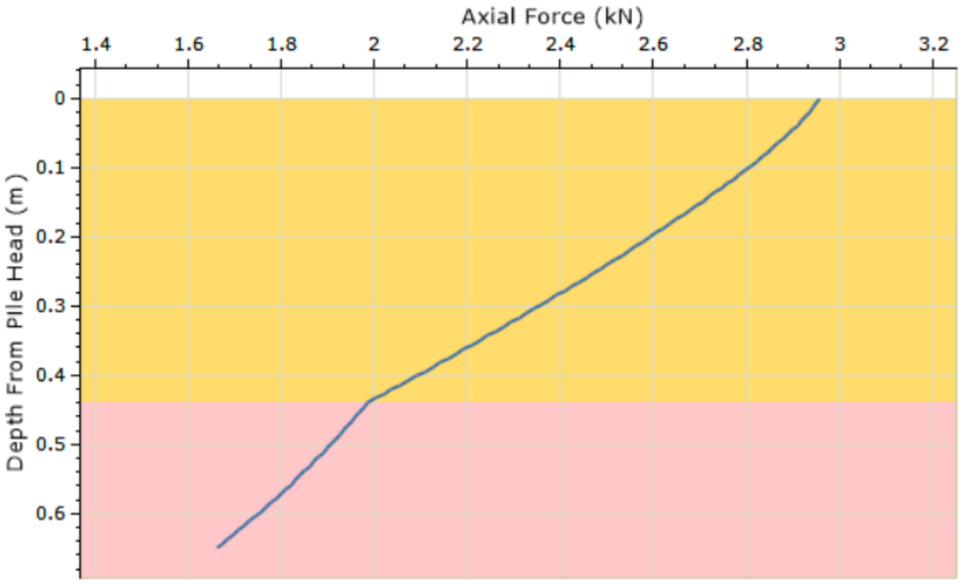

Figure 33. RSplie model-Case study 1 (P = 2.9 kN)-Axial Force

❖ **Case study 2:** RSPile was used to simulate the test conducted by Poulos (1989).

There are 3 cases of the applied load, P =1000 kN, P = 2000 kN, P = 2500 kN. Those loads were simulated to RSPile software. The models and results of each case are demonstrated below.

- **The applied load: P = 1000 kN**

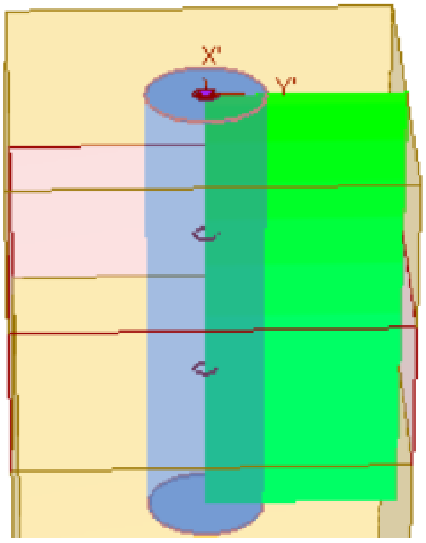

Figure 34. RSplie model-Case study 2 (P = 1000 kN)

