Abstract

The incompressible Bingham fluid can be found in various chemical, metal, and food industries, e.g., margarine, mayonnaise and ketchup. The flow field of a Bingham fluid is divided into two regions: the first is an unyielded zone where the fluid is at rest or undergoes a rigid motion, and the second where the fluid flows like a viscous liquid. In the unyielded zone, the second invariant of the extra stress tensor is less than or equal to the yield stress and a constitutive relation does not exist. In the yielded region, this invariant exceeds the yield stress and a constitutive relation exists for the extra stress tensor. Thus, the location and shape of the yield surface(s), i.e. the interface between these two sets, is also a part of the solution of flow problems of such fluids. The hydrodynamic and thermal effects are two interesting aspects arising in the research of the flows of incompressible Bingham fluids. This thesis is concerned with macroscopic and mesoscopic numerical investigations of isothermal (Lid-driven cavity, Steady flow in a pipe) and non-isothermal (Natural convection in an enclosure and mixed convection in a lid-driven cavity) problems of Bingham fluids. Firstly, a general mesoscopic method based on Lattice Boltzmann Method (LBM) has been derived which has the advantage to be applied to all fluids, whether they be Newtonian, or power law fluids, or viscoelastic or viscoplastic fluids. In fact, an innovative model for the distribution functions, which leads to the conservation of mass, momentum, and energy equations, and applicable to incompressible fluids without any drawbacks, is introduced. The Finite Difference Lattice Boltzmann Method (FDLBM) and the Thermal Difference Discrete Flux Method (TDDFM) are derived, using vector analysis and linear algebra. The applied algorithm for solving the main equations of the discrete particle distribution function in the FDLBM and the internal energy distribution function in the TDDFM is explained. The required equations for compressible fluids are also derived and thereafter the method is extended to the flows of every type of fluid, compressible or incompressible, in three dimensions. In addition, the Courant-Friedrichs-Lewy (CFL) condition for the method is derived. The mesoscopic and macroscopic numerical methods for the cited isothermal and non-isothermal problems have been described in detail, using the Bingham model and also the regularisation based on the Papanastasiou model. The lid-driven cavity flow and the steady flow in a pipe have been studied by other researchers before, employing ALM (Augmented Lagrangian Method) and OSM (Operator Splitting Method) respectively and the obtained results by the mesoscopic method in the problems are compared with them. In the case of the natural convection in a cavity, the problem had not been considered at the start of the thesis by other researchers and therefore this problem has been solved by applying the OSM to the Bingham model has been utilised in the constitutive equations. Thereafter, the mesoscopic method has been applied to the non-isothermal problem to simulate the natural convection in a cavity and mixed convection in a lid-driven cavity, employing the Bingham and regularised Papanastasiou models.