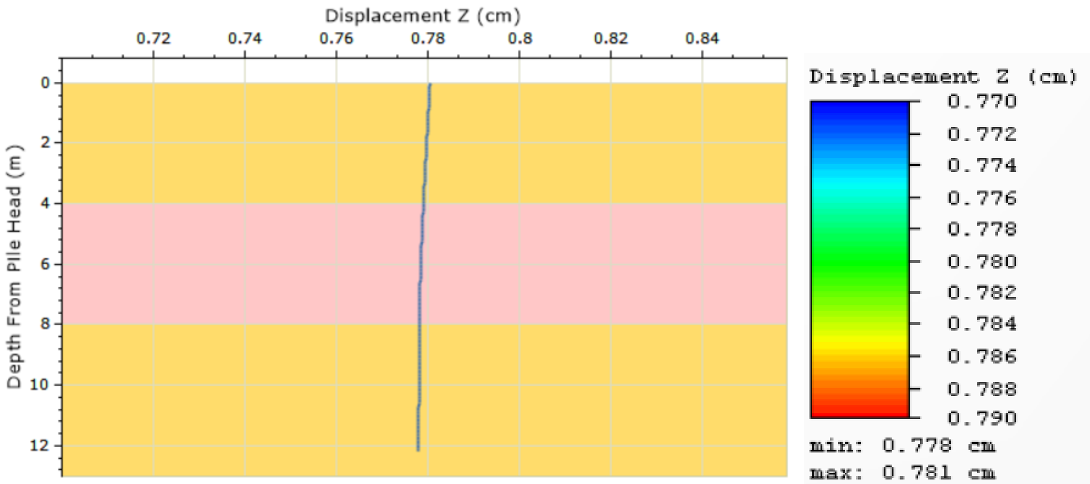

Figure 35. RSplie model-Case study 2 (P = 1000 kN)-Displacement

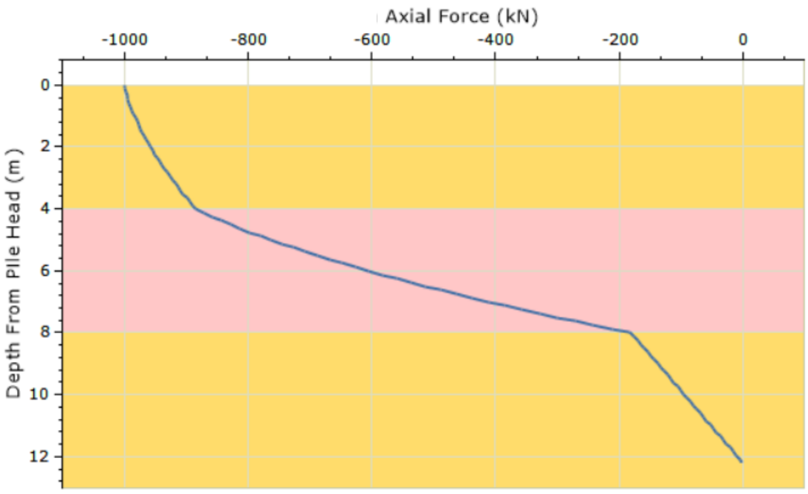

Figure 36. RSplie model-Case study 2 (P = 1000 kN)-Axial Force

- **The applied load: P = 2000 kN**

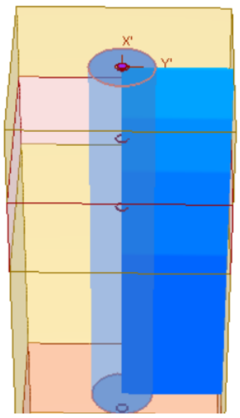

Figure 37. RSplie model-Case study 2 (P = 2000 kN)

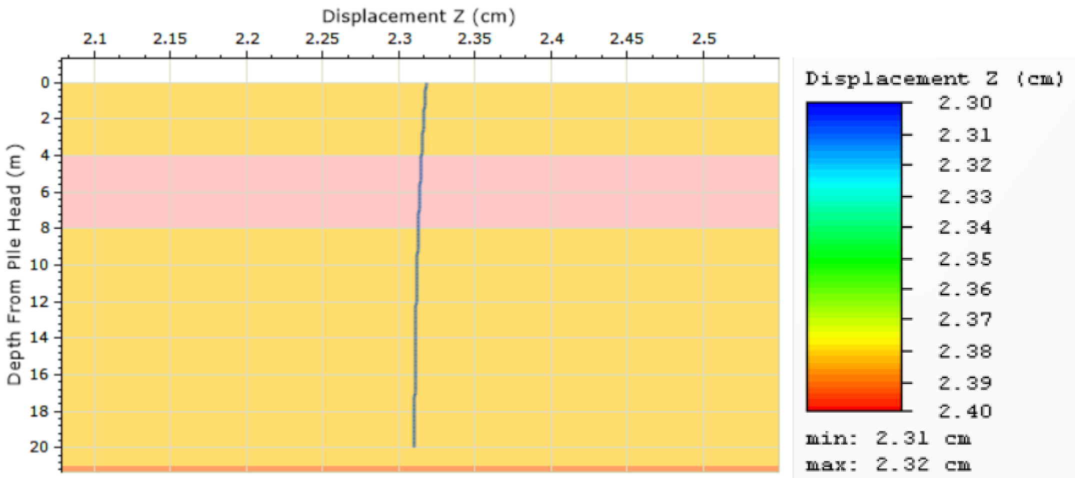

Figure 38. RSplie model-Case study 2 (P = 2000 kN)-Displacement

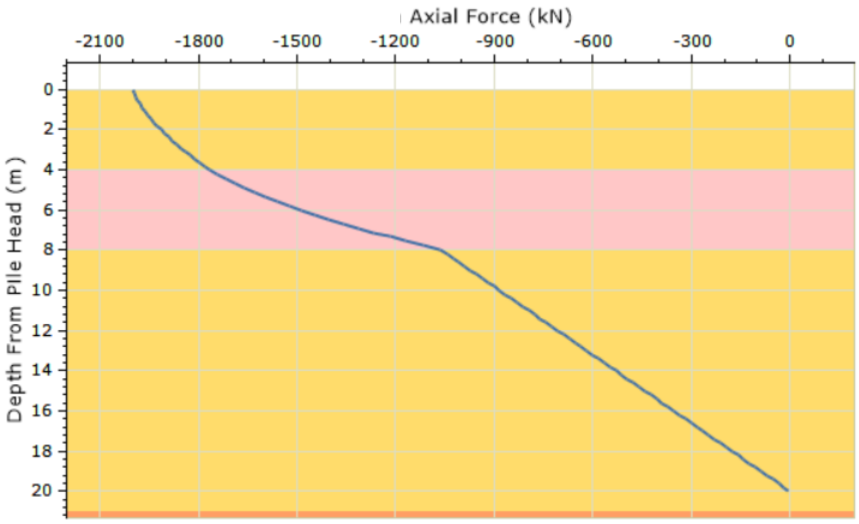

Figure 39. RSplie model-Case study 2 (P = 2000 kN)-Axial Force

- **The applied load: P = 2500 kN**

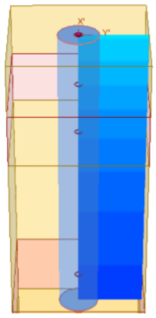

Figure 40. RSplie model-Case study 2 (P = 2500 kN)

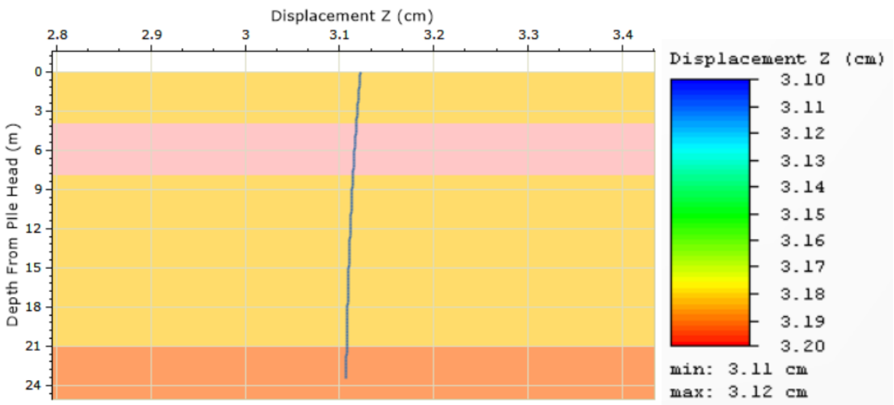

Figure 41. RSplie model-Case study 2 (P = 2500 kN)-Displacement

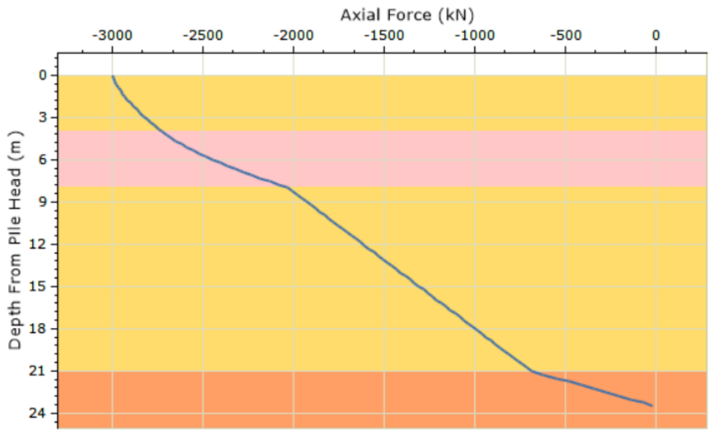

Figure 42. RSplie model-Case study 2 (P = 2500 kN)-Axial Force

❖ **Case study 3:** RSPile was used to simulate the test conducted by Poulos (1989).

This case study investigated the effect of diameter on pile-soil interaction. Diameters were changed from d = 0.5 m to d = 2m. The models and results of each case are illustrated below.

- **Diameter: d = 0.5 m**

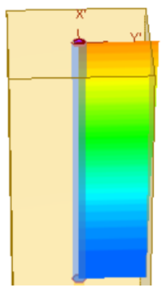

Figure 43. RSplie model-Case study 3 (d = 0.5 m)

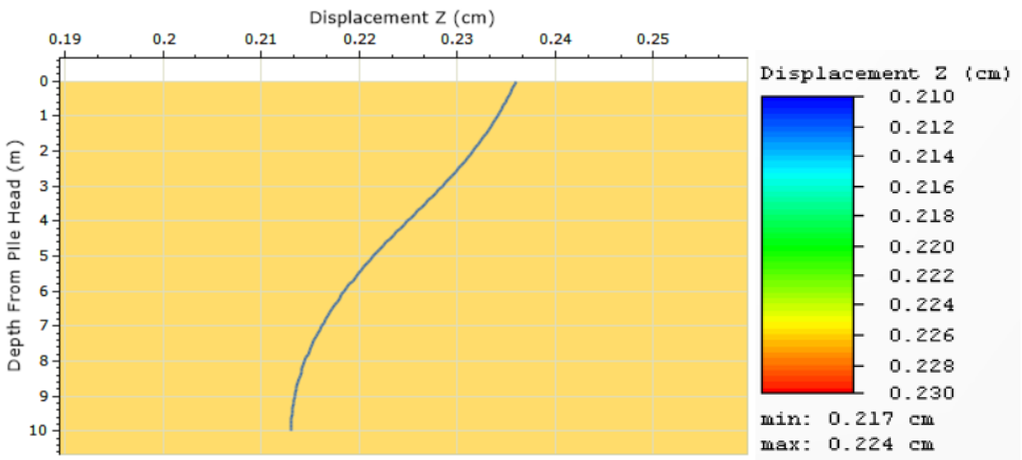

Figure 44. RSplie model-Case study 3 (d = 0.5 m)-Displacement

- **Diameter: d = 1 m**

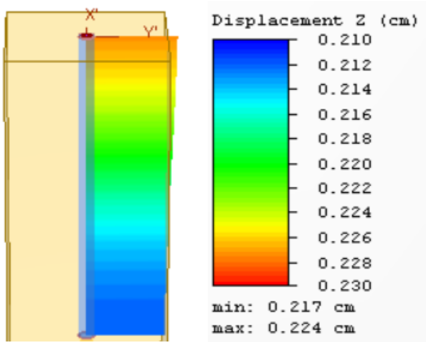

Figure 45. RSplie model-Case study 3 (d = 1 m)

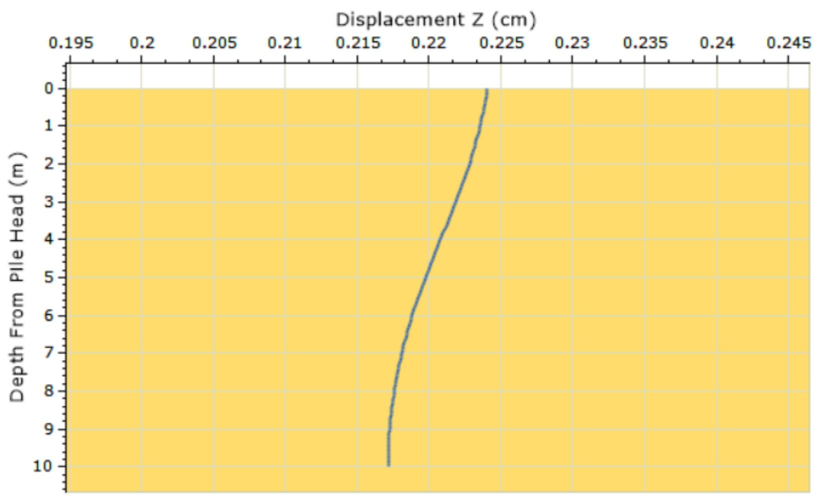

Figure 46. RSplie model-Case study 3 (d = 1 m)-Displacement

- **Diameter: d = 1.5 m**

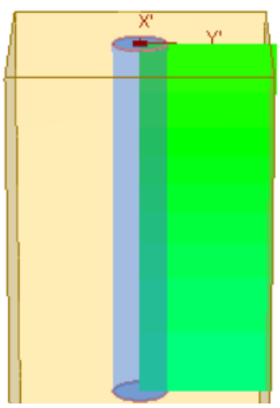

Figure 47. RSplie model-Case study 3 (d =1.5 m)

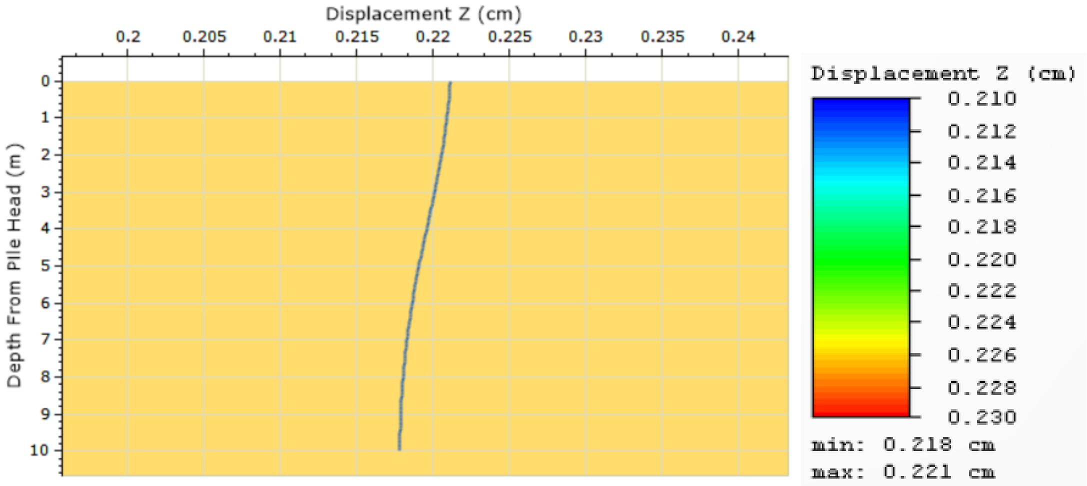

Figure 48. RSplie model-Case study 3 (d =1.5 m)-Displacement

- **Diameter: d = 2 m**

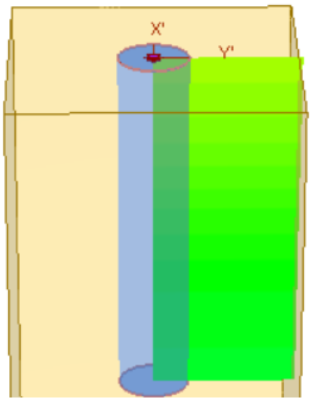

Figure 49. RSplie model-Case study 3 (d = 2 m)

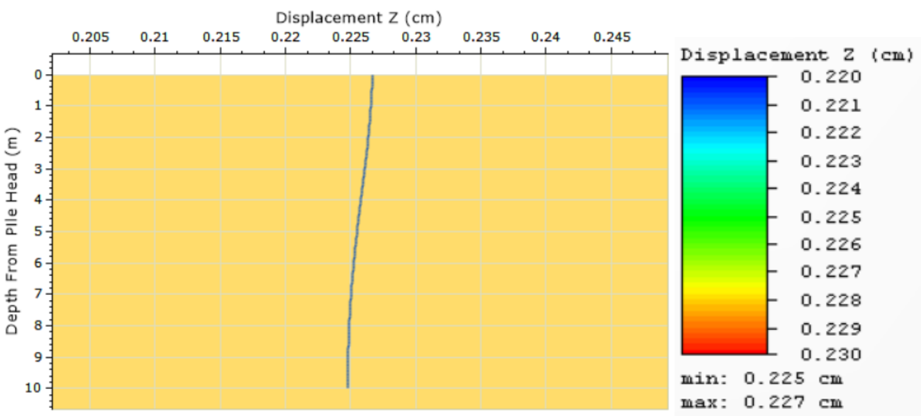

Figure 50. RSplie model-Case study 3 (d =2 m)-Displacement

❖ **Case study 4:** RSPile was used to simulate the test conducted by Kuwabara (1991).

This case study investigated the effect of the distance from the pile axis to the calculated position (r) via the ratio of r and r_o (r_o is the pile's radius). There are 6 cases of r/r_o were implemented. The models and results of each case are presented below.

- **Ratio: r/r_o = 1**

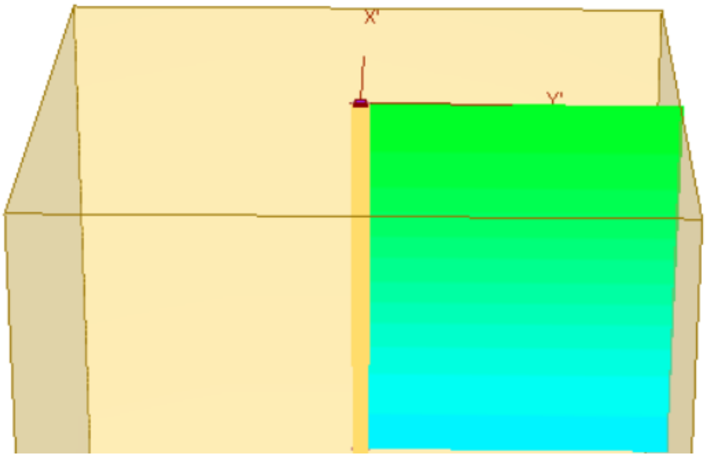

Figure 51. RSplie model-Case study 4 (r/r_o = 1)

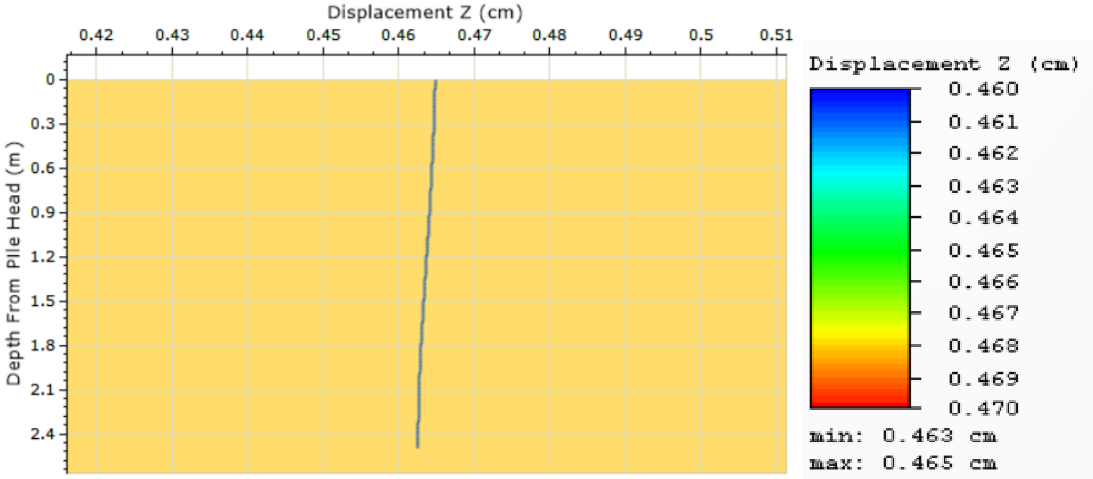

Figure 52. RSplie model-Case study 4 (r/r_o = 1)-Displacement

- **Ratio: $r/r_o = 2$**

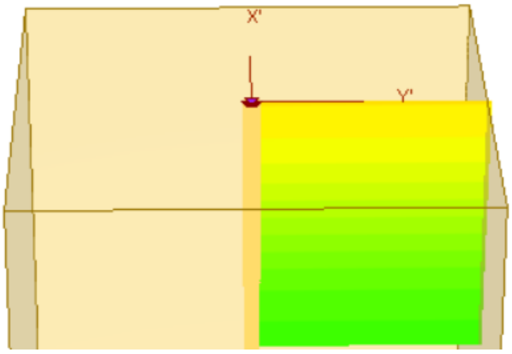

Figure 53. RSplie model-Case study 4 (r/r_o = 2)

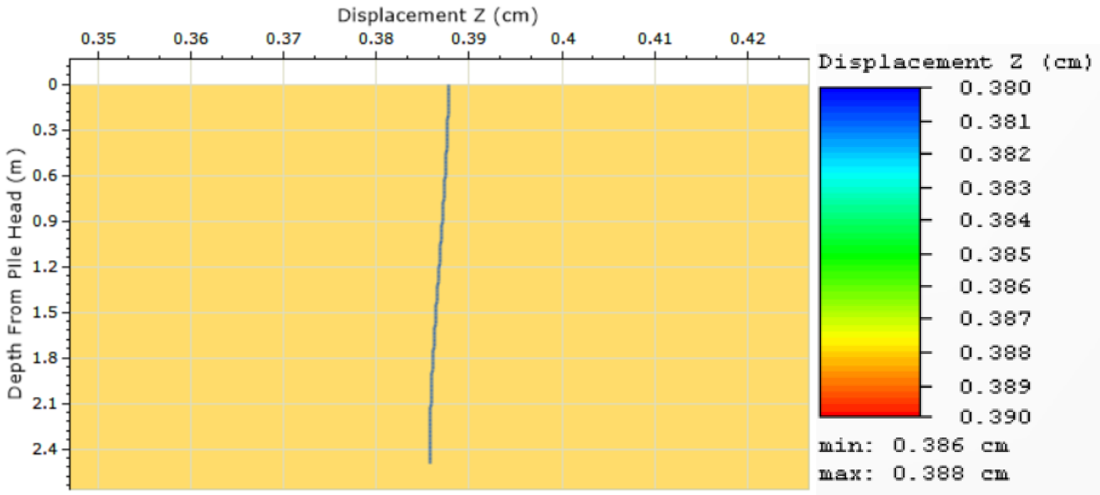

Figure 54. RSplie model-Case study 4 (r/r_o = 2)-Displacement

- **Ratio: $r/r_o = 5$**

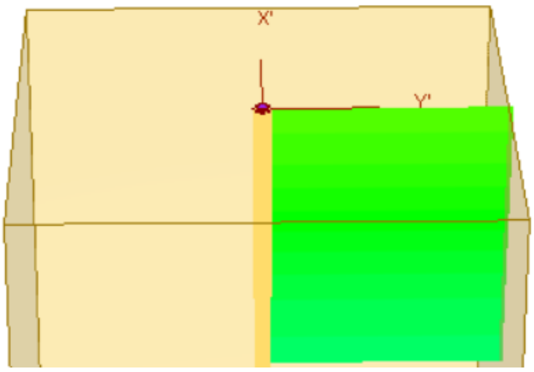

Figure 55. RSplie model-Case study 4 (r/r_o = 5)

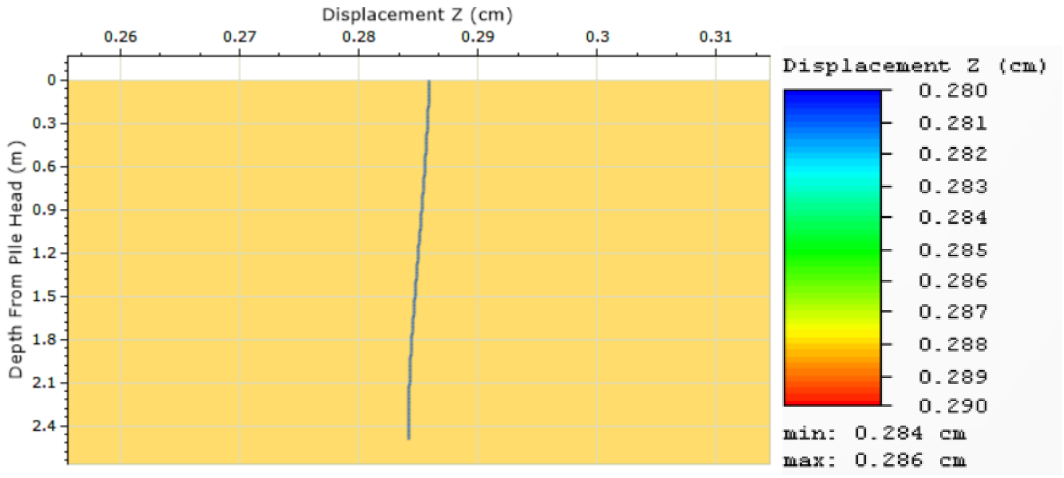

Figure 56. RSplie model-Case study 4 (r/r_o = 5)-Displacement

- **Ratio: r/r_o = 10**

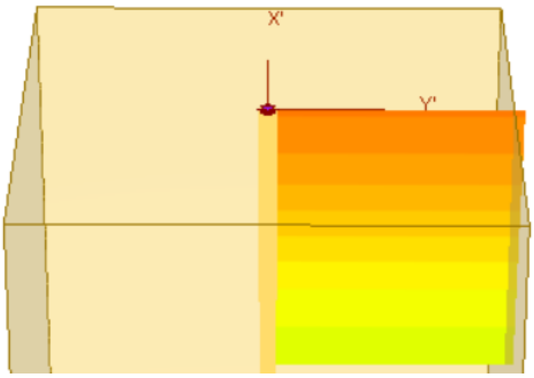

Figure 57. RSplie model-Case study 4 (r/r_o = 10)

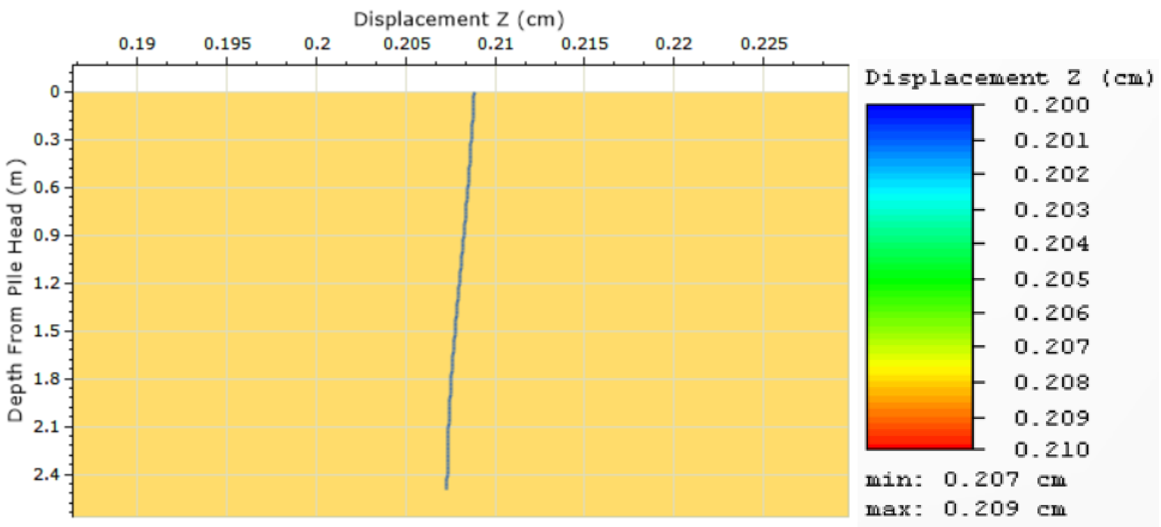

Figure 58. RSplie model-Case study 4 (r/r_o = 10)-Displacement

- **Ratio: $r/r_o = 20$**

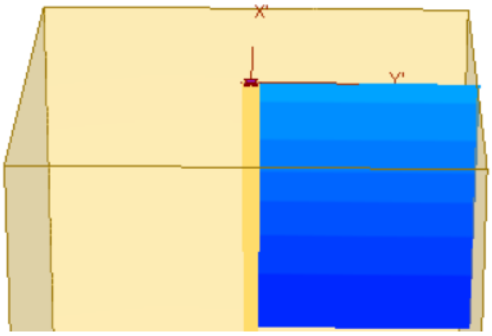

Figure 59. RSplie model-Case study 4 (r/r_o = 20)

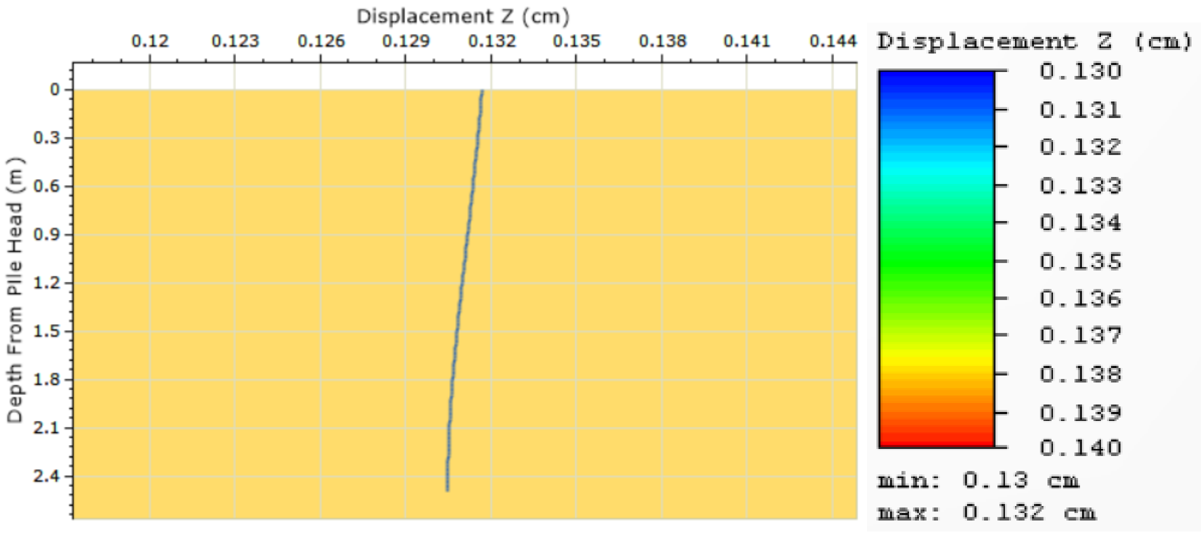

Figure 60. RSplie model-Case study 4 (r/r_o = 20)-Displacement

- **Ratio: $r/r_o = 50$**

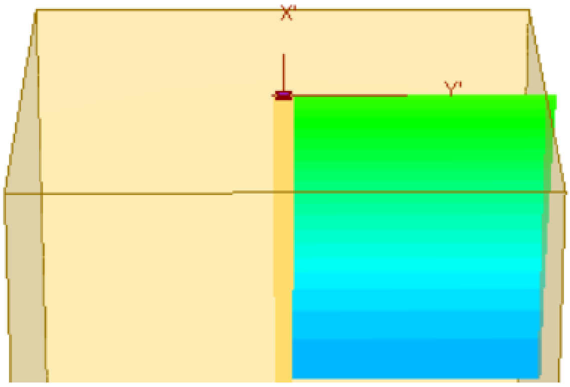

Figure 61. RSplie model-Case study 4 (r/r_o = 50)

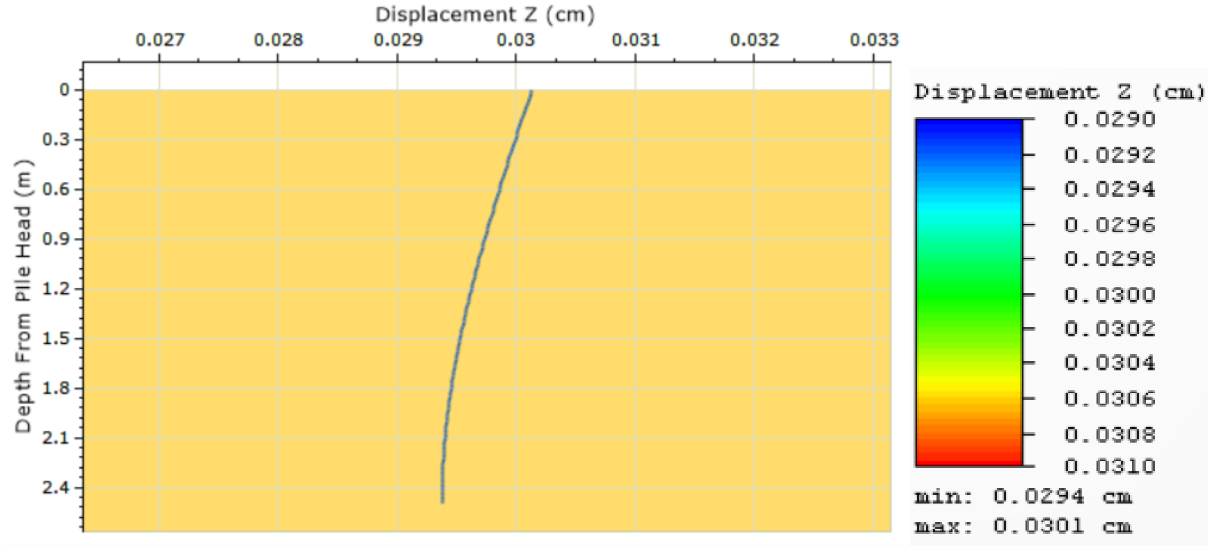

Figure 62. RSplie model-Case study 4 (r/r_o = 50)-Displacement